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Probabilistic Geometric Design of Highways: A Review

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Abstract

The design input parameters such as vehicular operating speed, deceleration rate and driver's perception reaction time considered in the geometric design of highways exhibit considerable uncertainty in practical scenario. As such, in this context, the probabilistic approach or reliability based approach is essential in highway geometric design which is capable of incorporating the factor of uncertainty of design input parameters. The objective of this paper is, therefore, to present a state of the art review on the works dealing with reliability based methodology in highway geometric design and discuss future scope of development. Prominently, in the present study, the process of probabilistic or reliability based design in the reference of sight distance considerations and horizontal alignment have been reviewed, and thereby, some recently developed parameters such as $L_{front\text{-}eye}$ (the distance from the front of the car to the driver's eye) and considerations as reported in various literature are recommended to be introduced in the framework of reliability based highway geometric design practice. Further, it is recommended in line with the reliability based design practice to incorporate the methodology to establish the link between Probability of Noncompliance (P_{nc}) and collision frequency obtained from accident data. Also, it is recommended that a reliability based optimization technique can act as a viable tool for ensuring uniform level of safety on a reliability based designed road.

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1. Introduction

The design input parameters such as vehicular operating speed, deceleration rate and driver's perception reaction time considered in the geometric design of highways exhibit considerable uncertainty in practical scenario (Lerner 1995, Fambro et. al. 1997,Hussein et. al. 2014). However, the current geometric design procedures are deterministic in nature and this renders the current geometric design specifications not fully effective to address all the requirements

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to ensure safety of the designed highway elements. From the perspective of maintaining adequate level of safety for all road users, adopting a holistic design methodology for highway geometric design is the need of the hour and it is in this context that the probabilistic approach or reliability based approach is essential, which is capable of incorporating the factor of uncertainty of design input parameters in highway geometric design. Considerable research has been conducted in the field of introducing probabilistic approach in highway geometric design in the form of reliability based design. The objective of this paper is, therefore, to present a state of the art review on the works dealing with reliability based methodology in highway geometric design and future scope of development. It is a reported fact that the horizontal curves account for a majority of accidents in a highway compared to other highway elements (Torbic et. al. 2004) and in this context, the sight distance considerations play a vital role to ensure a safely designed highway. As such, this paper lays emphasis on the studies conducted on reliability based design methodologies for horizontal curves by various researchers. Further, this study attempts to connect some more studies conducted on correlating the output of reliability based design with crash data (accident frequency and location), so as to establish how effective reliability based design can be in addressing the real issue of ensuring safety for road users. Finally, considering the diverse facets of developments in reliability based design and the studies on correlation with accident data, a conceptual framework of parameters and considerations from recent studies are proposed to be adopted for a reliability based safety correlated design practice for geometric design of highways.

Nomenclature

SSD	Stopping Sight Distance
ASD	Available Sight Distance
P_{nc}	Probability of Noncompliance
MCS	Monte Carlo Simulation
FORM	First Order Reliability Method

2. Background

1.1 Concept of Reliability and Related Terminologies

Reliability can be defined as the probability that the system in concern will perform its function satisfactorily without fail over its design life (Kottegeda and Rosso 1997). It may be mathematically expressed as,

$$R = Prob.(Supply > Demand) \tag{1}$$

If Supply and Demand are considered as 'X' and 'Y' respectively, then the parameter 'Z' given by,

$$Z = X - Y \tag{2}$$

is called the Performance Function / Safety Margin. Thus, Reliability (R) can also be expressed as

$$R = Prob.(Z > 0) \tag{3}$$

The Probability of Noncompliance (P_{nc}) can be expressed as,

$$P_{nc} = 1 - R = Prob.(Z < 0) \tag{4}$$

The parameter P_{nc} is considered as an important measure for safety implications in reliability based geometric design of highways.

