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## OPTIMAL MAINTENANCE AND RENEWAL STRATEGY DUE TO RAIL TRACK GEOMETRY

ANTÓNIO RAMOS ANDRADE (CORRESPONDING AUTHOR), INSTITUTO SUPERIOR TÉCNICO (IST), TECHNICAL UNIVERSITY OF LISBON, EMAIL: ANDRADE@CIVIL.IST.UTL.PT

PAULO FONSECA TEIXEIRA, INSTITUTO SUPERIOR TÉCNICO (IST), TECHNICAL UNIVERSITY OF LISBON, EMAIL: PFT@IST.UTL.PT

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# OPTIMAL MAINTENANCE AND RENEWAL STRATEGY DUE TO RAIL TRACK GEOMETRY

*António Ramos Andrade (corresponding author)*

*Dipl-Eng, PhD Student*

*CESUR, Department of Civil Engineering*

*Instituto Superior Técnico (IST), Technical University of Lisbon*

*Av. Rovisco Pais 1049-001 Lisboa, Portugal.*

*Phone: +351-21-8418425, Fax: +351-21-849 7650, Email: andrade@civil.ist.utl.pt*

*Paulo Fonseca Teixeira*

*PhD, Dipl-Eng., Assistant Professor*

*CESUR, Department of Civil Engineering*

*Instituto Superior Técnico (IST), Technical University of Lisbon*

*Av. Rovisco Pais 1049-001 Lisboa, Portugal.*

*Phone: +351-21-8418266, Fax: +351-21-849 7650, Email: pft@ist.utl.pt*

ABSTRACT

The aim of the present paper is to discuss a model to optimize a maintenance and renewal strategy to control rail track geometry degradation, addressing the practical need of the Portuguese Infrastructure Manager to set Alert and Intervention Limits for planned maintenance decisions. The resulting model is a Bayesian Decision Process, which incorporates a Bayesian Hierarchical model for rail track geometry degradation and comprises life-cycle costs, in particular renewal costs, planned maintenance costs, unplanned maintenance costs, planned delay costs and unplanned delay costs.

*Keywords: railway infrastructure, maintenance and renewal strategy, Bayesian Decision Process.*

# 1- INTRODUCTION

Optimizing maintenance and renewal actions related to rail track geometry is a complex problem in railway infrastructure management. The European Standard EN 13848-5 (2008) put forward Alert and Intervention Limits (AL and IL) for some indicators for planned maintenance decisions, as the standard deviation of the longitudinal levelling defects (filtered in the wavelength range 3-25 m). However, the EN 13848-5 considers these limits as purely indicative and suggests that each European Infrastructure Manager should select at the national level their own limits according to four dimensions of track geometry impact, namely: i) safety, ii) ride quality, iii) lower life-cycle costs and iv) availability. Therefore, the main objective of the present paper is developing a model for the Portuguese Infrastructure Manager (REFER, E.P.) to support setting these limits as part of an optimized maintenance and renewal strategy using a Bayesian Decision Process with imprecise information on the degradation model parameters.

Following previous research work on developing a Bayesian predictive model for rail track geometry degradation and a bi-objective optimization model for associate maintenance and renewal actions, we will try to incorporate the Bayesian learning mechanism into the typical concepts of Decision Processes. We will start by revising the Markov Decision Process (MDP) and two famous extensions: i) the partially observed Markov Decision Process (POMDP) and ii) the Markov Decision Process with uncertain or imprecise transition probabilities (MDPIP). Afterwards, we discuss the differences between such approaches and our approach, addressing as well the need for extensive simulation to achieve an optimal strategy.

The outline of the present paper is as follows: section 1 introduces the need for research on an optimal maintenance and renewal strategy due to rail track geometry. Section 2 discusses the role of the European Standard EN 13848 on the process of planning maintenance and renewal actions related to rail track geometry. Then, section 3 provides details on the hierarchical Bayesian model to predict the evolution of rail track geometry degradation.

Section 4 discusses the typical concepts and ideas on Decision Processes, such as the well-known Markov Decision Process (MDP) and some of its extensions. Section 5 will then propose a model to optimize the maintenance and renewal strategy, incorporating the Bayesian Learning mechanism provided by the Bayesian Hierarchical model. Moreover, section 6 briefly discusses future experimental tests to compare different choices of the Alert and Intervention Limits. Finally, section 7 discusses the main conclusions and further research needed in this topic.

# 2- PLANNING MAINTENANCE AND RENEWAL ACTIONS IN RAIL TRACK

Rail Track as a transport infrastructure system evolves with time. As more trains pass above a given track section, the rail track geometric quality degrades and eventually a maintenance or renewal action should be planned in order to restore the desired track geometry condition. In simple terms, the main quality indicator that European Infrastructure Managers (IM) monitor in order to plan maintenance and renewal actions related to rail track geometry, according to a guide on best practices for rail track geometry durability (UIC 2008), is the standard deviation of longitudinal level defects filtered in the short-wavelength (3-25m) - $SD_{LL}$. In practical terms, this indicator should be kept lower than a certain safety limit. Although several other indicators of track geometry defects (besides the $SD_{LL}$) should be monitored and also kept under certain limits, they are only considered for unplanned maintenance actions.

In fact, according to the European standard EN 13848-5 (2008) on the track geometric quality levels for railway track, three main levels are defined:

- IAL – Immediate Action Limit: refers to the value which, if exceeded, requires making speed restrictions or immediate correction of track geometry;
- IL – Intervention Limit: refers to the value which, if exceeded, requires corrective maintenance before the immediate action limit is reached;
- AL – Alert Limit: refers to the value which, if exceeded, requires that track geometry condition is analyzed and considered in the regularly planned maintenance operations.

Immediate Action Limits (IAL) provided in the EN 13484-5 are considered normative, as they take into account the track/vehicle interaction derived from experience and from theoretical considerations of the wheel-rail interaction, and provide the highest admissible limits to ensure safety and ride comfort. Note that the derivation of these limits through physical tests with different vehicles up to the point of derailment would not be a feasible option due to the high costs involved.

Intervention Limits (IL) and Alert Limits (AL) are purely indicative, reflecting common practice among most European Infrastructure Managers. They are mainly linked with planned maintenance policy of each Infrastructure Manager, which should supposedly take into account four dimensions of track geometry maintenance: safety, ride quality, lower life-cycle costs and availability. In fact, IL and AL values for some defects are even expressed as a range rather than as a discrete value. Moreover, IAL, IL and AL values for each indicator vary depending on the maximum permissible speed. The definition of such limits took into account two main previous documents: the ORE Question B55 report No. 8 from 1983 and the EN 14363 (2005) on specifications for testing for the acceptance of running characteristics of railway vehicles.

