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The value of reliability: an equilibrium approach

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Abstract

This paper extends the theory on the valuation of travel time reliability, currently limited to the case with a single traveler, by considering interactions between individuals. To be more specific, we examine how the Nash equilibrium in departure times is modified when stochastic travel times are introduced in the standard bottleneck model with *N* morning commuters. For individuals with (α,β,γ) preferences, we find that the primary impact of travel time variability, modeled here under the form of a uniform random delay, is peak spreading. Departures are spread more evenly around the peak than when travel times are perfectly reliable, which reduces congestion. This mitigating phenomenon leads to a lower value of reliability (VoR) than the one given by non-equilibrium approaches. When the maximum random delay is small enough compared to the total length of the rush hour, the VoR is even null. These results entail two caveats as far as cost-benefit analysis is concerned: first, VoR based on non-equilibrium approaches overestimate the cost of unreliability as they fail to capture equilibrium mechanisms. Conversely, VoR based on (α,β,γ) preferences likely underestimate this same cost.

Keywords: Value of reliability; Nash equilibrium; Bottleneck model

1. Introduction

Travel time variability is an important element in people's travel decisions. Bates et al. [1] recapitulate the two main explanations that one may find in a now relatively large body of literature (*e.g.* [2], [3]). First, unreliable travel times are detrimental to scheduling or even doing one's activities, as it may cause one to be early or late at his destination (missing a flight because of a delay in public transit well exemplifies this point). Secondly, people could also be averse to travel time variability *per se*, as it would generate some form of stress or anxiety, or because it would cause some additional cognitive burden to the planning of activities.

The question of evaluating the cost associated to travel time unreliability is on the other hand relatively recent. In their review on this issue, Bates et al. [1] underlined the lack of research as well as the absence of the issue of reliability in most evaluation methodologies of road and public transport schemes at the time. The situation has progressed since, in particular thanks to additional theoretical works on the topic. In line with the previous but less general work of Noland and Small

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[4], Fosgerau and Engelson [5] and Fosgerau and Karlström [6] have studied the departure time choice in the case of a single traveler facing variable travel times. A different utility function is used in each paper, including the standard (α,β,γ) preferences proposed by Small [7]. In both papers, the optimal time of departure is computed and the resulting value of reliability (VoR) is derived. These papers have been quite influential in designing evaluation methodologies that would take the issue of travel time reliability into account, as they offered a theoretical foundation as well as a relatively simple formula for the VoR. The case with multiple travelers has yet to be properly considered, however. Siu and Lo [8] represents a first endeavor in this direction, but the authors fail to properly derive the value of reliability.

This paper attempts to address this lack and provide a theoretical framework to compute the value of reliability in the case with multiple travelers. To be more specific, we examine how the Nash equilibrium in departure times is modified when a random delay is introduced in the standard bottleneck model with N morning commuters. The delay follows a uniform law and individuals have (α,β,γ) preferences. Given these assumptions, we find that the primary impact of travel time variability is peak spreading. Departures are spread more evenly around the peak than when travel times are perfectly reliable, which reduces congestion. This mitigating phenomenon leads to a lower VoR than the one given by non-equilibrium approaches. When the maximum random delay is small enough compared to the total length of the rush hour, the VoR is even null. These results entail two caveats as far as cost-benefit analysis is concerned: first, VoR based on non-equilibrium approaches overestimate the cost of unreliability as they fail to capture equilibrium mechanisms. Conversely, VoR based on (α,β,γ) preferences likely underestimate this same cost.

The paper is structured as follows. The theoretical model is exposed in Section 2. Section 3 summarizes the main findings when travel times are certain. We then compute in Section 4 the equilibrium solution when a uniform random delay is introduced, which includes the derivation of the equilibrium trip cost, and thus the VoR. Section 5 uses the results of Section 4 to analyze the impact of the unreliability level on the equilibrium features (departure rate, congestion and equilibrium cost). Finally, Section 6 offers some concluding remarks and highlights some directions for future research.

2. The model

In the bottleneck model of road congestion with peak-load demand, a fixed number N of individuals commute every day from point A (home) to point B (work). Each one travels with his own car, along the single road that joins A and B. The road starts with a bottleneck with fixed capacity (or service rate) s. Accordingly, whenever the departure rate $r(t)$ gets greater than s, a queue develops at the bottleneck entrance (if it does not yet exist).

