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Empirical Investigation of the Existence of Class-Wise Fundamental Diagram for a Heterogeneous Traffic Stream

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Abstract

Multiclass traffic flow models have achieved potential research attention due to the increased travel cost and congestion consequent to the interaction of multiple vehicle classes moving on the same road space. The existing theories and models are unable to explain the unique dynamics associated with the heterogeneous traffic stream. The main challenges in modeling such traffic stream are the varying physical and the dynamical properties of vehicles and dissimilarity in the response of vehicles classes towards the same traffic concentration. Since different types of vehicles occupy the same road space, the conventional measures of traffic concentration would not effectively characterize the heterogeneous traffic stream. Present study considers Area Density, Area Flow, and Road space Freeing Rate (RFR) for characterizing the heterogeneous, no lane-disciplined traffic stream. One of the primary requirements for the multiclass traffic flow modeling is the existence of the class-wise fundamental diagram. There is no empirical evidence available in the literature for the existence of the class-wise fundamental diagram. Present study critically investigates the existence of the class-wise fundamental diagram for an urban traffic stream composed of heterogeneous vehicle mix. The findings from this study indicate that there exists a class-wise fundamental relationship between the RFR of each vehicle class and the Area Density of that particular vehicle class as well as the total Area Density of the traffic stream.

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Keywords: Multiclass traffic model, Heterogenous Traffic, Macroscopic Traffic Models, Traffic Stream Characteristics

1. Introduction

Traffic flow theory for heterogeneous traffic stream is still in its infancy due to the limited understanding of the chaotic traffic dynamics. Such unique dynamics associated with heterogeneous traffic stream is the consequence of the

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presence of a variety of vehicle classes. Varying physical and dynamic properties of the vehicle classes lead to a gap-filling behavior, consequently the violation of the lane-discipline. Most of the past macroscopic traffic flow models were developed based on an assumption that the traffic is composed of homogeneous vehicles with comparable behavior and are essentially in the car-following conditions. Moreover, most of the efforts in building a mathematical relationship between the speed and the volume or density were applicable only to an uninterrupted traffic flow conditions (Jain and Coifman, 2005; May, 1994). Unfortunately, the existing models could not capture the unique dynamics of the heterogeneous traffic stream as they violate the above assumptions. Recently, the multiclass traffic flow models have achieved potential research interest due to its capability of describing the dynamics of different vehicles classes and their interactions. These models were able to describe the puzzling traffic phenomena such as the two capacity conditions, hysteresis, platoon dispersion, and so on, which were unanswered by the conventional models. Basically, the multiclass models subdivide the traffic flow into flows of different user classes and model the dynamics of each homogeneous subgroups and their interactions. Each class encompasses a group of driver-vehicle entities that entitles a similar speed choice. Positively, most of the existing multiclass traffic flow models capture the driver differences in a homogeneous traffic stream that lead to different speed choices (e.g., Gupta and Katiyar, 2007; Logghe and Immers, 2003; Wong and Wong, 2002).

For a heterogeneous traffic stream, that is prevalent in most of the developing countries, the traffic is composed of different vehicle classes that have varying physical as well as dynamical properties. No significant research attempts have been exclusively made to investigate the possibilities of multiclass modeling of such traffic stream considering both the varying physical as well as the dynamical properties of the vehicle classes. One of the difficulties in dealing with such a traffic stream is the direct relationship of the vehicles' physical dimensions to the traffic concentration. The conventional definitions of the traffic stream characteristics are unable to capture this aspect. Recently, Suvin and Mallikarjuna, (2018) have redefined Edie's generalized definitions (Edie, 1963) of the traffic stream characteristics by considering both the physical as well as the dynamic properties of the vehicles. A detailed discussion of the modified definitions is given in Section 4.

One of the fundamental assumptions of the multiclass traffic flow modeling is the existence of class-wises Fundamental Diagrams (FD). Though the multiclass traffic flow theory theoretically proves the existence of class-wise FD by considering the conservation of vehicles classes, the empirical existence of such a relationship is certainly a question. Furthermore, the multiclass consideration of the heterogeneous traffic stream is outside the scope of the existing framework of the multiclass traffic modeling. Hence, before endeavoring to adopt multiclass traffic flow theory for heterogeneous traffic flow conditions, it is important to critically investigate the existence of class-wise FD for a heterogeneous traffic stream. Present study hypothesizes that there exists a class-wise fundamental diagram for a heterogeneous, no lane-disciplined traffic stream and empirically investigates the validity of the hypothesis.

The remainder of the paper is organized as follows. Section 2 briefly reviews the multiclass traffic flow theory and its advancements. In Section 3, the data collection procedure and the post-processing of the data are described. Section 4 discusses the methodology of the present study. The findings from this study are discussed in Section 5. Section 6 summarizes and concludes the paper.