Nevertheless, the European Standard clarifies that IM should define their own IL and AL values as part of their planned maintenance policy according to their preferences on the four above-mentioned dimensions of track geometry maintenance. Therefore, the distinction between planned maintenance and unplanned maintenance is then in the hands of the IM. Although there seems to be a consensus among European IMs that planned maintenance decisions should be based on the $SD_{LL}$ (UIC 2008). However, the standard deviation for the short wavelength of horizontal alignment defects ($SD_{HA}$) seems to play a similar role for some IMs and it is usually included as part of decision rules for their policy on planned maintenance. This option is quite reasonable in part due to the fact that these two indicators ($SD_{LL}$ and $SD_{HA}$) are extremely correlated with vertical and horizontal forces respectively, which are proxies of vertical and horizontal accelerations felt by the passenger and thus, of ride quality (see for example Esveld (1990) or Lichtberger (2005)). Therefore, the definition of the Intervention and Alert Limits according to the preferences of the decision maker[1] (i.e. the Portuguese railway Infrastructure Manager: REFER) are a gap in current research and practice, and thus represents a major research opportunity.

In 2009, the Portuguese Infrastructure Manager published the standard IT.VIA.018 (REFER 2009) on the limits for rail track geometric indicators based on the two European standards EN 13231 and EN 13848 and another Portuguese standard IT.VIA.002. The standard IT.VIA.018 provides limit values for the indicators of rail track geometry defects for new lines/renewals and for maintenance decisions. It was elaborated inside REFER using their past experience and their perception/intuition on the effect of the four dimensions that a given planned maintenance criteria should consider. This process mainly resulted in the adoption of the highest bound of the ranges recommended by the European Standard EN 13848 for the alert limits for planned maintenance criteria ($SD_{LL}$), extending the first maintenance cycle to its maximum, while disregarding higher risks of unplanned maintenance and their impacts in availability in the short-, medium- and long-terms.

[1] Here, the decision problem is formulated from a single agent perspective. Nevertheless, it is important to refer that there are other agents involved in this process, such as: operators (e.g. reliability in wheel defects), regulator entity (e.g. safety assurance), maintenance contractors (e.g. reliability in tamping recuperation) or even the end users/passengers (e.g. perceived ride quality, safety and performance).

Table I – Standard deviation limits of the longitudinal leveling defects for different trains speeds and quality levels according to the EN 13848-5 (2008).

| Standard deviation limits of the longitudinal leveling defects | AL (mm) |
|---|---|
| V ≤ 80 km/h | 2.3 – 3.0 |
| 80 < V ≤ 120 km/h | 1.8 – 2.7 |
| 120 < V ≤ 160 km/h | 1.4 – 2.4 |
| 160 < V ≤ 230 km/h | 1.2 – 1.9 |
| 230 < V ≤ 300 km/h | 1.0 – 1.5 |

For more details on the discussion related with track geometry degradation and all the indicators monitored, we remit the curious reader to our previous papers on these topics. (Andrade and Teixeira (2011a, forthcoming_a)).

# 3- HIERARCHICAL BAYESIAN MODELLING OF RAIL TRACK GEOMETRY DEGRADATION

This section contains a detailed view on a forthcoming paper (Andrade and Teixeira (forthcoming_b)), discussing a hierarchical Bayesian model for rail track geometry degradation. It was first approached in simple terms in Andrade and Teixeira (2012) and the inclusion of this section is justifiable as it allows the reader to gain a full understanding of the methodology for optimizing a maintenance and renewal strategy in section 5.

Let us first review the main assumptions on rail track geometry degradation. A typical assumption on statistical modelling of rail track geometry degradation is considering that the $SD_{LL}$, which would be hereafter represented as $y_{svkl}$, at inspection l for track section k from track segment v from area s, is normally distributed with mean $m_{svkl}$ and variance $\sigma_s^2$, i.e. $y_{svkl} \sim N(m_{svkl}, \sigma_s^2)$. Figure 1 supports the reader to spot the meaning of indices s,v, and k for a typical double track line:

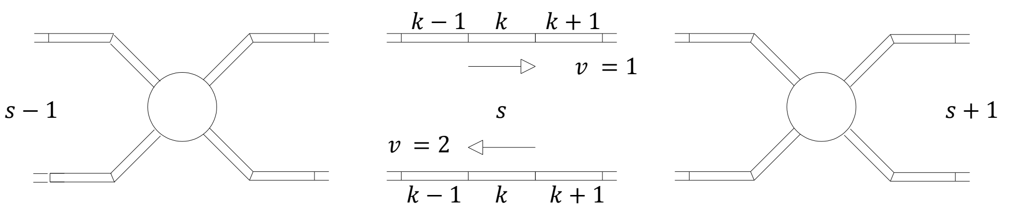


Figure 1 – Schematic representation of a typical double track line and indices from area s, segment v and track section k.

Then, some assumptions on its mean value result from a combination of factors:

1) A constant linear evolution with accumulated tonnage ($T_{svkl}$ – accumulated tonnage since last tamping or renewal operation), given by the deterioration rate - $\beta_{svk}$, assuming different values for each track section k in track segment v for area s.

2) An initial standard deviation of longitudinal level defects, given by the initial quality - $\alpha_{svk}$, assuming different values for each track section k in track segment v for area s.
3) A disturbance effect ($\delta_{sv}$) of the initial standard deviation of longitudinal defects after each tamping operation, i.e. it does not recover to its initial value $\alpha_{svk}$, but it is affected by a rate $1+\delta_{sv}$, given by $\delta_{sv}$, assuming different values for each track segment v for area s. Therefore, note that at each new tamping cycle the initial quality would be $\alpha_{svk}(1+\delta_{sv})^{N_{svkl}}$, in which $N_{svkl}$ is the number of tamping operations conducted since last renewal.
4) Distinction between renewed track sections ($R_{svkl}=1$) and non-renewed track sections ($R_{svkl}=0$) is assured through the separation of the initial quality and the deterioration rates for a non-renewed track section k from segment v for area s – $\alpha'_{svk}$ and $\beta'_{svk}$ respectively; whereas the disturbance effect ($\delta_{sv}$) is considered the same for renewed and non-renewed track sections.

Therefore, having in mind the above-mentioned assumptions, we may write the mean $m_{svkl}$ as:

$$m_{svkl} = [\alpha_{svk}(1+\delta_{sv})^{N_{svkl}} + \beta_{svk}T_{svkl}] \cdot R_{svkl} + [\alpha'_{svk}(1+\delta_{sv})^{N_{svkl}} + \beta'_{svk}T_{svkl}] \cdot (1-R_{svkl}) \quad (1)$$

In this expression, we should regard $\alpha, \beta, \alpha', \beta'$ and $\delta$ as parameters, to which should be assigned a hierarchical probability structure, whereas $N, T$ and $R$ should be regarded as known variables. Figure 2 provides a graphical representation of the intended behavior of the standard deviation of longitudinal level defects expressed by the equation above.