In the standard setting, the travel time is composed of two elements: a fixed component, representing the time necessary to go from *A* (or more accurately from the bottleneck exit) to *B*, and a variable one, representing the time spent in the queue (if any). Both terms are deterministic. We add a stochastic component so as to introduce day-to-day variability in travel times. This term takes the form of a random delay, noted ε_t, which may vary over the course of the day. Accordingly, the travel time $\tilde{T}$ takes the following form:

$$\tilde{T}(t) = T_0 + Q(t)/s + \varepsilon_t \tag{1}$$

a ~ being used to underline the stochastic nature of a variable. T_0 is the free flow travel time, while $Q(t)/s$ is the congestion delay, equal to the ratio between the queue length $Q(t)$ and the service rate s. $Q(t)$ is linked to the departure rate and the service rate through the following relationship:

$$\begin{cases} Q'(t) = \left[\dfrac{r(t)-s}{s}\right]^+ & \text{if } Q(t) = 0 \\ Q'(t) = \dfrac{r(t)-s}{s} & \text{if } Q(t) > 0 \end{cases} \tag{2}$$

Strictly speaking, ε_t encompasses any phenomenon leading to travel time variability other than those related to fluctuations in the level of travel demand (N) or in the actual capacity of the bottleneck (s). This includes weather conditions and variations in the driving speed after exiting the bottleneck among other things.[1] The role of the provision of dynamic information regarding ε_t is not considered in this paper, however. This *de facto* excludes dynamic strategies based on such information (like choosing to leave when ε_t appears to be low, for instance). The time dynamics of ε_t has therefore no importance here, and any behavior can be assumed as long as the basic assumptions mentioned further still hold.[2]

Given these elements, the question is how individuals choose their departure time. As is common in this strand of literature (e.g. [9], [10]), we assume that individuals have (α,β,γ) preferences. Under this formulation, the generalized cost of travel is linear in travel time and piecewise linear in schedule delay with respect to the preferred arrival time:

$$\tilde{C}(t) = \alpha\tilde{T}(t) + \beta\left[t^* - t - \tilde{T}(t)\right]^+ + \gamma\left[t + \tilde{T}(t) - t^*\right]^+ \tag{3}$$

[1] This formulation may also be considered as a simplified way of representing all kinds of hazard, including accidents, fluctuations in the level of demand and so on. The extent to which not explicitly representing variations in N or s alters results will be the object of future work.

[2] If one wants to maintain some basic flow properties (such as First In First Out), some constraints must be imposed on the time dynamics of ε_t. In particular, if time independence is assumed, overtakings may occur, in contradiction with the FIFO principle.

where $\tilde{C}(t)$ is the trip cost when leaving at time t and $t^* - t - \tilde{T}(t)$ is the so called schedule delay (early if positive, late if negative), measured relatively to a fixed target arrival time t^*. As is also usual in the literature, we further assume $\beta \le \alpha$.

Each individual seeks to minimize his trip cost by adjusting his departure time. A Nash equilibrium is reached when no individual has an incentive to change his departure time. The equilibrium is fully characterized by the departure rate function $r(t)$, with the condition that the expected trip cost $\mathbf{E}\left[\tilde{C}(t)\right]$ must be minimum and constant on the set $\{t \,/\, r(t)>0\}$.

3. The certain case

We start by recalling the main findings when travel times are perfectly reliable, in other words when $\varepsilon_t=0 \;\; \forall t\in\Re$.[3] First, one can show the existence and uniqueness of a pure Nash equilibrium regarding the choice of departure time. The main characteristics of this equilibrium are as follows:

- Individuals leave home in a continuous (but not homogenous) way between t_s and t_e.
- There is congestion throughout the whole rush hour except at the very beginning and at the very end, and the bottleneck always operates at full capacity. Consequently, the length of the rush hour is: $t_s - t_e = N/s$.
- The departures rate varies so as to equalize the trip cost throughout the rush hour. This entails the following dynamics regarding travel time:

$$\begin{cases} T'(t) = \dfrac{\beta}{\alpha - \beta} & \text{for } t \in \left[t_s, t_p\right] \\ T'(t) = -\dfrac{\gamma}{\alpha + \gamma} & \text{for } t \in \left[t_p, t_e\right] \end{cases} \qquad (4)$$

Outside the rush hour $[t_s, t_e]$, despite enjoying no congestion the cost of travel is greater as the schedule delay is too high. During the first part of the rush hour, *i.e.* the interval $[t_s, t_p]$, the departure rate is greater than s. Queue length increases and is maximal at the peak t_p. Waiting time is also greatest at t_p, but conversely the schedule delay is minimal (it is null actually). People leaving before the peak are thereby early while those leaving after the peak are late.

