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ITERATIVELY IMPROVING THE BASE MATRIX OF THE IPF METHOD FOR ESTIMATING TRANSIT ROUTE-LEVEL OD FLOWS FROM APC DATA

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ABSTRACT

An algorithm based on the widely used Iterative Proportional Fitting (IPF) method is proposed to estimate bus route-level origin-destination (OD) passenger flow matrices from boarding and alighting data for time-of-day periods in the absence of good a priori estimates of the flows. The algorithm takes advantage of the large quantities of boarding and alighting data that are routinely collected by transit agencies using Automatic Passenger Counter (APC) technologies. An arbitrarily chosen OD matrix is used as the base matrix required to initialize the algorithm, and the IPF method is applied with bus trip boarding and alighting data and the base matrix to produce an estimate of the OD flow matrix for each trip. The trip-level OD flow matrices are then aggregated across bus trips over a time-of-day period to produce an estimate of the period-level OD flow matrix, which in turn is used as the base matrix for the subsequent iteration until convergence. The method is applied to an operational bus route using APC data collected during two quarters where directly observed OD passenger flows are available to represent the ground-truth. For both quarters the developed method produces better estimates than the traditional application of the IPF method when using the uninformative null matrix as the base without updating.

Keywords: Transit Route-level OD Flow Estimation, IPF Method, APC data, Empirical Evaluation.

1. INTRODUCTION

Transit route-level passenger Origin-Destination (OD) flow matrices provide quantitative information on passenger flows between boarding and alighting stops on a single bus route, which can be used for ridership forecasting, service planning, and operations control (Cortes *et al*., 2011; Site and Filippi, 1998; Tirachini *et al*., 2011). OD flow matrices are often estimated from data collected by large-scale onboard surveys, but such surveys are costly and time consuming. They can also suffer from response biases and a variety of sampling errors (Furth and Navick, 1992). In contrast, relatively accurate passenger boarding and alighting counts at stops along a bus route can be easily collected. Such counts contain indirect OD flow information. For a bus trip, the sums of OD flows originating from and destined to a bus stop equal the boarding and alighting counts at that bus stop, respectively. However, a given set of boarding and alighting counts can correspond to many feasible OD matrices. As a result, different methods can be and have been developed to estimate an OD matrix that is consistent with the boarding and alighting counts, assumed behavioral characteristics, and, possibly, additional information, such as data collected from a small sample size onboard survey (Ben-Akiva, 1987; Ben-Akiva *et al*., 1985; Furth and Navick, 1992; Hazelton, 2010; Kikuchi and Perincherry, 1992; Li and Cassidy, 2007; Simon and Furth, 1985).

The Iterative Proportional Fitting (IPF) method (Bacharach, 1970; Ben-Akiva *et al*., 1985) is arguably the state-of-the-practice method used to estimate OD flows from boarding and alighting data. The IPF method has been shown to produce relatively good OD flow estimates in empirical studies (McCord *et al*., 2010; Mishalani *et al*., 2011). Under certain assumptions, the IPF method is theoretically equivalent to many methods proposed in the literature (Li and Cassidy, 2007; Tsygalnitsky, 1977), and empirical studies have shown the similarity in the results produced by the IPF method and other more complex methods (Ben-Akiva *et al*., 1985; Lu, 2008). Relationships between the IPF and other methods are summarized in Ji (2011).