2. Background

Development of traffic flow models dates back to 1950's by the introduction of the first dynamic traffic flow model by Lighthill and Whitham (1955) and (Richards, 1956), hence called the LWR model. The LWR model describes the traffic on a link using the conservation law, an equilibrium speed-density relationship, and the fundamental equation of the traffic flow. This equilibrium relation $Q_e(k)$ is better known as the fundamental diagram for the traffic stream (Logghe and Immers, 2003). The LWR model is known to have limitations such as; i) the equilibrium speed-density relationship is the only mode to measure the speed and no fluctuations of speed around the equilibrium values are allowed (Gupta and Katiyar, 2007), ii) the LWR model is not applicable to the non-equilibrium conditions like stop and go, and heterogeneous traffic compositions (Logghe and Immers, 2008). Many efforts have been made on extending the LWR model to capture the various empirically observed traffic dynamics. In fact, several researchers (e.g., Kerner and Konhäuser, 1993; Payne, 1971; Phillips, 1979; Zhang, 1998) have suggested the higher order traffic flow models meant to overcome the drawbacks of the LWR model. Yet, these models could not explain some perplexing traffic phenomena observed on the highway, such as the two-capacity or reversed-lambda state, hysteresis, platoon dispersion, and so on.

In order to model the above dynamics, researchers started developing the multi-class continuum models considering different vehicle classes having varying dynamic properties (Fan and Work, 2015a, 2015b; Gupta and Katiyar, 2007; Hoogendoorn and Bovy, 2000; Logghe and Immers, 2008; Ngoduy, 2011; van Lint et al., 2008; van Wageningen-Kessels, 2016; van Wageningen-Kessels et al., 2014; Wong and Wong, 2002). Separating the user classes and their specific flow characteristics, researchers were able to improve the accuracy and the descriptive power of the macroscopic traffic flow models (Hoogendoorn and Bovy, 1998). However, the analyses were limited to a small set of specific problems and not all the currently known models were included in multiclass modelling (van Wageningen-Kessels, 2016).Wong and Wong (2002) presented a multiclass traffic flow model by extending the LWR model for a different user classes having different speed choices. Chanut and Buisson (2003) have contributed an approach in which vehicles are differentiated by their lengths as well as their speed choices in the free-flow conditions. Gupta and Katiyar (2007) proposed a new higher order continuum model by extending Berg's model (Berg et al., 2000) for a heterogeneous traffic stream. Gupta and Katiyar also differentiated the vehicle classes based on the speed choices. van Lint et al., (2008) proposed a new multiclass model named "FASTLANE" that specified the heterogeneity of the traffic stream in a more sensible way. FASTLANE differs from previous multiclass first-order macroscopic traffic models in estimating the traffic characteristics in terms of a state-dependent passenger-car equivalents. Later on van Wageningen-Kessels et al., (2014) showed the distinctive properties of the FASTLANE in a simulation environment. A numerical scheme for solving the multi-class extension of the LWR model was proposed by Zhang et al., (2009) which was a high-order weighted essentially non-oscillatory (WENO) scheme. Ngoduy, (2011) showed that both of the hysteresis transitions and the wide scattering can be reproduced by a multiclass first-order model with a stochastic setting in the model parameters. Nair et al., (2011) developed a multiclass traffic flow model for heterogeneous traffic without lane discipline based on an analogy of fluid flow through a porous medium. The model was able to explain the creeping behavior (special case of overtaking where small vehicles will be moving even after all the larger vehicles are totally stopped) of the smaller vehicles through the available gaps (pores) in the congested traffic stream. Fan and Work, (2015a) presented a multi class model to capture overtaking and creeping in highly heterogeneous traffic stream.

Having an efficient and reliable traffic flow model is the prerequisite for the development of an effective transport management system from a macroscopic perspective (Kotsialos and Papageorgiou, 2001). Empirical validation of the macroscopic models measures the degree of accuracy of any macroscopic traffic flow model (Papageorgiou, 1998). Thus, before employing a traffic flow model in practice, it is important to calibrate it against the real traffic data. Literature clearly states that, apart from the dynamic characteristics of the vehicle classes, it is important to consider the physical aspects of a vehicle class for a better representation of the heterogeneous traffic stream. It is also evident from the literature that the existence of class-wise fundamental diagram is an important criterion for the multi-class traffic flow modelling. However, no empirical evidence is shown in the literature for the existence of class-wise fundamental diagram. Most of the above modeling attempts were theoretical or assumed a certain fundamental diagram for the vehicle classes. Hence there is a clear scope for the research in the direction of the empirical investigation of the class-wise fundamental diagram after appropriately characterizing the heterogeneous traffic stream.

3. **Data Collection and Post-Processing**

Traffic data have been collected by recording the video footages from the heterogeneous, no lane-disciplined traffic stream. An urban midblock section located at Dispur, Assam was selected for the data collection. A camera was placed over a nearby foot-over-bridge in such a way that the camera field of view could cover the maximum road length. The video was recorded for a duration of 2-hours during which traffic was moving in both the free and forced conditions. From the video footages, the vehicle trajectories were extracted using an image-processing tool, the Traffic Data Extractor (TDE) (Munigety et al., 2014). The extracted trajectories were reconstructed using the methodology proposed by Suvin and Mallikarjuna, (2018b). Figure 1 shows a sample of the trajectory data considered in the present study. The average traffic composition observed during the 2-hr period was, 51.6% LMV, 32.3% Bike, 8.6% HMV, and 7.5% Auto- Rickshaw.