One can push calculations further and establish the following relationships:

$$\begin{cases} t_s = t^* - T_0 - \dfrac{\gamma}{\beta+\gamma}\dfrac{N}{s} \\ t_p = t_s + \dfrac{\alpha-\beta}{\alpha}\dfrac{\gamma}{\beta+\gamma}\dfrac{N}{s} \\ t_e = t^* - T_0 + \dfrac{\beta}{\beta+\gamma}\dfrac{N}{s} \end{cases} \quad \text{and} \quad \begin{cases} T(t_p) = T_{\max} = T_0 + \dfrac{1}{\alpha}\dfrac{\beta\gamma}{\beta+\gamma}\dfrac{N}{s} \\ C_{eq} = \alpha T_{\max} = \alpha T_0 + \dfrac{\beta\gamma}{\beta+\gamma}\dfrac{N}{s} \end{cases} \qquad (5)$$

[3] This case was extensively studied in the literature (see [9], [10], among others).

4. Equilibrium solution with a uniform random delay

We now turn to the case of stochastic travel times. As previously pointed out by Bates et al. [1], the most general case where the set of distributions of $(\varepsilon_t)_{t\in\Re}$ has no specific properties does not yield any significant result. To carry the analysis further, we make the following simplifying assumptions:

- $(\varepsilon_t)_{t\in\Re}$ follow a uniform law;
- they all have the same mean T_ε and the same standard deviation σ_ε.

These assumptions, in particular that of a uniform law and that of a constant standard deviation, are both strong and little plausible. On the other hand, they lead to closed-form solutions unlike most other distributions and make the model easier to follow, hence this choice. Relaxing these assumptions will be the object of future work.[4]

The distribution function of the normalized random variable $\hat{\varepsilon}_t = (\varepsilon_t - T_\varepsilon)/\sigma_\varepsilon$ is noted *f*, and its cumulative distribution function *F*. $\hat{\varepsilon}_t$ being a uniform random variable with zero mean and unit standard deviation, we have:

$$\begin{cases} f(x) = \dfrac{\mathbf{1}_{[-\sqrt{3};\sqrt{3}]}(x)}{2\sqrt{3}} \\ F(x) = \left(\dfrac{1}{2} + \dfrac{x}{2\sqrt{3}}\right)\mathbf{1}_{[-\sqrt{3};\sqrt{3}]}(x) + \mathbf{1}_{]\sqrt{3};+\infty[}(x) \end{cases} \quad \text{where } \mathbf{1}_A(x) \text{ is the characteristic function of } A.$$

4.1. Expected trip cost

In a context of random travel times, each individual seeks to minimize his expected trip cost. For a departure at time *t*, the expected cost is:

$$\mathbf{E}\left[\tilde{C}(t)\right] = \alpha\bar{T}(t) + \beta\int_{-\infty}^{\frac{t^*-t-\bar{T}(t)}{\sigma_\varepsilon}} \left[t^* - t - \bar{T}(t) - \sigma_\varepsilon u\right] f(u)du + \lambda\int_{\frac{t^*-t-\bar{T}(t)}{\sigma_\varepsilon}}^{+\infty} \left[t + \bar{T}(t) + \sigma_\varepsilon u - t^*\right] f(u)du \quad (6)$$

where $\bar{T}(t) = T_0 + Q(t)/s + T_\varepsilon$ is the expected travel time. Using $\int_{-\infty}^{+\infty} uf(u)du = 0$, (6) can be rewritten as:

$$\mathbf{E}\left[\tilde{C}(t)\right] = \alpha\bar{T}(t) + (\beta+\gamma)\sigma_\varepsilon\left[m(t)F(m(t)) + G(m(t))\right] - \gamma\sigma_\varepsilon m(t) \quad (7)$$

where $G(x) = \int_x^{+\infty} uf(u)du$ is a positive function and $m(t) = \left[t^* - t - \bar{T}(t)\right]/\sigma_\varepsilon$ is often referred to as the normalized safety margin (e.g. [6]).

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[4] Preliminary findings show that the results presented here are quite robust.