The inputs to the IPF method include a vector of boarding and alighting counts and a base OD matrix. The base (also referred to as seed) OD matrix can be considered an initial estimate of the OD flows, where this initial estimate would be derived, for example, from an outdated OD matrix, recent onboard surveys, or some travel demand model. In the absence of any *a priori* information, the null OD flow matrix is typically used as the base matrix. The null OD matrix is constructed by assigning equal values (*e.g.*, one) to all feasible OD pairs and zeros to all infeasible OD pairs. However, a null base generally does not reflect actual passenger travel patterns. The null base implies that a passenger is equally likely to travel along any feasible OD pair and that an onboard passenger is equally likely to alight at a given downstream stop, regardless of the stop at which he or she boarded (Furth and Navick, 1992). Li and Cassidy (2007) pointed out that the equally likely alighting assumption is not reasonable for bus routes serving activity centers, such as shopping centers and metro stations. In addition, if passengers prefer walking or biking for short travel distances, onboard passengers who recently boarded the bus would be less likely to alight at a given bus stop than onboard passengers who have already travelled longer distances. The structure of the base OD matrix used as input to the IPF method is known to severely restrict the value that can be added from information on the structure of the marginal (*e.g.*, boarding and alighting counts) data (Bishop *et al*., 1975). Therefore, the poor behavioral assumptions of the null OD flow matrix limit the quality of the estimates produced using the IPF method with a null base, referred to henceforth as the IPF-null method. For example, Simon and Furth (1985) demonstrated empirically that the IPF-null method tends to overestimate both short and long trips.

Since many transit agencies now deploy Automatic Passenger Counter (APC) technologies to collect boarding and alighting counts at stops on a regular basis, large quantities of boarding and alighting data can be used to estimate route level passenger OD flow matrices, and improved route-level OD flow estimation methods can conceivably be developed when no *a priori* information is available to form a good base OD matrix. Some methods have recently been proposed (Hazelton, 2010; Ji *et al*., 2012) to take better advantage of the rich information contained in large sets of boarding and alighting data. Although these methods show promise, they are sufficiently different from the widely used and familiar IPF method that their attractiveness for immediate use by transit and planning agencies could be limited. Furthermore, the computational performance of at least one of these methods is burdensome when large APC datasets are available, thus limiting its appeal for wide-scale applications.

In this paper, a procedure that relies on the IPF method as a building block is proposed to estimate route-level OD matrices with large quantities of boarding and alighting data when a good base matrix is not available. Given the intuitive nature in which the well-known and frequently used IPF method is incorporated, the developed Iterative Proportional Fitting-Iterative Base (IPF-IB) method is attractive for practical use. In the next section, the IPF-IB method is described. Following that, an empirical study demonstrating the improved estimation accuracy offered by the IPF-IB method is presented. The paper concludes with a summary and discussion of future research.

2. METHODOLOGY

The objective is to estimate a route-level OD flow matrix for a time-of-day period where passenger travel patterns are relatively stable. Ji *et al*. (2011) provide an approach for identifying time-of-day periods that can be considered homogeneous in OD flow patterns. An OD flow pattern is captured by a probability OD flow matrix (Ji, 2011; Ji *et al*., 2012; Ji *et al*., 2011; Mishalani *et al*., 2011), a cell value of which represents the probability that a randomly selected passenger travels from the origin stop to the destination stop corresponding to that cell.

Even though the IPF algorithm has been described extensively in the literature [5], given the central role it plays in the IPF-IB method, it is described briefly to facilitate the presentation of the IPF-IB method. As discussed above, the inputs to the IPF method are boarding and alighting counts for every stop along a bus route and a base OD flow matrix. Beginning with the two-dimensional base OD flow matrix, the IPF method iteratively adjusts the matrix by row and column until the marginal row and column totals of the updated OD flow matrix converge to the given boarding and alighting counts, respectively. When the algorithm converges, the relationship between the updated OD flow matrix and the base OD flow matrix is given by:

$$\hat{T}(i,j) = k_i T_0(i,j) k_j \tag{1}$$

where $\hat{T}(i,j)$ and $T_0(i,j)$ are the updated and the base passenger flows travelling from stop i to stop j, respectively. The variables k_i and k_j are factors for row i and column j, respectively. The row and column factors are determined through the iterations described above. Finally, the marginal row and column totals of the updated OD flow matrix will be approximately equal to the given boarding and alighting counts, where the degree of approximation depends on the convergence criterion. That is:

$$\sum_j \hat{T}(i,j) \simeq b_i \;\; i = 1,\ldots,S \tag{2a}$$

$$\sum_i \hat{T}(i,j) \simeq a_j \;\; j = 1,\ldots,S \tag{2b}$$

where b_i and a_j are the boarding count at stop i and alighting count at stop j, respectively, and S is the number of bus stops along the given bus route.