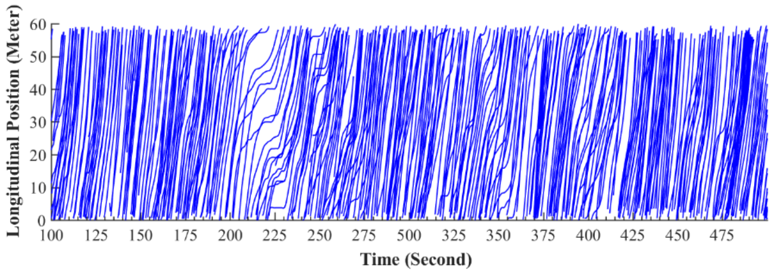


Figure 1: Sample Trajectory Data Considered in the Present Study

4. **Methodology**

This section is divided into three subsections. In the first part, a brief introduction of the concepts of the multiclass traffic flow theory is given. The second part discusses the process of the characterization of the heterogeneous traffic stream. In the third part, the calibration procedure of the fundamental diagram is explained.

Multiclass Traffic Flow Theory

Multiclass traffic flow theory states that the conservation equation could be applied separately to different vehicle classes. Besides, the multiclass traffic flow modeling also assumes the distribution of speed around the equilibrium speed, corresponding to a total density, due to the presence of '*M*' user classes with different speed choice behavior. It has been assumed that the variation of the speed around the mean speed decreases with the increasing density due to the tighter interaction between the vehicle classes. Total density corresponding to a multiclass traffic stream is given by,

$$\rho(x,t) = \sum_{m=1}^{M} \rho_m(x,\mathrm{t}) \tag{1}$$

Where, $\rho_m(x,\mathrm{t})$ is the density of user class '*m*' in the time space domain. The traffic stream characteristics such as the flow, speed, and the density of a particular vehicle class are related as,

$$q_m(x,t) = \rho_m(x,t) \times v_m(x,t) \qquad \forall m = 1,2,3,....,M \tag{2}$$

Based on the driving behavior of a particular vehicle class, the conservation law could be applied to each vehicle class as,

$$\frac{\partial \rho_m(x,t)}{\partial t} + \frac{\partial q_m(x,\mathrm{t})}{\partial x} = 0 \qquad \forall m = 1,2,3,....,M \tag{3}$$

Equation (3) states that the density changes according to the balance between the inflow and outflow of vehicles of user class *m* along a topographically homogeneous highway section.

As discussed earlier, the conventional measures of the traffic stream characteristics could not be applied to the heterogeneous traffic stream since the dimensions of the vehicle and the no lane-disciplined driving significantly influence the traffic characteristics. Moreover, the hindrance to the nearby traffic by the presence (time spent in the traffic stream) of a smaller vehicle (say, Bike) and a larger vehicle (say, Truck) would be significantly different. Considering this fact, the present study adopted the modified generalized definitions proposed by Suvin and Mallikarjuna, (2018b) for defining the traffic stream characteristics. A detailed discussion of the modified definitions is given in the following section.

Modified Generalized Definitions for Heterogeneous Traffic Stream

In order to capture the variability in the vehicles' dimensions and the no lane-disciplined driving actions, Suvin and Mallikarjuna, (2018b) have modified Edie's generalized definitions of the traffic stream characteristics by incorporating an additional dimension of the space. All the heterogeneous traffic dynamics were assumed to take place in a three-dimensional time-space continuum, which encompasses two dimensions of space and a time dimension. The width of the vehicle, as well as the road, has been incorporated into Edie's generalized definition for capturing the vehicle dimensions and the no lane-disciplined actions, respectively. Figure 2 shows the three-dimensional trajectory and the time-space continuum considered in the definitions. The newly defined traffic stream characteristics were named as the ‘Area Density’, ‘Area Flow’, and the ‘Road Space Freeing Rate (RFR)’.