Calculation yields $G(x)=\frac{3-x^2}{4\sqrt{3}}\mathbf{1}_{\left[-\sqrt{3};\sqrt{3}\right]}(x)$, hence the following relationships:

$$\begin{cases}\mathbf{E}\left[\tilde{C}(t)\right]=\alpha\bar{T}(t)+\beta\left[t^*-t-\bar{T}(t)\right] & \text{if } m(t)\geq\sqrt{3}\\ \mathbf{E}\left[\tilde{C}(t)\right]=\alpha\bar{T}(t)+\frac{\beta-\gamma}{2}\left[t^*-t-\bar{T}(t)\right]+\frac{\beta+\gamma}{4}\frac{3\sigma_\varepsilon{}^2+\left[t^*-t-\bar{T}(t)\right]^2}{\sqrt{3}\sigma_\varepsilon} & \text{if } -\sqrt{3}\leq m(t)\leq\sqrt{3}\\ \mathbf{E}\left[\tilde{C}(t)\right]=\alpha\bar{T}(t)+\gamma\left[t+\bar{T}(t)-t^*\right] & \text{if } m(t)\leq-\sqrt{3}\end{cases}$$

(8)

When the normalized safety margin is both positive (negative) and large enough in absolute value, one is sure to be early (late) whatever the actual travel time, and we find again the same formula as in the certain case. When choosing a smaller safety margin (in absolute value), one is exposed to being either early or late depending on the delay he is subjected to, which gives rise to a different formula.

4.2. Travel time dynamics

Let us note $[t_s^v, t_e^v]$ the rush hour period for stochastic travel times. At equilibrium, the expected trip cost is constant throughout this period. This implies the following relationships:

$$\forall t\in\left[t_s^v,t_e^v\right]\quad\begin{cases}\bar{T}'(t)=\frac{\beta}{\alpha-\beta} & \text{if } m(t)\geq\sqrt{3}\\ \bar{T}'(t)=\dfrac{\frac{\beta+\gamma}{2}\frac{m(t)}{\sqrt{3}}+\frac{\beta-\gamma}{2}}{\alpha-\left\{\frac{\beta+\gamma}{2}\frac{m(t)}{\sqrt{3}}+\frac{\beta-\gamma}{2}\right\}} & \text{if } -\sqrt{3}\leq m(t)\leq\sqrt{3}\\ \bar{T}'(t)=-\frac{\gamma}{\alpha+\gamma} & \text{if } m(t)\leq-\sqrt{3}\end{cases}\qquad(9)$$

Again, when the safety margin is large enough in absolute value, the expected travel time varies like in the certain case. For intermediate values of the normalized safety margin, the travel time derivative is smoothed compared to the certain case.[5]

When $|m(t)|\leq\sqrt{3}$, the above differential equation can be integrated, and we find that:

$$\bar{T}(t)=t^*-t-\frac{2\sqrt{3}\sigma_\varepsilon}{\beta+\gamma}\left[\alpha+\frac{\gamma-\beta}{2}-\sqrt{\left(\alpha+\frac{\gamma-\beta}{2}\right)^2-\frac{\beta+\gamma}{\sqrt{3}}\left(A-\frac{\alpha}{\sigma_\varepsilon}t\right)}\right]\qquad(10)$$

where A is a constant to be determined using boundary conditions.

[5] This can be seen by the fact that when $m(t)$ shifts from $\sqrt{3}$ to $-\sqrt{3}$, $\overline{T'}(t)$ shifts continuously from $\beta/(\alpha-\beta)$ to $-\gamma/(\alpha+\gamma)$ (and not in a discontinuous manner as it occurs in the certain case).

4.3. Equilibrium solutions

Given the above elements, there are four kinds of solutions, depending on the magnitude of σ_ε as well as the relative magnitude of β and γ.

4.3.1. Case 1: $\sqrt{3}\sigma_\varepsilon \le \frac{\gamma}{\beta+\gamma}\frac{N}{s}$ *and* $\sqrt{3}\sigma_\varepsilon \le \frac{\beta}{\beta+\gamma}\frac{N}{s}$

Case 1 corresponds to the set of equilibrium solutions verifying $|m(t_s^v)|\geq\sqrt{3}$ and $|m(t_e^v)|\geq\sqrt{3}$. In case 1, the maximum random delay (equal to $\sqrt{3}\sigma_\varepsilon$) is relatively small. The first commuters are thus still sure to be early at their destination whatever the actual random delay (which stems from $|m(t_s^v)|\geq\sqrt{3}$), and similarly the last ones remain sure to be late (as $|m(t_e^v)|\geq\sqrt{3}$). People leaving near the peak may switch from being early to being late and vice versa however, depending on the delay experienced.