Based on APC data only, the IPF-IB method relies on the IPF method to determine a probability OD flow matrix p by applying the following steps:

Step 0: Arbitrarily choose a base OD matrix, p^0 (a null base can be used); set counter $h = 1$.

Step 1: Apply the IPF method with p^{h-1} as the base and the boarding and alighting counts from each bus trip i, $i = 1, \ldots, N$ (*i.e.*, number of bus trips), to produce N trip-level OD volume flow matrices.

Step 2: Aggregate (sum) the trip-level volume OD flow matrices determined in Step 1 by OD pair across N bus trips to determine an aggregated volume OD matrix, and then normalize the aggregated matrix by the total passenger volume to determine an estimate of the probability OD flow matrix p^h.

Step 3: If the procedure converges, or if a specified number H of iterations has been conducted, stop; if not, set $h = h + 1$ and go to Step 1.

Several convergence criteria could be used. The criterion used in this study is one where the absolute value of the maximum change, among all OD pairs, in the probability OD flows from one iteration to the next is less than a threshold specified as input.

In essence, the approach updates the base at each iteration by using the estimated matrix in an iteration as the base in the next iteration. Other than aggregating trip-level OD matrices into a probability OD matrix for a time-of-day period, which is straightforward (McCord *et al.*, 2010), the approach simply uses the well-known and commonly used IPF method repeatedly. As such, the IPF-IB method is easy to implement for practical use.

Furthermore, the method is theoretically appealing. Ji (2011) has shown that the IPF-IB method produces a solution that maximizes the joint likelihood function of the trip-level volume OD flow matrices and the period-level probability OD flow matrix given the collection of boarding and alighting counts across all bus trips. Consequently, the IPF-IB method considers the associations among the OD flow matrices across bus trips while the IPF method does not. Notice, given the nature of the IPF-IB algorithm, the estimated OD flows on all bus trips contribute to the probability OD flow estimates, which in turn influence the OD flow estimates for each bus trip in the next iteration, and so on and so forth. That is, the probability OD flow matrix at a given iteration acts as the medium through which the OD flow pattern on one bus trip provides valuable information for the OD flow estimation for all bus trips in subsequent iterations. Eventually, better probability OD flow estimates are obtained as a result.

3. EMPIRICAL EVALUATION

3.1. Data Description

The data used in this study were collected through The Ohio State University's Campus Transit Lab (CTL, 2012). The CTL is based on the Campus Area Bus Service (CABS), a transit service owned and operated by The Ohio State University (OSU) that serves approximately four million passengers annually on seven routes in the vicinity of the OSU campus. To investigate the performance of the IPF-IB method data from the Campus Loop South (CLS) route are used. The CLS route is 8.34 km long and operates on a predetermined schedule with established time points where buses are routinely held to maintain schedule adherence. The route follows a loop structure that serves multiple classroom and academic facilities (agricultural campus, engineering campus, central campus), a large medical complex (hospitals and research labs), and student housing regions. The route traverses roadways of varying traffic characteristics, ranging from large parking lots to off-campus high traffic-volume intersections to high pedestrian-volume campus crosswalks and intersections. Therefore, the empirical results derived are expected to be meaningful to transit agencies operating longer routes in urban areas.

The CLS route serves 20 bus stops. Four park-and-ride stops in the west campus region are aggregated into a pseudo stop considered to be the terminal of the route, where bus trips begin and end. The beginning and ending terminal is treated as two separate stops, one restricted to boarding activity at the beginning of the bus trip and one restricted to alighting activity at the end of the trip. Given this aggregation, the route is considered to serve 18 stops and 153 feasible OD pairs. (Feasible OD pairs are those that reflect passenger travel in the direction of the route and, naturally, do not include boarding and alighting at the same stop.)