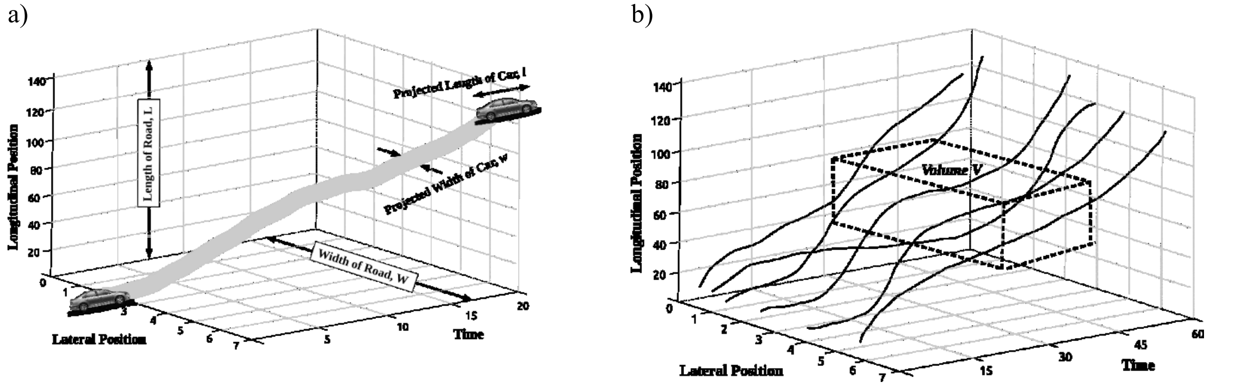


Figure 2: a) The Path Followed by a Car in the Time Space Continuum; b) Three Dimensional Trajectories and the Time Space Continuum

The area density that measures the crowdedness of the traffic and is defined as,

$$Area\ Density = \frac{Sum\ of\ the\ areas\ of\ the\ 'time\text{-}space'\ region\ occupied\ by\ each\ vehicle}{Volume\ of\ the\ 'time\text{-}space'\ continuum} \tag{4}$$

$$\rho_A = \frac{\sum_{i=1}^{n} t_i \times w_i}{L \times W \times T} \tag{5}$$

Similarly, the area flow, which is the demand for the supplied infrastructure is defined as,

$$Area\ Flow = \frac{Sum\ of\ the\ areas\ of\ the\ projected\ path\ of\ each\ vehicle}{Volume\ of\ the\ 'time\text{-}space'\ continuum} \tag{6}$$

$$q_A = \frac{\sum_{i=1}^{n} d_i \times w_i}{L \times W \times T} \tag{7}$$

The ratio of the area flow to the area density is termed as the road space freeing rate (RFR) and is defined as,

$$RFR = \frac{q_A}{\rho_A} = \frac{\sum d_i \times w_i}{\sum t_i \times w_i} \tag{8}$$

Using the above definitions, the traffic stream characteristics were obtained from the extracted trajectories. The class-wise Area Flow, RFR, and Area Densities were also calculated by applying the same definitions to the class-wise data. The existing single regime models were calibrated using the empirical data and the details of the calibration procedures are discussed in the following section.

Calibration of the Traffic Flow Models

Calibration of the mathematical models with empirical data is the process of estimating the model parameters as accurate as possible. Therefore, the calibration is a critical process that decides model prediction capability. Several optimization tools have been employed as a calibration method to minimize the deviations between the empirical data and the model form. Qu et al., (2015) have stated that the inaccuracy of single-regime models arise not solely because of the improper functional forms but also by the bias in the collected data. It is evident from many of the empirical macroscopic traffic studies that the traffic data is heavily biased to the region below a certain density value (e.g., Gomes and Horowitz, 2009; Wu et al., 2011). Considering the possibility of bias in the data, Qu et al., (2015) proposed a novel calibration approach, so-called Weighted Least Square Method (WLSM), that assigns certain weights to each data points based on the distance to the nearby points on the left and right side of the subject point (Figure 3). The WLS method was able to resolve the model calibration issues due to the selection bias in the data sample. Present study follows the WLSM for the model calibration.

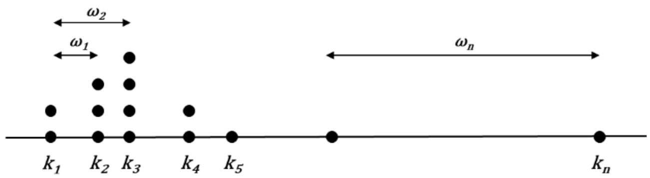


Figure 3: General Weight Determination Method

The procedure of the WLSM is as follows. For all the speed-density observations (v_i,k_i), where k_i and v_i are the i^{th} density and speed observations, a weight w_i is assigned based on the closeness of nearby observations. The step-by-step procedure followed in the weight estimation is shown below.

Step 1: Rank the observations in terms of their densities. We thus have,

(v_1,k_1), (v_2,k_2), (v_3,k_3), (v_4,k_4),…., (v_i,k_i),…, (v_n,k_n),

Where, $k_1 \leq k_2 \leq \ldots\ldots. \leq k_i \leq \ldots.. \leq k_n$

Step 2: Define $\hat{u}$ as the largest index 'i' that corresponds to the same density as k_1, that is,

$$\hat{u} := \arg\max\left\{i = 1,2,...,n \mid k_i = k_1\right\} \tag{9}$$

Then,

$$\varpi_i = \frac{k_{\hat{u}+1} - k_1}{\hat{u}}, \qquad i = 1,2,...,\hat{u} \tag{10}$$

Step 3: Define $u = \hat{u} + 1$, define $\hat{u}$ as the largest index 'i' that corresponds to the same density as k_u, that is,

$$\hat{u} := \arg\max\left\{i = u, u+1, \mathrm{u}+2,...,n \mid k_i = k_u\right\} \tag{11}$$

$$\varpi_i = \frac{k_{\hat{u}+1} - k_{u-1}}{2(\hat{u} - u + 1)}, \qquad i = u, u+1, \mathrm{u}+2,...,\hat{u} \tag{12}$$

And repeat Step 3. Else,

$$\varpi_i = \frac{k_m - k_{u-1}}{m - u + 1}, \qquad i = u, u+1, u+2,...,n \tag{13}$$

And stop.