One can show that the equilibrium solution of case 1 verifies:

$$\begin{cases} t_s^v = t^* - (T_0 + T_\varepsilon) - \frac{\gamma}{\beta+\gamma}\frac{N}{s} \\ C_{eq}^v = \alpha(T_0 + T_\varepsilon) + \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s} \end{cases} \tag{11}$$

Compared to the certain case, the timing of the rush hour is left unchanged and the equilibrium trip cost also remains the same. As we will see in section 5, departure rates are different from the certain case however, leading to a different congestion profile.

4.3.2. Case 2: $\beta > \gamma$ *and* $\frac{\gamma}{\beta+\gamma}\frac{N}{s} \le \sqrt{3}\sigma_\varepsilon \le \frac{1}{4}\frac{\beta+\gamma}{\gamma}\frac{N}{s}$

Case 2 regroups solutions verifying $|m(t_s^v)| \geq \sqrt{3}$and $|m(t_e^v)| \leq \sqrt{3}$ (which requires that $\beta > \gamma$). Compared to the previous case, travel time variability has reached a level such that the very first commuter may now be late. People leaving at the end of the rush hour remain sure to be late though, whatever the delay.

In case 2, the equilibrium solution is characterized by:

$$\begin{cases} t_s^v = t^* - (T_0 + T_\varepsilon) + \sqrt{3}\sigma_\varepsilon - 2\sqrt{\frac{\gamma}{\beta+\gamma}\sqrt{3}\sigma_\varepsilon\frac{N}{s}} \\ C_{eq}^v = \alpha(T_0 + T_\varepsilon) + \gamma\left(\frac{N}{s} + \sqrt{3}\sigma_\varepsilon - 2\sqrt{\frac{\gamma}{\beta+\gamma}\sqrt{3}\sigma_\varepsilon\frac{N}{s}}\right) \end{cases} \tag{12}$$

4.3.3. Case 3: $\gamma > \beta$ *and* $\frac{\beta}{\beta+\gamma}\frac{N}{s} \le \sqrt{3}\sigma_\varepsilon \le \frac{1}{4}\frac{\beta+\gamma}{\beta}\frac{N}{s}$

Case 3 is analogous to case 2 with the difference that $\gamma > \beta$ instead of the opposite. Consequently, it is now people leaving at the end of the rush hour who may be early or late while people leaving at the beginning of the rush hour remain always early. In case 3, the equilibrium solution is characterized by:

$$\begin{cases} t_s^v = t^* - (T_0 + T_\varepsilon) - \sqrt{3}\sigma_\varepsilon + 2\sqrt{\dfrac{\beta}{\beta+\gamma}\sqrt{3}\sigma_\varepsilon \dfrac{N}{s}} - \dfrac{N}{s} \\ C_{eq}^v = \alpha(T_0 + T_\varepsilon) + \beta\left(\dfrac{N}{s} + \sqrt{3}\sigma_\varepsilon - 2\sqrt{\dfrac{\beta}{\beta+\gamma}\sqrt{3}\sigma_\varepsilon \dfrac{N}{s}}\right) \end{cases} \tag{13}$$

4.3.4. Case 4: $\sqrt{3}\sigma_\varepsilon \geq \frac{1}{4}\frac{\beta+\gamma}{\beta}\frac{N}{s}$ *and* $\sqrt{3}\sigma_\varepsilon \geq \frac{1}{4}\frac{\beta+\gamma}{\gamma}\frac{N}{s}$

Finally, case 4 is the set of equilibrium solutions complying with $|m(t_s^v)| \leq \sqrt{3}$ and $|m(t_e^v)| \leq \sqrt{3}$. In case 4 all N individuals may be early or late (relatively to their preferred arrival time t^*) depending on the random delay they are subjected to. Those leaving at the beginning of the rush hour are naturally more likely to be early, and vice versa. One can show the following results for case 4:

$$\begin{cases} t_s^v = t^* - (T_0 + T_\varepsilon) - \dfrac{1}{2}\dfrac{N}{s} + \sqrt{3}\sigma_\varepsilon \dfrac{\beta-\gamma}{\beta+\gamma} \\ C_{eq}^v = \alpha(T_0 + T_\varepsilon) + \sqrt{3}\sigma_\varepsilon \dfrac{\beta\gamma}{\beta+\gamma} + \dfrac{\beta+\gamma}{16\sqrt{3}\sigma_\varepsilon}\dfrac{N^2}{s^2} \end{cases} \tag{14}$$