Large-scale onboard survey data are used to construct ground-truth OD flow matrices to which the estimated OD flow matrices are compared for performance evaluation. In this study, onboard survey data collected from 48 bus trips in the relatively homogenous 8-10 a.m. period on weekdays during four academic quarters are used. The survey design was identical to that described in McCord *et al*. (2010), where boarding passengers are each handed a card labeled according to the boarding stop, which they return when alighting from the bus. By filing the card according to the appropriate alighting stop, the passenger's origin and destination can be obtained. More than 95% of the passengers traveling on the targeted trips were successfully sampled in this way.

The numbers of surveyed bus trips and passengers are summarized in Table 1 by quarter. (Onboard surveys were not conducted during Winter 2010.) Considering the ridership levels and the numbers of stops served, the size of the onboard OD flow survey is substantially larger than that of a typical onboard survey conducted in practice. Therefore, the OD flow patterns deduced are expected to be reasonable estimates of the true flow patterns. For each bus route and each quarter, surveyed OD flow matrices are aggregated for each OD pair across bus trips, and the resulting aggregated OD flow matrix is normalized by the total number of boarding passengers. This normalized OD flow matrix is used as the underlying, true probability OD flow matrix.

All buses operating on CABS bus routes are equipped with APC systems. APC data from 464 and 584 weekday bus trips during Autumn 2009 and Spring 2010, respectively, in the 8-10 a.m. period are used as input to the OD estimation. The number of bus trips and passengers, as measured by APCs, are also shown in Table 1 by quarter. The APC data are subject to realistic measurement errors. Bus trips with extremely large APC measurement errors have been excluded, and the APC boarding and alighting counts on the remaining bus trips were balanced such that the total boarding count equals the total alighting count and no negative load exists between any two consecutive bus stops for each bus trip. A description of the balancing procedure can be found in (Furth *et al*., 2005).

Table 1 – Number of Bus Trips and Passengers in the Field OD Survey and APC Datasets

Quarter	Field OD Flow Surveys		APC	
	Bus trips	Passengers	Bus trips	Passengers
Winter 2009	13	816	560	31,380
Spring 2009	13	762	500	22,976
Autumn 2009	10	666	464	27,701
Spring 2010	12	786	584	29,350

3.2. Performance Measures

The accuracy of the IPF-IB method is evaluated by comparing the OD flow estimates produced to the ground-truth (field) OD flows. For comparative purposes, similar comparisons are made for the IPF-null method. Two versions of the IPF-null method could be considered to estimate a period-level probability OD flow matrix from trip-level vectors of boarding and alighting data. In the first version (see, *e.g.*, Ben-Akiva (1987) and Ben-Akiva *et al*. (1985)), the trip-level boarding and alighting vectors in the period are first summed to produce an aggregate boarding vector and an aggregate alighting vector. The IPF method is then applied with a null base and the aggregate boarding and alighting vectors to estimate a period-level OD volume matrix. Finally, the estimated OD volumes are normalized by the total volume to determine the probability OD flow matrix for the period. In the second version (see, *e.g.*, Mishalani *et al*. (2011) and Ji *et al*. (2011, 2012), the IPF method is first applied to each bus trip separately with a null base and the trip-level boarding and alighting vectors to estimate an OD volume matrix for each trip. The resulting trip-level OD volume matrices are then summed to produce a period-level volume OD flow matrix. Then, as in the first method, the OD volumes are normalized by the total volume to produce the probability OD flow matrix for the period. Numerical studies, which are beyond that scope of this paper, support the advantages of this second version. Therefore, the second version is used as the implementation of the IPF-null method in the empirical study.

To assess the overall accuracy of an estimated matrix, the difference between the estimated matrix and the corresponding true matrix (*i.e.,* that obtained from the onboard surveys for the corresponding route and quarter) is calculated. The Hellinger Distance, *HD*, metric is used for this purpose, which is defined by:

$$HD = \sqrt{\sum_{i=1}^{S-1} \sum_{j=i+1}^{S} \left(\sqrt{\hat{p}(i,j)} - \sqrt{p(i,j)} \right)^2} \tag{3}$$

where $\hat{p}(i,j)$ represents the probability OD flow from stop *i* to stop *j* estimated from one of the procedures considered, $p(i,j)$ represents the underlying true probability OD flow from stop *i* to stop *j*, and *S* represents the number of bus stops. The *HD* metric is commonly used to quantify differences between two probability distributions (Yang *et al.*, 2000). Smaller *HD* values indicate better performance. (Other metrics, such as the Euclidean Distance, were also used. The trends in the results were found to be the same, regardless of the metric used.)