Apart from the above considerations, present study utilizes a robust least squares approach to remove the outliers in the speed observations (Cleveland, 1979). It is usually assumed that the errors follow a normal distribution, and the occurrence of the extreme values are rare. The main disadvantage of using simple least-squares fitting is its sensitivity towards the outliers. Outliers have a large influence on the fit because squaring the residuals magnifies the effects of these extreme data points. To minimize the influence of outliers, the robust weighted least-squares regression was considered. This method minimizes the weighted sum of squares, where the weight given to each data point depends on how far the point is from the fitted line. Points near the line get full weight and those are farther from the line get a reduced weight. Robust fitting with bi-square weights uses an iteratively reweighted least-squares algorithm, and follows this procedure:

Step 1: Fit the model by weighted least squares.

Step 2: Compute the adjusted residuals and standardize them. The adjusted residuals are given by

$$r_{adj} = \frac{r_i}{\sqrt{1-h_i}} \tag{14}$$

r_i is the usual least-squares residual ($r_i = y_i - \hat{y}_i$) and h_i is the leverage that adjust the residuals by reducing the weight of high-leverage data points. The standardized adjusted residuals are given by

$$u = \frac{r_{adj}}{Ks} \tag{15}$$

'K' is a tuning constant equal to 4.685, and 's' is the robust variance given by $\frac{MAD}{0.6745}$ based on the idea that E[MAD] = 0.6745 for the standard normal conditions, where MAD is the median absolute deviation of the residuals (Holland and Roy, 1977; Rousseeuw and Leroy, 1987).

Step 3: Compute the robust weights as a function of u. The bi-square weights are given by

$$w_i = \begin{cases} \left(1-(u_i)^2\right)^2 & |u_i| < 1 \\ 0 & |u_i| \geq 1 \end{cases} \tag{16}$$

Step 4: If the fit converges, stop the iteration. Otherwise, perform the next iteration of the fitting.

In this study, different single regime traffic flow models were calibrated against the empirical data using the Robust WLSM. The selection of the macroscopic traffic flow models for calibration was done by considering different model categories proposed by Carey et al.,(2012). Carey et al. has reviewed different macroscopic traffic flow models and categorized them based on certain properties of the models. We have considered models from these different categories to ensure that the fit of a model with the empirical data is not only due to the functional form of the model. Table 1 shows the different models considered for the calibration and the category. Calibration was performed for each models considering the class-wise RFR and area density as well as the class-wise RFR and the total area density of the traffic stream. The calibration results and the discussions are given in the following section.

Table 1: Macroscopic Single Regime Models Considered for Calibration

Model Category	Model	Model Form
Flow-Density functions that has the jam density at +∞, unless truncated	i. Drake's Model	$v = v_f \times \left[\exp\left(-\frac{1}{2}\times\left(\frac{k}{k_c}\right)^2\right)\right]$
	ii. Papageorgiou's Model	$v = v_f \times \left[\exp\left(-\frac{1}{m}\times\left(\frac{k}{k_c}\right)^m\right)\right]$
Flow-Density functions that has the jam density and a parameter for gradient at jam density	iii. Newell's Model	$v = v_f \times \left[1-\exp\left(\frac{c_j}{v_f}\times\left(1-\frac{k_j}{k}\right)\right)\right]$
Non-classical Flow-Density functions	iv. Del Castillo's Model	$v = v_f \times \left[1-\exp\left(1-\exp\left(\frac{c_j}{v_f}\times\left(\frac{k_j}{k}-1\right)\right)\right)\right]$

5. **Result and Discussion**

The calibration results support the hypothesis that there exists a class-wise fundamental diagram for a heterogeneous, no lane-disciplined traffic stream. Figure 4 shows the calibrated models with the class-wise RFR and Area Density. Table 2 shows the fit statistics corresponding each models calibrated with the class-wise RFR and Area Density. It is apparent that most of the models give satisfactory fit to the empirical observations. Similarly Figure 5 and Table 3 shows the calibration results of the models with the class-wise RFR and the total Area Density of the traffic stream. The fit results are satisfactory and support the hypothesis of this study.