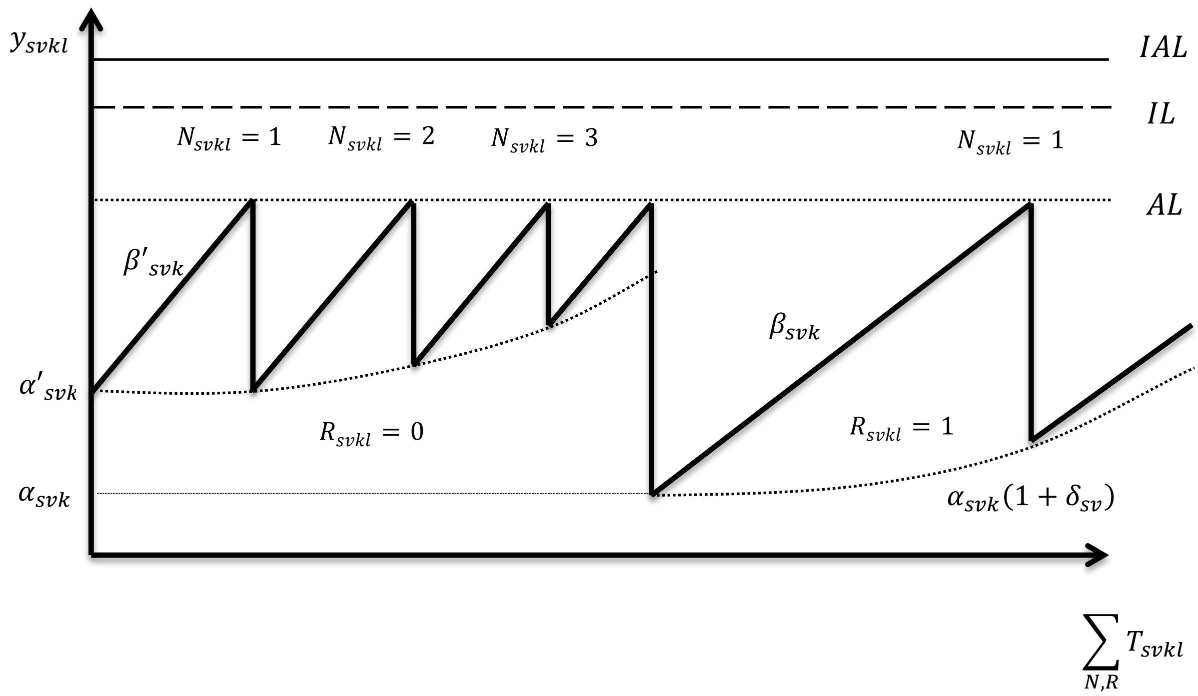


Figure 2 - Graphical representation of the track geometry degradation model for a given track section k in segment v in area s.

The model allows to predict the future state of the track for a maintenance and renewal plan $(\mathrm{N},\mathrm{R})$ based on observed data and a future usage $(\mathrm{T})$, and thus a typical transportation demand model can be hierarchically assigned to $\mathrm{T}$. In the present paper we are then interested in finding a strategy to assign a maintenance and renewal plan $(\mathrm{N},\mathrm{R})$ based on observed data and predicted state of rail track geometry degradation.

In order to be parsimonious in modelling, we considered (Gaussian) Conditional Autoregressive (CAR) terms for the spatial interactions between degradation rates and initial qualities for consecutive track sections. Therefore, the initial quality $\alpha_{svk}$ for a given track section k in segment v in area s is modelled as a combination of two additive components: an average value $\alpha_{sv}$ and a spatially correlated term $\varepsilon_{\alpha_{svk}}$ so that: $\alpha_{svk} = \alpha_{sv} + \varepsilon_{\alpha_{svk}}$. For $\varepsilon_{\alpha_{svk}}$, we then assign a conditional probability structure such as $\varepsilon_{\alpha_{svk}} | \varepsilon_{\alpha_{sv(-k)}}, \sigma_\alpha^2 \sim \mathrm{N}(\bar{\varepsilon}_{\alpha_{svk}}, \sigma_\alpha^2/n_{svk})$, in which $\bar{\varepsilon}_{\alpha_{svk}} = \sum_{j\in\mathcal{N}_{svk}} \varepsilon_{\alpha_{svj}}/n_{svk}$, $\mathcal{N}_{svk}$ denotes the set of track sections which are considered neighbors to track section k (in segment v in area s), and $n_{svk}$ is the number of neighbors of track section k (in segment v in area s), and finally $\varepsilon_{\alpha_{sv(-k)}}$ is the vector with all components $\varepsilon_{\alpha_{svk}}$ from segment v in area s except the component related to track section k. In Andrade and Teixeira (forthcoming_b), we mainly compared two well-known CAR structures: the first-order random walk $(\mathrm{RW}(1))$ and the second-order random walk $(\mathrm{RW}(2))$ as hierarchical structures for $\alpha, \beta, \alpha', \beta'$, and found that the first-order random walk $(\mathrm{RW}(1))$, defined by considering as neighbors structure $n_{svk} = 1$ for $k = 1$ and $k = K_s$, and $n_{svk} = 2$ for $k = 2, \dots, K_s - 1$, and $\bar{\varepsilon}_{\alpha_{svk}} = \varepsilon_{\alpha_{sv(k+1)}}$ for $k = 1$, $\bar{\varepsilon}_{\alpha_{svk}} = \left(\varepsilon_{\alpha_{sv(k-1)}} + \varepsilon_{\alpha_{sv(k+1)}}\right)/2$ for $k = 2, \dots, K_s - 1$, and $\bar{\varepsilon}_{\alpha_{svk}} = \varepsilon_{\alpha_{sv(k-1)}}$ for $k = K_s$, showed that the first-order random walk exhibited lower DIC (Deviance Information Criterion), and thus it was considered the more appropriate structure to statistically model rail track geometry degradation.

Moreover, for the disturbance effect of the initial quality after each tamping operation $(\delta_{sv})$, we define a typical probability structure expressing vague information on that parameter, i.e. $\delta_{sv} \sim \mathrm{N}(0, \sigma_\delta^2)$. Finally, for each variance component in each hierarchical structure, we finalize by assigning inverse gamma distributions to each component, i.e. $\sigma_s^2 \sim \mathrm{IG}(c_0, d_0)$, $\sigma_\delta^2 \sim \mathrm{IG}(c_1, d_1)$, $\sigma_\alpha^2 \sim \mathrm{IG}(c_2, d_2)$, $\sigma_\beta^2 \sim \mathrm{IG}(c_3, d_3)$, $\sigma_{\alpha'}^2 \sim \mathrm{IG}(c_4, d_4)$ and $\sigma_{\beta'}^2 \sim \mathrm{IG}(c_5, d_5)$, where $\mathrm{IG}(c, d)$ denotes an inverse gamma distribution with shape parameter $c$ and scale parameter $d$, whose density is proportional to $x^{-(c+1)} \exp\left(-\frac{d}{x}\right)$, $x > 0$. The choice of assigning inverse gamma distributions 'is an attempt at non-informativeness within the conditional conjugate family' [27], which mainly translates into full conditional posterior distributions for each variance component within the same distributional family, i.e. also inverse gamma distributions, as later seen in the Appendix for i), iii), v), vii), ix) and xi). This choice is not only attractive for pedagogical purposes, but it is also a common choice in many BUGS software applications.