5. Impact of travel time variability on the equilibrium solution: analysis in an applied case

We will now focus the analysis on an applied case as a way to discuss the main results of our model. We choose the following values for the various parameters:

- schedule preferences: $\alpha = 1.2$, $\beta = 1$, $\gamma = 3$
- PAT: $t^* = 9.5$
- expected free flow travel time: $T_0 + T_\varepsilon = 0.5$
- travel demand: $N = 100$
- transportation supply: $s = 100$

While not entirely realistic, this set of parameters aims to represent a somewhat plausible situation. The preferred arrival time is set to 9.30 am, with an expected commute time of half an hour in the absence of congestion. Travel demand and transportation supply are equal, resulting in a rush hour that is one hour long. Finally, the utility parameters α, β and γ are in the ratio 1.0:0.8:3.0 indicat*ed* by *B*ates et al. [1] as typical from the literature.

Keeping these values fixed, we analyze how the equilibrium is modified when travel variability (which is controlled by σ_ε) increases. While the exact values of the parameters influence the magnitude of the various phenomena that we will observe, all results can be considered as general to the case $\gamma > \beta$. The other case $\gamma < \beta$ is symmetrical and is therefore not examined.[6]

In the present bottleneck model with random uniform delays, travel time variability has a threefold impact:

- peak reduction;
- peak spreading;
- peak shifting.

The first two phenomena occur immediately. Peak shifting only occurs when travel time unreliability exceeds a certain threshold.

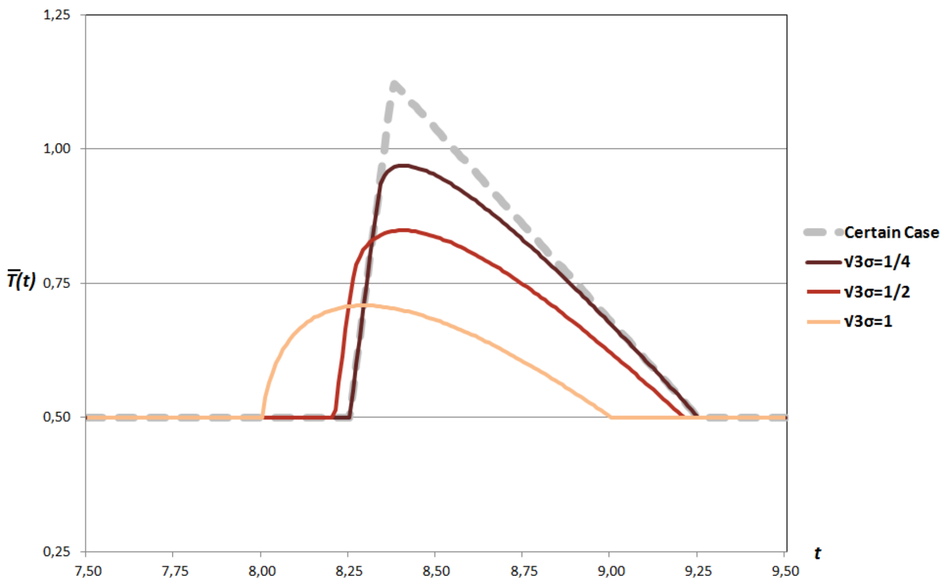

Figure 1: Expected travel time according to the variability level σ_ε

[6] The special case $\beta = \gamma$ is also not exposed here. In sum, when $\beta = \gamma$, travel time variability leads to peak spreading and peak reduction, but no peak shifting occurs. Whatever the level of σ_ε, $t_s^v = t_s = t^* - T_0 - T_\varepsilon - 1/2.N/s$ and $t_e^v = t_e = t^* - T_0 - T_\varepsilon + 1/2.N/s$.