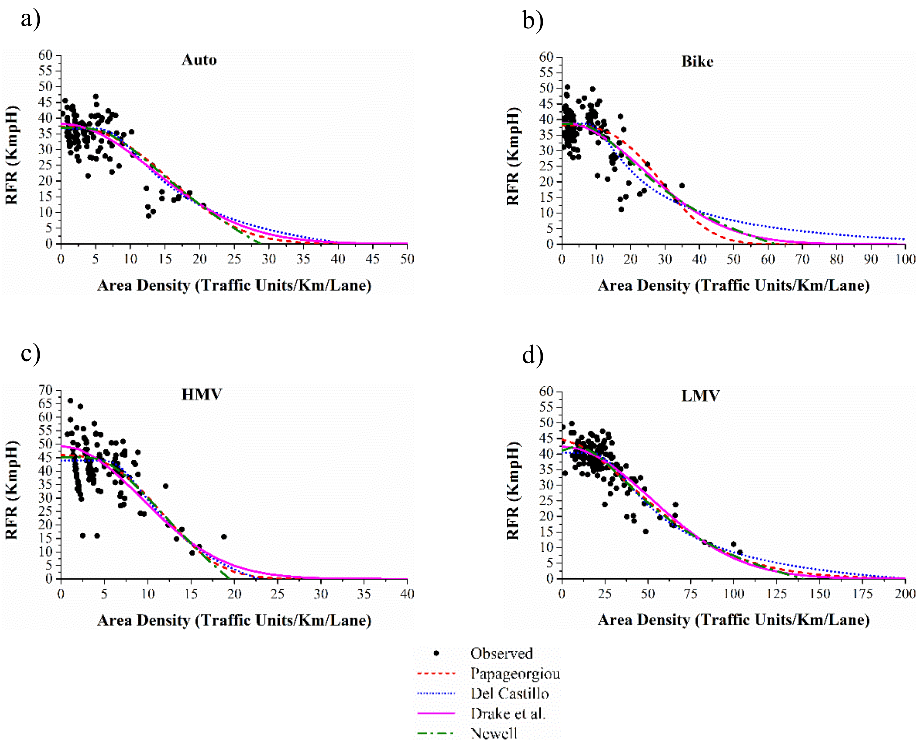
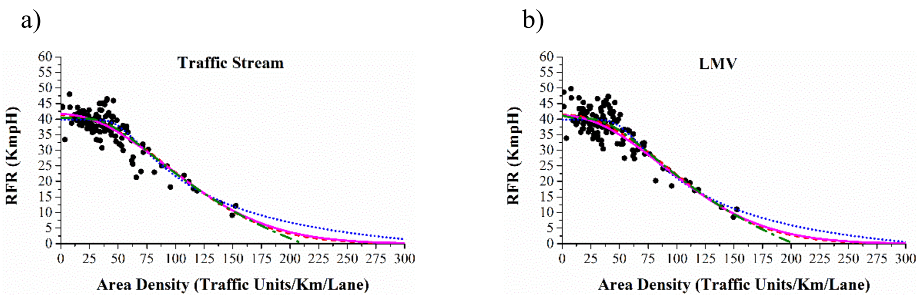


Figure 4: Variation of the RFR of a Particular Vehicle Class with the Area Density of that Particular Vehicle Class

Table 2: Result of Model Calibration with the Empirical Data Considering Class-wise Density

Model	Auto-Rickshaw		Bike		HMV		LMV	
	R^2	***RMSE***	***R^2***	***RMSE***	***R^2***	***RMSE***	***R^2***	***RMSE***
Drake et al (1967)	0.8852	1.629	0.8841	1.8340	0.8479	2.390	0.9545	2.342
Papageorgiou (1989)	0.8895	1.605	0.8983	1.731	0.856	2.336	0.9464	2.552
Newell (1961) & Franklin (1961)	0.8975	1.546	0.8882	1.816	0.8558	2.337	0.9487	2.498
Del Castillo & Benitez (1995)	0.9012	1.5780	0.8841	1.8480	0.8650	2.261	0.9587	2.239



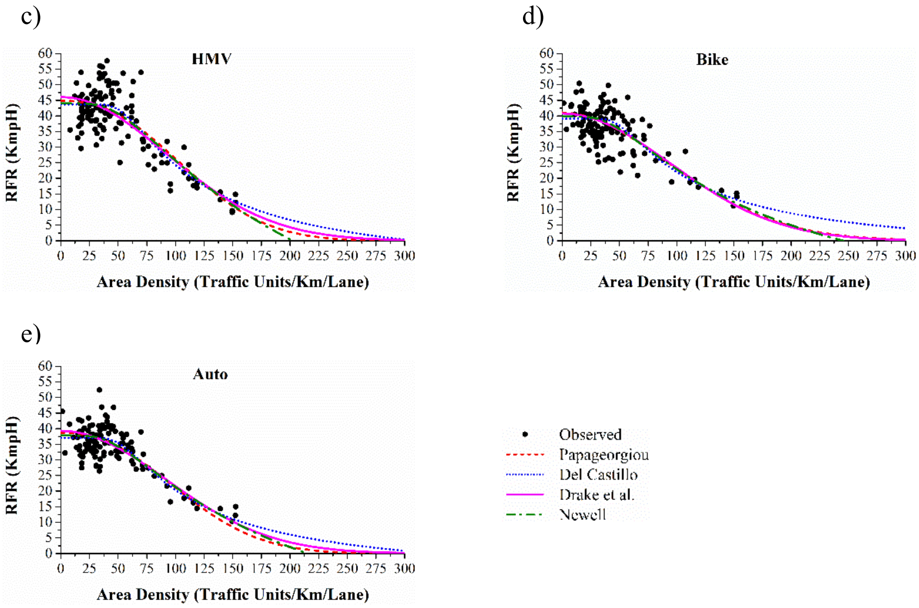


Figure 5: Variation of the RFR of a Particular Vehicle Class with the Total Area Density of the Traffic Stream