Therefore, to derive the joint posterior distribution, prior independence is usually assumed amongst the model parameters so that the joint posterior density is then proportional to:

$$\prod_{s=1}^{S}\prod_{v=1}^{V_s}\prod_{k=1}^{K_s}\prod_{l=1}^{L_s}\left\{\frac{1}{\sigma_s}\exp\left(-\frac{1}{2}\left(\frac{y_{svkl}-m_{svkl}}{\sigma_s}\right)^2\right)\right\}\cdot\prod_{s=1}^{S}\left\{\left(\frac{1}{\sigma_s^2}\right)^{c_0+1}\exp\left(-\frac{1}{\sigma_s^2}d_0\right)\right\}\cdot$$
$$\prod_{s=1}^{S}\prod_{v=1}^{V_s}\left\{\frac{1}{\sigma_\delta}\exp\left(-\frac{1}{2}\left(\frac{\delta_{sv}}{\sigma_\delta}\right)^2\right)\right\}\cdot\left\{\left(\frac{1}{\sigma_\delta^2}\right)^{c_1+1}\exp\left(-\frac{1}{\sigma_\delta^2}d_1\right)\right\}\cdot$$
$$\prod_{s=1}^{S}\prod_{v=1}^{V_s}\prod_{k=1}^{K_s}\left\{\frac{\sqrt{n_{svk}}}{\sigma_\alpha}\exp\left(-\frac{1}{2}\left(\frac{\varepsilon_{\alpha_{svk}}-\bar{\varepsilon}_{\alpha_{svk}}}{\sigma_\alpha}\sqrt{n_{svk}}\right)^2\right)\right\}\cdot\left\{\left(\frac{1}{\sigma_\alpha^2}\right)^{c_2+1}\exp\left(-\frac{1}{\sigma_\alpha^2}d_2\right)\right\}\cdot$$
$$\prod_{s=1}^{S}\prod_{v=1}^{V_s}\prod_{k=1}^{K_s}\left\{\frac{\sqrt{n_{svk}}}{\sigma_\beta}\exp\left(-\frac{1}{2}\left(\frac{\varepsilon_{\beta_{svk}}-\bar{\varepsilon}_{\beta_{svk}}}{\sigma_\beta}\sqrt{n_{svk}}\right)^2\right)\right\}\cdot\left\{\left(\frac{1}{\sigma_\beta^2}\right)^{c_3+1}\exp\left(-\frac{1}{\sigma_\beta^2}d_3\right)\right\}\cdot$$
$$\prod_{s=1}^{S}\prod_{v=1}^{V_s}\prod_{k=1}^{K_s}\left\{\frac{\sqrt{n_{svk}}}{\sigma_{\alpha'}}\exp\left(-\frac{1}{2}\left(\frac{\varepsilon_{\alpha'_{svk}}-\bar{\varepsilon}_{\alpha'_{svk}}}{\sigma_{\alpha'}}\sqrt{n_{svk}}\right)^2\right)\right\}\cdot\left\{\left(\frac{1}{\sigma_{\alpha'}^2}\right)^{c_4+1}\exp\left(-\frac{1}{\sigma_{\alpha'}^2}d_4\right)\right\}\cdot$$
$$\prod_{s=1}^{S}\prod_{v=1}^{V_s}\prod_{k=1}^{K_s}\left\{\frac{\sqrt{n_{svk}}}{\sigma_{\beta'}}\exp\left(-\frac{1}{2}\left(\frac{\varepsilon_{\beta'_{svk}}-\bar{\varepsilon}_{\beta'_{svk}}}{\sigma_{\beta'}}\sqrt{n_{svk}}\right)^2\right)\right\}\cdot\left\{\left(\frac{1}{\sigma_{\beta'}^2}\right)^{c_5+1}\exp\left(-\frac{1}{\sigma_{\beta'}^2}d_5\right)\right\}\cdot$$
$$\prod_{s=1}^{S}\prod_{v=1}^{V_s}\{P[\alpha_{sv}]\cdot P[\beta_{sv}]\cdot P[\alpha'_{sv}]\cdot P[\beta'_{sv}]\} \quad (2)$$

In which:

$m_{svkl}=[\alpha_{svk}(1+\delta_{sv})^{N_{svkl}}+\beta_{svk}T_{svkl}]\cdot R_{svkl}+[\alpha'_{svk}(1+\delta_{sv})^{N_{svkl}}+\beta'_{svk}T_{svkl}]\cdot(1-R_{svkl})$, $\alpha_{svk}=\alpha_{sv}+\varepsilon_{\alpha_{svk}}$, $\alpha'_{svk}=\alpha'_{sv}+\varepsilon_{\alpha'_{svk}}$, $\beta_{svk}=\beta_{sv}+\varepsilon_{\beta_{svk}}$ and $\beta'_{svk}=\beta'_{sv}+\varepsilon_{\beta'_{svk}}$. In order to ensure that the CAR model structures are identifiable, we follow the typical constraint suggested by Besag and Kooperberg [28] that is to impose that the $\sum_k \varepsilon_{\alpha_{svk}}=0$ and use a flat prior for the constant $\alpha_{sv}$ on the whole real line. This is also adopted for $\beta$, $\alpha'$ and $\beta'$ CAR structures. Note that flat priors are improper distributions (i.e. do not integrate to one, but assume a constant value everywhere, attempting to describe vague or no prior information on that parameter).

As the joint posterior is rather complex, the full conditional posterior distribution is derived in the Appendix, so that a Gibbs sampling strategy can iteratively draw for each parameter and use them as current values for each conditional posterior distribution. In terms of simulation details, we found that the proposed Gibbs sample is stable for MCMC samples were of size 20,000, taking every tenth iteration (thin=10) of the simulated sequence, after 10,000 iterations of burn-in period. Initial values were set: for the variance terms $\sigma_s^2$, $\sigma_\alpha^2$, $\sigma_\beta^2$, $\sigma_{\alpha'}^2$, $\sigma_{\beta'}^2$ and $\sigma_\delta^2$ equal to 10 (which is equivalent to precision $(1/\sigma^2)$ equal to 0.1), for the spatially correlated terms $\varepsilon_{\alpha_{svk}}$, $\varepsilon_{\beta_{svk}}$, $\varepsilon_{\alpha'_{svk}}$ and $\varepsilon_{\beta'_{svk}}$ equal to 0, and finally, for each parameter $\alpha_{sv}$, $\beta_{sv}$, $\alpha'_{sv}$, $\beta'_{sv}$ and $\delta_{sv}$ equal to 0..

A simple analysis of some results from the application of the proposed hierarchical Bayesian models explored in the previous section on a representative segment of a sample from the historical data from the main Portuguese line (Lisbon-Oporto). This historical data mainly refers to: i) the inspection records from the EM 120 vehicle to get the standard deviation of longitudinal level defects with respect to 200-m long track sections ($y_{svkl}$), ii) the operation records to get the accumulated tonnage ($T_{svkl}$), and finally to iii) the maintenance records to get the past maintenance and renewal actions ($N_{svkl}, R_{svkl}$).