The HD metric is then used to construct a measure of relative performance, *RP*, given by (McCord *et al.*, 2010):

$$RP = \frac{HD_{null} - HD}{HD_{null}} \tag{4}$$

where HD_{null} is the value of the *HD* metric obtained when using the null matrix as the estimate, and *HD* is the value of this metric for the OD flow estimates produced with some estimation method. (The IPF-null and IPF-IB methods are used in this study.) Thus, the *RP* measure quantifies the improvement of the estimated OD flow matrix, compared to using the null OD matrix as an estimate, as a proportion of the measure of dissimilarity between the null and ground-truth (field observed) matrices. Low *RP* values indicate little improvement in performance, and *RP* values close to one indicate great improvement in performance. For example, the relative performance value would be zero if the null OD flow matrix is used as an "estimate", and it would be one if the normalized field OD flow matrix is used as an estimate. (*RP* values could be less then zero if the estimated OD matrix was "farther" than the null matrix from the field OD flow matrix.)

3.3. Empirical results

The *RP* values, by quarter, are presented in Figure 1. The values for the IPF-IB method are consistently higher than those for the IPF-null method for all quarters, demonstrating the superior accuracy of the IPF-IB method. Quantitatively, the differences in the *RP* values produced when using the IPF-null method and when using the IPF-IB method range from

0.094 to 0.095. In all cases, the IPF-IB estimates were produced in less than 10 seconds using a duo-core MacBook Pro with a 2.7 GHz processor.