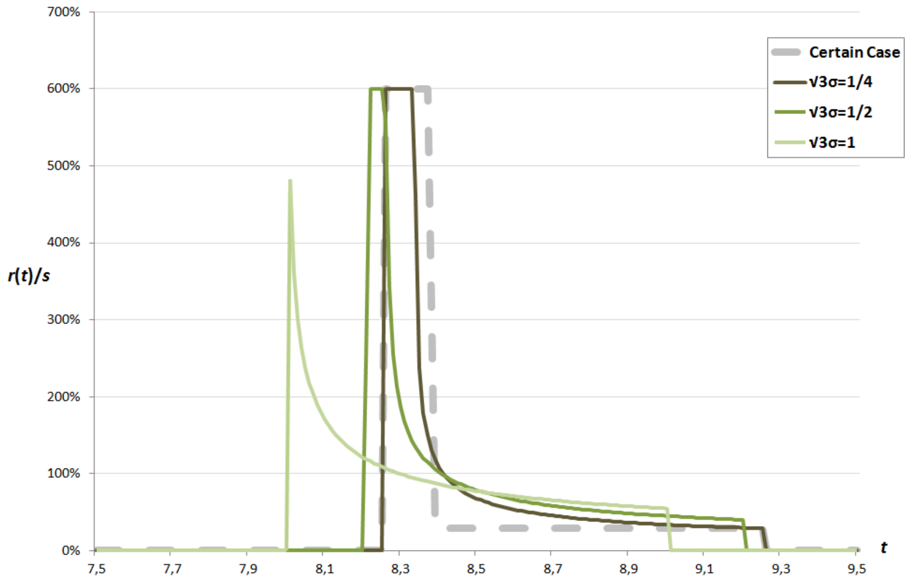

Figure 2 : Departure rate according to the variability level σ_ε

These various points are illustrated in Figure 1. As long as $\sqrt{3}\sigma_\varepsilon \leq 1/4$ (condition for being in case 1), the timing of the rush hour is unchanged but the congestion peak is flattened as σ_ε grows. Beyond this threshold, the rush hour shifts earlier and earlier as variability increases. Congestion also keeps on decreasing.

The analysis of the departure rate sheds light on these results. Starting from $\sigma_\varepsilon = 0$ (no variability), an increase in σ_ε causes people leaving just before and after the peak to adjust their time of departure (Figure 2). Instead of the two plateaus, the normalized departure rate $r(t)/s$ smoothly decreases from $\alpha/(\alpha\text{-}\beta)$ to $\alpha/(\alpha+\gamma)$. Indeed, travel time unreliability is most costly for people leaving exactly at the peak, for whom schedule disutility was null otherwise. For the others, the fact that they have chosen not to be on time and to enjoy a lower congestion level gives them some slack as far as the random delay is concerned: for instance, people leaving after the peak and thus normally getting late at their destination will benefit from a situation where the travel time is shorter than usual. Far from the peak, people are not impacted by the unreliability of travel times, hence stable departure rates at the beginning and at the end of the rush hour.

When unreliability exceeds the threshold associated to case 2, changes become more significant. People being especially averse to being late ($\gamma > \beta$), they start leaving earlier. The departure rate is also further homogenized. The level at the beginning of the rush hour remains the same though.

Finally, when $\sqrt{3}\sigma_\varepsilon \geq 1$ (case 4), unreliability is so great that the whole curve is flattened.

We now turn our attention to the influence of σ_ε on the expected trip cost. For low values of σ_ε (as long as we stay in case 1 to be specific, which for our applied case corresponds to $\sqrt{3}\sigma_\varepsilon \leq 1/4$), the expected trip cost remains constant (Figure 3). When $\sqrt{3}\sigma_\varepsilon \geq 1/4$, $C_{eq}^{\,v}$ starts to increase. Asymptotically, the cost increases linearly.

Two points are especially interesting. First, when σ_ε is small enough, the cost of unreliability is zero. Individual adjustments at the level of the departure time manage to perfectly offset the cost of uncertainty. This result stems from the form of the utility function. Indeed, one must keep in mind that as (α,β,γ) preferences are piecewise linear, people who stay on either side of the kink (*i.e.* people who are always early or always late whatever the random delay) are risk neutral. This technicality is the very reason why the expected trip remains constant for low variability levels.

Secondly, it is only for a relatively high value of σ_ε that the expected trip cost starts to increase. For $\gamma > \beta$, this occurs when $\sqrt{3}\sigma_\varepsilon \geq \beta/(\beta+\gamma).N/s$. For the typical values of β and γ mentioned in Bates et al. [1], $\beta/(\beta+\gamma)$=0.8/(0.8+3) ≈0.2. This means that the maximum random delay must be greater than the fifth of the length of the morning "rush hour" (here in the sense of the whole timespan used by people to commute). For the Paris area, this "rush hour" lasts for 4 to 5 hours (Figure 4). The maximum random delay would therefore have to be at least one hour long, which is quite unrealistic.