Table II provides estimates for the posterior parameters for a representative track segment.

Table II – Estimates of the posterior parameters for a representative track segment.

| $\alpha_{sv}$ (mm) | | $\beta_{sv}$ (mm/100MGT) | | $\alpha'_{sv}$ (mm) | | $\beta'_{sv}$ (mm/100MGT) | | $\delta_{sv}$ | |
|---|---|---|---|---|---|---|---|---|---|
| Mean | s.d. | Mean | s.d. | Mean | s.d. | Mean | s.d. | Mean | s.d. |
| 0.3102 | 0.012 | 1.460 | 0.081 | 1.381 | 0.015 | 6.247 | 0.179 | 0.0015 | 0.009 |

Regarding Table II, note that both the parameters related to renewal track sections, i.e. initial quality $\alpha_{sv}$ and deterioration rate $\beta_{sv}$ are respectively lower than for the non-renewed track sections. In fact, the deterioration rate for non-renewed track sections are on average at least four times higher than the deterioration rates for renewed track sections, whereas the initial quality for non-renewed track sections seem to be at least four times higher than for the renewed track sections. Moreover, note that the value for the disturbance effect seem to be very close to zero.

# 4- DISCUSSING TYPICAL DECISION PROCESSES

This section will discuss Decision Processes, particularly the Markov Decision Process (MDP) and its two main extensions: i) the Partially Observed Markov Decision Process (POMDP) and ii) the Markov Decision Process with Imprecise Probabilities (MDPIP). Finally, we put forward our Bayesian Decision Process (BDP) for maintenance and renewal of rail track geometry, discussing it with the typical machinery and concepts used for a MDP and highlighting the main differences between this BDP and the MDP, the POMDP and the MDPIP.

A Markov Decision Process (MDP) is a model for sequential decision making under uncertainty, which takes into account both the outcomes of current decisions and future decision making opportunities (Puterman 2005). Its key ingredients are:

1) A set of decision epochs or periods – $t \in \{1, 2, \dots, T\}$
2) A set of system states – $s \in \{s_1, s_2, \dots, s_N\}$
3) A set of available actions – $a \in \{a_1, a_2, \dots, a_M\}$
4) A set of state and action dependent immediate rewards or costs – $r(s, a)$
5) A set of state and action dependent transition probabilities – $p(s'|s, a)$

At each decision epoch, the decision maker (or agent) will choose an action $a$ from the set of available actions, and then the decision maker will receive a reward/cost $r(s, a)$ and the system will evolve to a possibly different state according to $p(s'|s, a)$. Note that the rewards/costs and the transition probabilities depend on both the state and the action chose by the decision maker. Then, one is interested to find decision rules $d$, which specify the action to be chosen at a particular time, depending on the current state or on the history of previous states and actions. Decision rules can be history-dependent or markovian, and deterministic or randomized. A decision rule is history-dependent if it depends on the past history of the system $h_t = (s_1, a_1, \dots, s_{t-1}, a_{t-1}, s_t)$, where history $h_t$ follows the recursion $h_t = (h_{t-1}, a_{t-1}, s_t)$. If the decision rule only depends on the current state of the system - $s_t$, then it is a markovian decision rule. In parallel, a deterministic decision rule specifies an action with certainty, whereas a randomized decision rule specifies a probability distribution on the set of actions. Finally, a policy or strategy $\pi$ specifies the decision rules to be used at all decision epochs, i.e. $\pi = (d_1, d_2, \dots, d_{N-1})$. A stationary policy or strategy is a policy that prescribes the same decision rules at all epochs, i.e. $\pi = (d, \dots, d)$.

The MDP is a very popular decision process due to the simple representation of its system dynamics. By assuming the Markovian property, i.e. $p(s_{t+1}|h_t) = p(s_{t+1}|s_t, a_{t-1})$, meaning that the probability of the next state conditioned on all the previous history of states and actions is equal to the probability only conditioned on the previous state and action, and $p(s_{t+1}|s_t, a_{t-1})$ is stationary if it does not depend on epoch $t$, and the transition probability can be defined $p(s'|s, a)$. Considering a discrete set of states, if we call $P_a$ to the transition probability matrix with $(s, s')$th component $p(s'|s, a)$, the Markovian property is very useful because it reduces the computation of the m-step transition probability $p(s_{t+m}|s_t, a)$ under the same action $a$ to the $(s, s')$th component of matrix ${P_a}^m$.

Typically one would be interested to assess the value of a certain policy and maximize it according to the preferences of the decision maker. One major distinction between various MDP's is the finite-horizon case ($T < \infty$) or the infinite-horizon case. Moreover, a discount factor $\lambda$ is usually considered to account for the different time of rewards, where $0 \leq \lambda < 1$, and it quantifies the value at period $t$ of a unit reward received in period $t + 1$. It plays the same role as the discount rate considered in typical project finance exercises. Finally, different optimality criteria can be defined, such as: the expected total reward criterion, the expected total discounted reward criterion or the average reward criterion.

After discussing above the typical concepts used in MDP, let us now explore the two extensions of the MDP. The Partially Observed Markov Decision Process (POMDP) is an extension of MDPs, in which we relax the assumption of certainty in the observed state, i.e. we assume that we do not observe directly the system state $s$, but instead a system output $o$ that has some relation with the true (unobserved) system state $s$, through an observation/measurement probability distribution $p(o|s, a)$. Smallwood and Sondik (1973) proposed one algorithm to find an optimal control for the POMDP for a finite horizon. Sondik (1978) extended the proposed POMDP to the infinite horizon case with discounted costs. The POMDP has had much attention of the researchers in many areas of application, from machine learning, artificial intelligence and also to infrastructure management. A very important work on this matter is the assessment of the introduction of new monitoring/inspection technologies with more accuracy as technology cycles evolve. The quantification of potential benefits/savings in a maintenance and renewal strategy can be quantified through a POMDP, as it was proposed in Madanat (1993).

According to Satia and Lave (1973), the Bayesian formulation for the MDP with uncertain transition probabilities was suggested for the first time by Silver (1963). The basic assumption within the Bayesian formulation for the MDP is that there is a prior probability distribution of transition probability rows $\boldsymbol{p_i^a}$, which is successively updated as the process evolves through the Bayes' rule. In simpler cases, where we assume that the transition probability matrix P has rows $\boldsymbol{p_i^a} = (p_{i1}^a, p_{i2}^a, \dots p_{iN}^a)$ which a priori follow a Dirichlet distribution, i.e. $\boldsymbol{p_i^a} \sim Dir(\boldsymbol{\alpha_i^a})$ with parameters $\boldsymbol{\alpha_i^a} = (\alpha_{i1}^a, \alpha_{i2}^a, \dots \alpha_{iN}^a)$, it can be shown that the posterior distribution also follows a Dirichlet distribution, i.e. $\boldsymbol{p_i^a}|\boldsymbol{x} \sim Dir(\boldsymbol{\alpha_i^{a\prime}})$, with updated parameters $\boldsymbol{\alpha_i^{a\prime}} = \boldsymbol{\alpha_i^a} + \boldsymbol{x}$, since the Dirichlet distribution is the conjugate prior distribution of the multinomial distribution $\boldsymbol{X}|\boldsymbol{p_i^a}$. This model is defined as Markov Decision Process with Imprecise transition Probabilities (MDPIP), but it has been mostly used assuming inequality conditions on the uncertain probabilities, which does not follow the Bayesian formulation. For more details, see White and Eldeib (1986 and 1994) and Nilim and El Ghaoui (2005).