Table 3: Result of Model Calibration with the Empirical Data Considering the Total Density

Model	Total Traffic		Auto-Rickshaw		Bike		HMV		LMV	
	R^2	*RMSE*	R^2	*RMSE*	R^2	*RMSE*	R^2	*RMSE*	R^2	*RMSE*
Papageorgiou (1989)	0.9586	2.369	0.8966	3.828	0.8773	3.899	0.8791	5.245	0.953	2.673
Newell (1961) & Franklin (1961)	0.9597	2.340	0.9009	3.748	0.8756	3.927	0.8831	5.157	0.9549	2.619
Drake et al (1967)	0.9605	2.302	0.8948	3.844	0.8809	3.827	0.8809	5.183	0.9541	2.631
Del Castillo & Benitez (1995)	0.9611	2.296	0.9137	3.497	0.8826	3.813	0.8917	4.964	0.9529	2.666

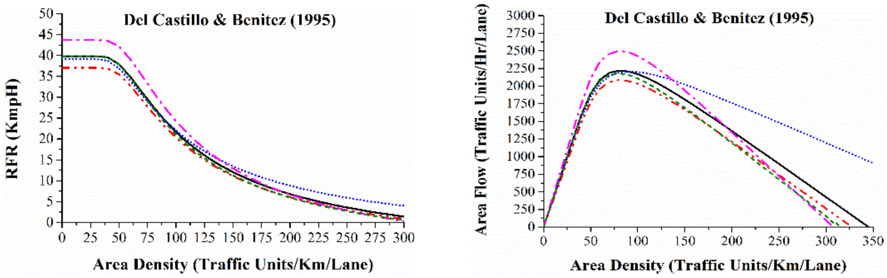
Figure 6 shows the multiclass fundamental diagram of the traffic stream calibrated with different macroscopic single regime models. This figure clearly shows the higher maneuverability of the Bike and its creeping behavior during higher density conditions. Further, we have investigated the additive property of the Area Densities of vehicle classes as shown in Equation (1). Corresponding to a given total Area Density (ρ_{Total}) the class-wise RFR ($RFR_m(\rho_{Total})$) was estimated from each of the model equations. The estimated $RFR_m(\rho_{Total})$ was applied to the class-wise model equations and estimated the class-wise Area Density (ρ_m **).** The class-wise Area Densities were added together to get $\sum\rho_m$ and compared with the ρ_{Total} . The difference between $\sum\rho_m$ and ρ_{Total} was measured with the Mean Absolute Percentage Error (MAPE) for all the models. Table 4Table 4: Error Analysis between the Total Area Density and the Class-Wise Area Densities ($\rho_{Total} = 50\ Traffic\ Units / Km / Lane$)

Model	Auto-Rickshaw		Bike		HMV		LMV		*MAPE*
	$RFR_m(\rho_{Total})$	ρ_m	$RFR_m(\rho_{Total})$	ρ_m	$RFR_m(\rho_{Total})$	ρ_m	$RFR_m(\rho_{Total})$	ρ_m	
Papageorgiou	34.17	7.35	35.32	14.48	40.45	6.61	35.87	27.72	12.32
Del Castillo	35.51	7.34	36.92	10.29	42.18	6.57	37.83	27.42	3.24
Drake et al.	33.84	6.64	35.47	10.5	39.76	6.11	35.04	34.04	14.58
Newell	34.32	7.54	35.80	10.88	40.80	6.47	36.45	27.17	4.12

Table *5* show that the MAPE for Del Castillo’s model is the least among all. Therefore, it can be stated that Del Castillo’s Model is showing the best results and the least error.

Figure 6 shows the multiclass fundamental diagram for the heterogeneous traffic stream calibrated with different models. Multiclass traffic flow theory assumes that, with an increment in the traffic density, the speed of all vehicle classes converges due to the higher interactions. The same could be observed in the figure except for the smaller vehicles, since they exhibit creeping behavior, which is another property of the heterogeneous traffic stream.

a)



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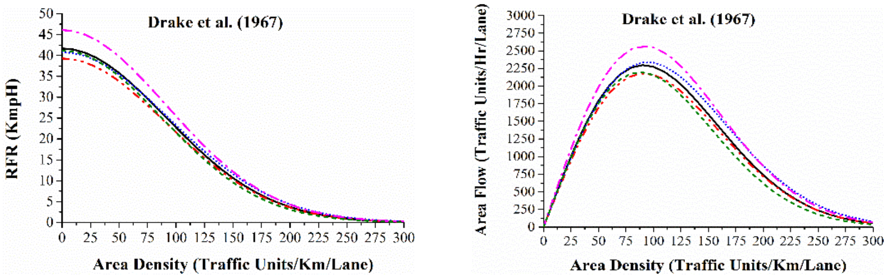