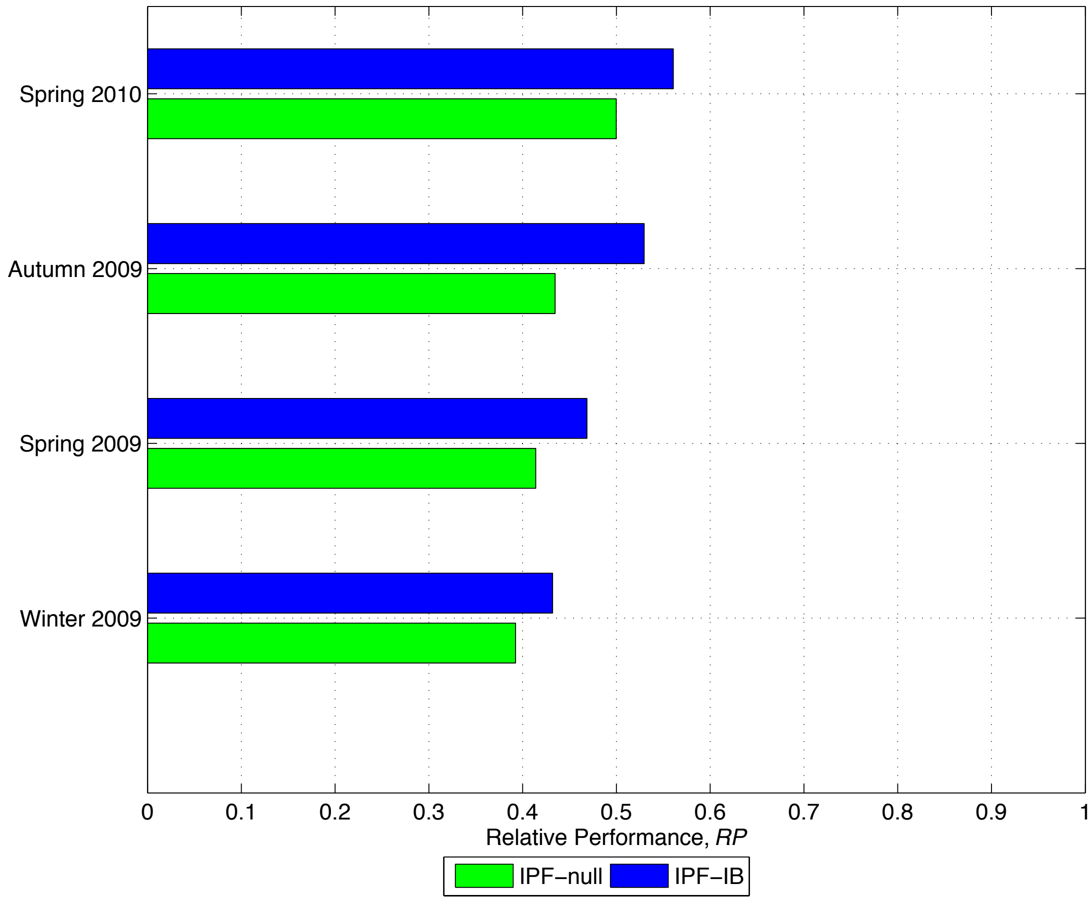

Figure 1 – *RP* Values for the IPF-IB and IPF-null Methods by Quarter

4. SUMMARY AND FUTURE RESEARCH

The IPF-IB is proposed as an easy-to-use method to estimate route-level OD flow matrices from large APC datasets for a time-of-day period. The IPF-IB method iteratively applies the IPF method to improve upon the base used when a good *a priori* estimate of the OD flow matrix is not available. Since the quality of the base used with the IPF method is known to affect the quality of the estimates produced, the improvement in the base offered by the iterations conducted in the IPF-IB method is expected to improve the quality of the estimates produced.

The expected improvement in the quality of the estimates is seen in an empirical study using APC data as inputs to the estimation and directly observed OD flows as ground-truth. Additional empirical studies would be warranted to confirm the encouraging results seen in this paper. Moreover, further methodological research would be warranted.

A null, "uninformative" OD matrix is used as a base when no better *a priori* estimate of the OD flow matrix is available. It would be worthwhile to investigate the ability of the IPF-IB method to produce improved estimates when an informative OD matrix is available. The IPF-IB method could be easily extended to incorporate onboard survey OD flow data in the estimation. In light of the time and effort that such onboard surveys require, it would be informative to investigate the effect of onboard OD flow survey data on the IPF-IB estimates.

APC systems are subject to measurement errors. The empirical results presented in this paper were based on data obtained from APC data collected from regular operations, but the magnitudes and biases in these errors could vary across various technologies and conditions. Moreover, the magnitudes and biases are expected to diminish as APC technologies are refined. Therefore, it would be informative to understand the effect of measurement errors, including their magnitudes and nature, on the improved performance offered by the IPF-IB method. Such understanding could inform the extent to which efforts should be made to reduce APC measurements errors for the purpose of OD flow estimation in light of the costs involved in achieving such reductions.

In this study large amounts of APC data for OD estimation were considered. Obtaining such quantities in a fairly short time-frame would not be difficult when all or a large proportion of buses are equipped with APC systems. However, in some transit agencies, only a subset of the fleet is equipped with APC technologies. Therefore, it would be valuable to investigate the effect of the quantity of APC data on the accuracy of OD flow estimates produced by the IPF-IB method.

As discussed in the introduction, methods of varying computational performance based on more complex formulations have been recently developed. Given the technical accessibility of the IPF-IB method associated with using the well-known IPF method as a foundation, it would be worthwhile to compare the performance of the simple IPF-IB method to the performance of the more complex methods.

Finally, this study demonstrated that the IPF-IB method produces more accurate OD flow estimates than the IPF-null method. It is worthwhile to investigate the implications of the improved accuracy on various planning and control applications that rely on OD flow estimates as inputs. That is, it is important to determine the extent to which the decisions that rely on OD flow estimates are improved as a direct result of the accuracy gains brought about by the IPF-IB method.

In closing, although future research is warranted, the results of the empirical study are encouraging in that the IPF-IB method can improve upon the estimation performance of the well-known IPF-null method. And, it does so with only marginal effort, negligible additional computational time and no additional data required.

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