When above-described POMDP and MDPIP are compared with the proposed Decision Process discussed in section 5, we find that we assume perfect inspection, meaning that there is no distinction between the observed outputs and the internal state of the system, as it is defined in the context of the POMDP. However, we will assume uncertainty associated with the parameters specifying the transition probabilities and thus the Hierarchical Bayesian model is playing the same role as the Bayesian formulation proposed by Satia and Lave (1973), though at a more complex level.

# 5- OPTIMIZING MAINTENANCE AND RENEWAL STRATEGY

This section will first identify the main ingredients of our proposed Bayesian Decision Process, using the concepts defined in section 4, and then apply them to the particular infrastructure management problem of searching for an optimal maintenance and renewal strategy due to rail track geometry degradation.

## States and Actions

The system states $s$ will be defined by the standard deviation of longitudinal levelling defects - $y_{svkl}$ as described in Section 4, where the subscript $l$ is again relative to the inspection record which is modelled as a decision period. Moreover, the level of maximum permissible speed is also part of the system state. As a simplification, five levels of maximum permissible speed are considered in light of the upper bounds of Table I for each speed group, i.e. 80, 120, 160, 230 and 300 km/h, which is modelled as $Z_{svl} = 0, 1, 2, 3, 4$ respectively for inspection $l$, track segment $v$ from area $s$. Therefore, a system state is defined by the combination of values $(y_{svkl}, Z_{svl})$ for every track section $k$ from track segment $v$ from area $s$, or simpler terms the combination $(y, Z)$, where we let the subscripts fall for simplicity.

## Model of dynamics

In terms of model dynamics, Section 3 provided a brief overview on the underlying degradation model – the Hierarchical Bayesian model to predict rail track geometry degradation. It is important to refer that a first approach was put forward in the WCTR 2010, in which the authors assess rail track geometry degradation, focusing in the Bayesian learning mechanism as new inspection data becomes available (Andrade and Teixeira

2012). Then, the authors provided a full revision of this first approach using a Hierarchical Bayesian model to include the spatial correlation structures between the degradation parameters of aligned and consecutive track sections. (Andrade and Teixeira forthcoming_b).

The Bayesian Hierarchical model allows assessing the posterior predictive distribution for future degradation $y_{svkl}$, given past history of states and actions, as well as for a proposed strategy on future actions $(N, R)$, through Gibbs sampling procedure contained in the Appendix. As this step would result in an unbearable computational time, particularly for a large network, a reasonable horizon and for every combination of future $(N, R)$ actions, we will make some simplifications.

## Reward (cost) model

First of all, any proposed reward model should take into account the preferences of the decision maker, and then assess, as objectively as possible, the impacts of the system in state s as we choose action a. In this case, we are facing a kind of cost model with negative rewards, and the objective will be to minimize these impacts.
Therefore, the main components of a cost model would be:

- Planned renewal costs
- Planned maintenance costs
- Unplanned maintenance costs
- Planned delay costs.
- Unplanned delay costs.

One important limitation is the consideration of deterministic values for every cost component, considering that there are no uncertainties associated with costs despite the one related with geometry degradation itself.

In terms of renewal and planned maintenance costs, we will assume that the renewal cost of a km of rail track is $c_R$ and that the planned maintenance cost is $c_{PM}$. In terms of unplanned maintenance costs, a very important step was previously taken in Andrade and Teixeira (forthcoming_a), where in succinct terms it was found that the probability that a given track section needed unplanned maintenance could be modelled through a logistic regression using as covariates the standard deviations of longitudinal levelling ($SD_{LL}$) and of horizontal alignment ($SD_{HA}$) defects, as well as the dummy variables: bridges (B) and switches (S), to include the effect of the presence of bridges and switches in a track section. We also showed that the coefficients would vary depending on the maximum permissible speed, and separate logistic regressions were estimated for each track section group and each quality limit (AL, IL and IAL). For instance, the probability that a given track section needs unplanned maintenance in the group 120-160 km/h (considering the IL limit as the criterion for all other indicators besides $SD_{LL}$ and $SD_{HA}$) can be estimated by the expression (Andrade and Teixeira (forthcoming_a):

$$p_{IL}(SD_{LL}, SD_{HA}, B, S) = \frac{1}{1+e^{-(-4.3+0.6\,SD_{LL}+1.0\,SD_{HA}-0.3\,B)}} \quad (3)$$

Other speed groups will have similar logistic expressions with different coefficients. Therefore, a straightforward way to estimate unplanned maintenance costs is considering the $SD_{LL}$ (or $y$ estimated from the Bayesian Hierarchical model) and for a certain speed group $(Z)$, i.e. with specific coefficients (-6.6, 0.7, 1.5, -0.6, 0.0) depending on the speed group, i.e. if we consider a fixed cost of unplanned maintenance $c_{UM}$, the expected cost of unplanned maintenance needs would be $p_{IL}.c_{UM}$.

In terms of delay costs, a major simplification is assumed: delay costs involved in this problem are due to infrastructure delays, and not due to operators' delays, nor to passengers' delays. This classification follows a regulatory perspective over the delay allocation component to the different agents responsible to cause it. Actually, this is a current research gap. Therefore, we assume that planned and unplanned delays are infrastructure delays from the responsibility of the Infrastructure Manager and are ideally penalized through a performance regime contracted with the IM and the regulatory entity. In the Portuguese case, the regulatory entity IMTT-URF for the first time (in June 2010) fixed a delay cost $(c_D)$ of 4 €/min for every passenger train and a 1.33 €/min for every freight train (IMTT-URF (2010)), and more recently refined these costs, distinguishing urban passenger trains from regional and intercity trains, respectively 4 €/min and 2.4 €/min; whereas the freight trains saw their delay costs reduced to 0.2 €/min (IMTT-URF (2011)). From our perspective, this regulatory signal had the purpose to potential the acceptability towards the delay penalty among the different operators, though it still seems low comparing it to the potential economic impacts for the end users/passengers.

Therefore, in terms of planned (infrastructure) delay costs, we proposed in Andrade and Teixeira (2011b) within a bi-objective optimization model that the planned delays $(D_P)$ should be computed in a simplified way (i.e. without the use of simulation software like Opentrack) as:

$$D_P = \sum_{s,v} D_{P_{sv}} = \sum_{s,v} \sum_{train} L_{sv} \left( \frac{1}{Sp_{sv}{}^{train}(Z)} - \frac{1}{Sp_{sv}{}^{train}} \right) \quad (4)$$

In which: $L_{sv}$ is the length of the track segment $v$ from area $s$, $Sp_{sv}{}^{train}$ is the maximum possible speed regarding the features of the infrastructure and the train, and $Sp_{sv}{}^{train}(Z)$ is the maximum speed regarding the choice of $Z$ (the speed group that track geometry defects should comply).