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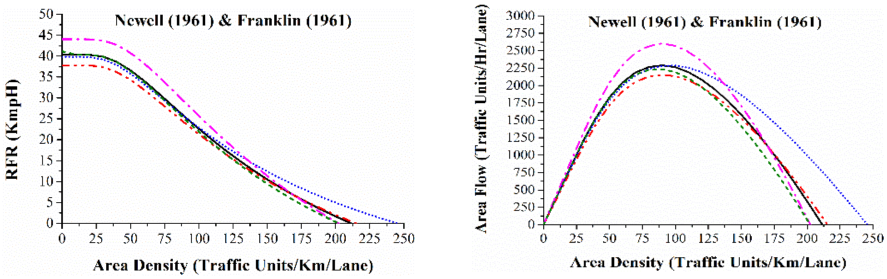

d)

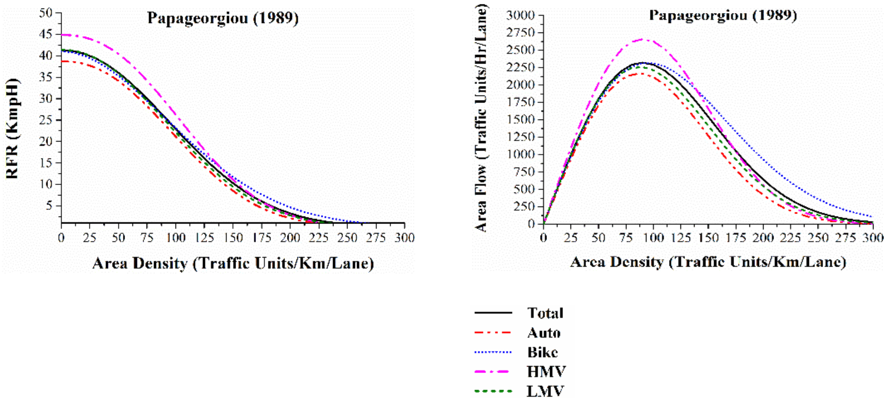

Figure 6: Speed and Flow Variation for Different Vehicle Classes in Response to the Total Density

Table 4: Error Analysis between the Total Area Density and the Class-Wise Area Densities ($\rho_{Total} = 50$ *Traffic Units / Km / Lane*)

Model	Auto-Rickshaw		Bike		HMV		LMV		*MAPE*
	$RFR_m(\rho_{Total})$	ρ_m	$RFR_m(\rho_{Total})$	ρ_m	$RFR_m(\rho_{Total})$	ρ_m	$RFR_m(\rho_{Total})$	ρ_m	
Papageorgiou	34.17	7.35	35.32	14.48	40.45	6.61	35.87	27.72	12.32
Del Castillo	35.51	7.34	36.92	10.29	42.18	6.57	37.83	27.42	3.24
Drake et al.	33.84	6.64	35.47	10.5	39.76	6.11	35.04	34.04	14.58
Newell	34.32	7.54	35.80	10.88	40.80	6.47	36.45	27.17	4.12

Table 5: Error Analysis between the Total Area Density and the Class-Wise Area Densities ($\rho_{Total} = 180$ *Traffic Units / Km / Lane*)

Model	Auto-Rickshaw		Bike		HMV		LMV		MAPE
	$RFR_m(\rho_{Total})$	ρ_m	$RFR_m(\rho_{Total})$	ρ_m	$RFR_m(\rho_{Total})$	ρ_m	$RFR_m(\rho_{Total})$	ρ_m	
Papageorgiou	3.88	26.57	6.93	39.66	5.18	18.13	4.68	114.4	10.42
Del Castillo	7.76	24.91	10.39	40.42	8.80	16.93	7.76	104.5	3.76
Drake et al.	5.65	26.23	6.67	45.05	6.76	18.52	5.06	106.8	9.22
Newell	4.99	24.96	7.66	44.5	4.45	17.86	3.64	118.3	14.23

6. Summary and Conclusions

Present study investigates the existence of the class-wise fundamental diagram for a heterogeneous, no lane-disciplined traffic stream. Various single regime traffic flow models were calibrated against the empirical data that collected from an urban arterial. The traffic stream characteristics were defined as proposed by Suvin and Mallikarjuna, (2018b). These definitions capture the vehicle heterogeneity as well as the no lane-disciplined actions. The models were calibrated using a Robust Weighted Least Square Method. The finding from this study indicates that there exist a class-wise fundamental diagram for a heterogeneous traffic stream and it follows the fundamental assumptions of the multiclass traffic flow theory.

The analysis shows that the Del Castillo's model gives a better representation of the heterogeneous, no lane-disciplined traffic stream. The fundamental assumption of the multiclass traffic flow theory, i.e., the total traffic density as the summation of the densities of individual vehicle classes was tested with different models. Del Castillo's model has produced the minimum MAPE for free flow and forced flow conditions. Besides, Del Castillo's model captures the seeping behavior of the smaller vehicles at higher density levels in a comparatively better way.

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