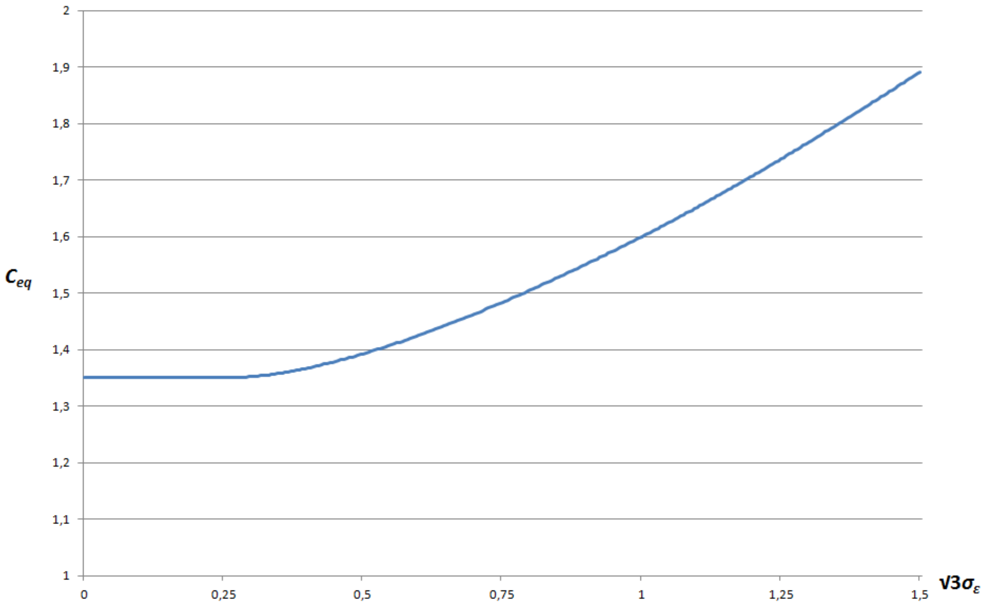

Figure 3: Expected trip cost according to the variability level σ_ε

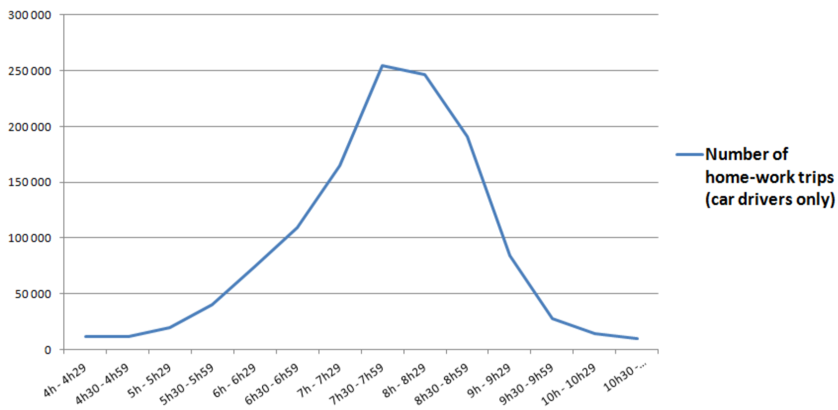

Figure 4: Number of home-work trips made by car (counted in number of cars), 2001 (Source: EGT 2001, DRIEA)

6. Conclusions

In this paper we have extended the theory on the valuation of travel time variability to the case with multiple travelers, thereby considering interactions between individuals and equilibrium mechanisms. To do so, we have amended the standard bottleneck model of peak-load congestion by introducing a random uniform delay. A key finding of this work is that equilibrium mechanisms mitigate the cost of travel time variability, the corollary being that single travel approaches overestimate the value of reliability (at least as far as the morning rush hour is concerned). As those latter approaches currently act as a reference on this topic, this calls for a new methodology as far as the evaluation of road and public transport schemes is concerned.

We have also shown that travel time variability entails peak spreading and peak shifting. It could therefore explain part of the changes observed in the morning traffic of several metropolitan areas over the last years. Finally, we have brought to light the fact that (α,β,γ) preferences lead to a value of reliability that is null when travel time variability is within a reasonable range. This relatively strong result questions the choice of (α,β,γ) preferences in an equilibrium framework, and utility functions featuring risk aversion in a clearer manner should be considered in future works.

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