Note that the computation of $D_P$ only depends on the choice of speed group – $Z$ and not on the standard deviation of longitudinal level defects – y. Finally, the computation of the planned delay costs would simply be $c_D D_P$ with the necessary conversion of units, or if one is interested in distinguishing different trains - $\sum_{train} c_D{}^{train} D_P{}^{train}$.

Finally, in terms of unplanned (infrastructure) delay costs, we assume that there is a similar logistic expression as the one before for $p_{IL}(SD_{LL}, SD_{HA}, B, S)$, due to unplanned maintenance needs, but here it is the IAL limit that is at stake, i.e. $p_{IAL}(SD_{LL}, SD_{HA}, B, S)$. For instance, the probability that a given track section needs speed restrictions causing unplanned maintenance delays in the group 120-160 km/h (considering the IL limit as the criterion for all other indicators besides $SD_{LL}$ and $SD_{HA}$) can be estimated by the expression (Andrade and Teixeira (forthcoming_a):

$$p_{IAL}(SD_{LL}, SD_{HA}, B, S) = \frac{1}{1+e^{-(-6.6+0.7\,SD_{LL}+1.5\,SD_{HA}-0.6\,B)}} \quad (5)$$

Therefore, if one assumes a typical value for an unplanned delay or a delay due to speed restriction equal to $D_U$, the expected unplanned delay cost would be $p_{IAL} c_D D_U$.

Note that these empirical relations with the different costs components depend on the distinction of planned and unplanned maintenance, which is also in the hands of the Infrastructure Manager. However, we believe that the planned maintenance criteria should be triggered by the standard deviations of longitudinal leveling and horizontal alignment defects at the Alert (AL) or Intervention Limits (IL), whereas the unplanned maintenance should be triggered by all indicators of track geometry defects at the Intervention Limits (IL). Finally, if the Immediate Action Limits (IAL) is reached by any indicator, then speed restrictions take place and thus, unplanned delays are affected. But, again, this distinction is in the hands of each Infrastructure Manager.

Another potential cost would be the inspection costs. However, we assume that any maintenance and renewal strategy would assume the same inspection costs, and thus this component is not included in the cost function.

## Objective criterion

The main objective criterion is then the minimization of the total expected discounted costs:

$$J^* = \max_{\pi} E[\textstyle\sum_{t=1}^{T} \lambda^{t-1}. r(s_t, a_t)] = \min_{\pi} E[\textstyle\sum_{t=1}^{T} \lambda^{t-1}. C_{total}(y_t, Z_t, a_t)] \quad (6)$$

In which: $\lambda$ is the discount factor and the function $C_{total}$ comprises the above-mentioned cost components, i.e. renewal costs, planned maintenance costs, unplanned maintenance costs, and planned delay costs and unplanned delay costs.

However, as the proposed decision model is not Markov, the optimal value $J^*$ and the associated optimal strategy cannot be computed in a straightforward way using the typical algorithms in the MDP context, such as the value iteration, policy iteration or modified policy iteration algorithms (see Puterman (2005) for further details), and thus we need a practical approach to search for an optimal strategy.

Therefore, our main simplification comes from an idea to search for an optimal strategy of the type of a control limit policy, typical in inventory models using policies with this simple structure. A control limit policy (Puterman 2005) is a deterministic Markov policy composed of decision rules of the form:

$$d_t(s) = \begin{cases} a_1, & s < s^* \\ a_2, & s \geq s^* \end{cases} \quad (7)$$

In which, $s^*$ is a control limit and $a_1$ and $a_2$ are distinct actions. The decision rule $d_t(s)$ should be interpreted as it is optimal to choose action $a_1$ when the system is at a state $s$ less than a certain limit $s^*$, and it is optimal to choose $a_2$ when the system is at a state equal or greater than $s^*$. Although there is no guarantee that the optimal policy for the problem described in this paper, the control limits policy is a very important structured policy because it is easily implemented and thus it is appealing to decision makers and allows a more efficient computation.

Intuitively, one may perceive the limit $s^*$ as the limit AL in hands of the Infrastructure Manager and try to compute $J_\pi$ for stationary policies with limits $s^*$ varying within the limit proposed in Table I for each speed group.

# 6- FUTURE EXPERIMENTAL RESULTS

This section explores the ideas for future experiments conducted using the model discussed in section 5. This section will be extended in future versions/reviews.

Take for instance, the example of a track section from the speed group 120-160 km/h so that the expressions for $p_{IL}$ and $p_{IAL}$ above presented are valid, and assume a train demand model corresponding to a constant annual tonnage of 10 MGT/year and a constant

For the evolution of rail track geometry, we run the Hierarchical Bayesian model on a particular exemplifying area. It mainly involves a double-track $(\mathrm{V_s} = 2)$ area of about 15 km $(\mathrm{K_s} = 74)$, from a total of 36 inspections $(\mathrm{L_s} = 36)$ for past inspection data, and using it to predict the next future values of $y_{svk}$ for the next 120 inspections onwards under a control limit policy of type $d_t(s)$ for choosing values for $(N, R)$. As the system is not Markovian, this is done in a step-by-step procedure where the posterior predicted mean for the next 120 inspections is used – $m_{svk}$. First, we set the AL limit or the $s^*$ control limit and when the first posterior predicted mean $m_{svk}$ is equal or greater than the AL limit, we add 1 to the previous N so that a maintenance action is performed. We re-run the hierarchical Bayesian model and get new predictions for future $m_{svk}$. We then stop as all the 120 predicted values respect the policy analysed. Then, using those predictions we compute $J$. We then vary the $s^*$ control limit, so that we can compute $J$ functions for each control limit $s^*$ within the limit in Table I, i.e. from 1.4 to 2.4.

We will then conduct sensitivity analysis on delay cost coefficients (e.g. 4€/min), i.e. on the signal that the regulatory entity can transmit to influence the maintenance and renewal strategy.

# 7- CONCLUSIONS AND FURTHER RESEARCH

At the research level, this is an innovative contribution to railway maintenance and renewal planning using a Bayesian Decision Problem approach. At the practice level, this framework would support the revision of the proposed limits for AL and IL values for the planned maintenance criteria in the Portuguese standard IT.VIA.018, balancing the dimensions referred in the EN 13848. At the policy level, we believe that our findings can suggest that regulatory entities would play an important role in the quantification of the value of time and in the definition of an optimized strategy.

Preliminary results have shown that two indicators of rail track geometry degradation, namely the standard deviations of longitudinal levelling and horizontal alignment defects (both filtered in the wavelength range 3-25 m), are not only the usual indicators for planned maintenance actions, but they are also reasonable predictors for unplanned maintenance actions regarding all track geometry defects. This paper still lacks a comprehensive exploration of the proposed model, which due to lack of time will be completed in the next versions/revisions.

Future research should also focus on a more solid theoretical foundation to enhance these improvements on railway infrastructure management, rather than simply on simulation results of the proposed model.

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# APPENDIX

Let $\theta$ be the vector of the model parameters, with elements $\sigma_s^2$, $\delta_{sv}$, $\sigma_\delta^2$, $\varepsilon_{\alpha_{svk}}$, $\sigma_\alpha^2$, $\varepsilon_{\beta_{svk}}$, $\sigma_\beta^2$, $\varepsilon_{\alpha'_{svk}}$, $\sigma_{\alpha'}^2$, $\varepsilon_{\beta'_{svk}}$, $\sigma_{\beta'}^2$, $\alpha_{sv}$, $\beta_{sv}$, $\alpha'_{sv}$ and $\beta'_{sv}$, with $s = 1, \dots, S$, $v = 1, \dots, V_s$, $k = 1, \dots, K_s$ . From the joint posterior, one can derive the full conditional posterior distributions (denoted below by $[j|\theta_{-j}]$), which are given by:

i) $\sigma_s^2|\theta_{-\sigma_s^2} \sim IG\left(c_0 + \frac{1}{2}V_sK_sL_s, d_0 + \frac{1}{2}\sum_{v,k,l}(y_{svkl} - m_{svkl})^2\right)$, $s = 1, \dots, S$;

ii) $\delta_{sv}|\theta_{-\delta_{sv}} \propto \exp\left(-\frac{1}{2\sigma_\delta^2}\delta_{sv}^2 - \frac{1}{2\sigma_s^2}\sum_{k,l}(y_{svkl} - m_{svkl})^2\right)$, $s = 1, \dots, S$, $v = 1, \dots, V_s$;

iii) $\sigma_\delta^2|\theta_{-\sigma_\delta^2} \sim IG\left(c_1 + \frac{1}{2}\sum_s V_s, d_1 + \frac{1}{2}\sum_{s,v}\delta_{sv}^2\right)$;

iv) $\varepsilon_{\alpha_{svk}}|\theta_{-\varepsilon_{\alpha_{svk}}} \propto \exp\left(-\frac{n_{svk}}{2\sigma_\alpha^2}\left(\varepsilon_{\alpha_{svk}} - \bar{\varepsilon}_{\alpha_{svk}}\right)^2 - \frac{1}{2\sigma_s^2}\sum_{s,v,k,l}(y_{svkl} - m_{svkl})^2\right)$, $s = 1, \dots, S$,

$v = 1, \dots, V_s$, $v = 1, \dots, V_s$;

v) $\sigma_\alpha^2|\theta_{-\sigma_\alpha^2} \sim IG\left(c_2 + \frac{1}{2}\sum_s V_sK_s, d_2 + \frac{1}{2}\sum_{s,v,k} n_{svk}\left(\varepsilon_{\alpha_{svk}} - \bar{\varepsilon}_{\alpha_{svk}}\right)^2\right)$

vi) $\varepsilon_{\beta_{svk}}|\theta_{-\varepsilon_{\beta_{svk}}} \propto \exp\left(-\frac{n_{svk}}{2\sigma_\beta^2}\left(\varepsilon_{\beta_{svk}} - \bar{\varepsilon}_{\beta_{svk}}\right)^2 - \frac{1}{2\sigma_s^2}\sum_{s,v,k,l}(y_{svkl} - m_{svkl})^2\right)$, $s = 1, \dots, S$,

$v = 1, \dots, V_s$, $v = 1, \dots, V_s$;

vii) $\sigma_\beta^2|\theta_{-\sigma_\beta^2} \sim IG\left(c_2 + \frac{1}{2}\sum_s V_sK_s, d_2 + \frac{1}{2}\sum_{s,v,k} n_{svk}\left(\varepsilon_{\beta_{svk}} - \bar{\varepsilon}_{\beta_{svk}}\right)^2\right)$

viii) $\varepsilon_{\alpha'_{svk}}|\theta_{-\varepsilon_{\alpha'_{svk}}} \propto \exp\left(-\frac{n_{svk}}{2\sigma_{\alpha'}^2}\left(\varepsilon_{\alpha'_{svk}} - \bar{\varepsilon}_{\alpha'_{svk}}\right)^2 - \frac{1}{2\sigma_s^2}\sum_{s,v,k,l}(y_{svkl} - m_{svkl})^2\right)$, $s = 1, \dots, S$,

$v = 1, \dots, V_s$, $v = 1, \dots, V_s$;

ix) $\sigma_{\alpha'}^2|\theta_{-\sigma_{\alpha'}^2} \sim IG\left(c_2 + \frac{1}{2}\sum_s V_sK_s, d_2 + \frac{1}{2}\sum_{s,v,k} n_{svk}\left(\varepsilon_{\alpha'_{svk}} - \bar{\varepsilon}_{\alpha'_{svk}}\right)^2\right)$

x) $\varepsilon_{\beta'_{svk}}|\theta_{-\varepsilon_{\beta'_{svk}}} \propto \exp\left(-\frac{n_{svk}}{2\sigma_{\beta'}^2}\left(\varepsilon_{\beta'_{svk}} - \bar{\varepsilon}_{\beta'_{svk}}\right)^2 - \frac{1}{2\sigma_s^2}\sum_{s,v,k,l}(y_{svkl} - m_{svkl})^2\right)$, $s = 1, \dots, S$,

$v = 1, \dots, V_s$, $v = 1, \dots, V_s$;

xi) $\sigma_{\beta'}^2|\theta_{-\sigma_{\beta'}^2} \sim IG\left(c_2 + \frac{1}{2}\sum_s V_sK_s, d_2 + \frac{1}{2}\sum_{s,v,k} n_{svk}\left(\varepsilon_{\beta'_{svk}} - \bar{\varepsilon}_{\beta'_{svk}}\right)^2\right)$

xii) $\alpha_{sv}|\theta_{-\alpha_{sv}} \propto \exp\left(\sum_{k,l}(y_{svkl} - m_{svkl})^2\right) \cdot P[\alpha_{sv}]$, $s = 1, \dots, S$, $v = 1, \dots, V_s$;

xiii) $\beta_{sv}|\theta_{-\beta_{sv}} \propto \exp\left(\sum_{k,l}(y_{svkl} - m_{svkl})^2\right) \cdot P[\beta_{sv}]$, $s = 1, \dots, S$, $v = 1, \dots, V_s$;

xiv) $\alpha'_{sv}|\theta_{-\alpha'_{sv}} \propto \exp\left(\sum_{k,l}(y_{svkl} - m_{svkl})^2\right) \cdot P[\alpha'_{sv}]$, $s = 1, \dots, S$, $v = 1, \dots, V_s$;

xv) $\beta'_{sv}|\theta_{-\beta'_{sv}} \propto \exp\left(\sum_{k,l}(y_{svkl} - m_{svkl})^2\right) \cdot P[\beta'_{sv}]$, $s = 1, \dots, S$, $v = 1, \dots, V_s$;