Another important measure for representing the reliability of a system is Reliability Index, which is denoted by β and can be defined as the ratio between the mean and standard deviation of the system if X and Y are normally distributed. The mathematical expression of β is shown in Equation 5 and a higher β value indicates higher reliability of a system.

$$\beta = \frac{E(X) - E(Y)}{\sqrt{Var(X) + Var(Y)}} \tag{5}$$

A more generalized expression of Performance Function, Z consisting of 'n' variables is given by,

$$Z = F(x_1, x_2, x_3, \ldots \ldots \ldots .., x_N) \tag{6}$$

In that case, Probability of Noncompliance (P_{nc}) is given by,

$$P_{nc} = \int_{F(x_1, x_2, x_3, \ldots\ldots\ldots, x_N) < 0} \ldots . \int g_{x_1 \ldots\ldots x_n}(x_1, \ldots \ldots x_2) dx_1, \ldots \ldots .., dx_n \tag{7}$$

where, $g_{x_1 \ldots\ldots x_n}(x_1, \ldots \ldots x_2) = joint\ probability\ function\ of\ the\ input\ variables\ (X_1, \ldots \ldots \ldots, X_n)$

3. Reliability based design methodology

3.1 *Sight Distance Considerations*

Sight distance has been reported as one of the most fundamental parameters to ensure safety criterion in geometric design of highways by various researchers (US DoT 1997, Wood and Donell 2017). Stopping Sight Distance (SSD) which is basically the summation of lag distance (perception-reaction distance) and braking distance, is given by the following equation (IRC 2001, AASHTO 2011, Fambro et. al 1997)

$$SSD = Vt_r + \frac{V^2}{2g(\frac{a}{g}+G)} \quad (8)$$

where, V (design speed) is in m/s,

t_r (perception-reaction time) is in seconds (2.5 s as standard value),

a is deceleration rate in m/s^2

g is acceleration due to gravity in m/s^2, and

G is ascending (positive) or descending (negative) gradient in decimal.

Various researchers have studied the variability of the design input parameters of sight distance, i.e. design speed, perception-reaction time and deceleration rate. The findings of these studies are summarized in Table 1.

Table 1. Uncertainty of Input Parameters for Stopping Sight Distance

Parameter	Mean	Std. Deviation	Reference
Operating Speed (for R = 200 - 900 m)	80.38 - 95.52 km/h	8.119 - 4.598 km/h	Hussein et. al. 2014
Perception-reaction time	1.5 s	0.4 s	Lerner 1995
Deceleration rate	4.2 m/s2	0.6 m/s2	Fambro et. al. 1997

As the calculation of perception-reaction time is based on non-distracted drivers, Bellinger et. al. (2009) and Phillip et. al. (2004) have reported the need to consider the effects of distraction (listening to music and talking with fellow vehicular occupants etc.) and fatigue (long drive and sleep deprivation) in the estimation of SSD. Further, in case of the determination of deceleration rate (equal to the product of friction coefficient and acceleration due to gravity), it is reported to be higher on tangents compared to that on horizontal curves, so also to be higher on dry pavements in comparison to that on wet pavements (Fambro et. al. 1997, Fitch et. al 2010). Further studies are, therefore, required to be conducted to develop methodologies to incorporate these issues in the consideration of deceleration rate for higher precision. Neuman (1999) reported the variation of SSD design values in comparison to that obtained from deterministic methodology. Fambro et. al. (2000) developed a model taking into consideration a wide range of field data for driver characteristics to address the variability issue and this model was developed for design of SSD, sag and crest vertical curves. In order to address the issue of safety, recent studies recommends the consideration of operating speed in Equation 8 instead of design speed so as to ensure design consistency which in fact is considered as a vital parameter of safety measure (Gibreel et. al 1999). That is to say, that the design speed in Equation 8 should be so chosen as to encompass the operating speed of almost all the vehicles on the designed road element and presently, the 85th percentile value is to be adopted as the design speed as per the current specifications (AASHTO 2011, AUSTROADS 2010). However, in the practical scenario, it is observed that the critical highway geometric elements such as the horizontal and vertical curves are not designed for the operating speed of vehicles due to various environmental and economic constraints. On the other hand, the standard value of 2.5 s for perception-reaction time is actually considered as 90th percentile value and the 10th percentile value of deceleration rate which governs the coefficient of longitudinal friction is adopted (AASHTO 2011). However, these percentile values of design speed, perception-reaction time and coefficient of longitudinal friction are considered deterministically for the calculation of SSD. As such, various researchers have recommended the application of reliability concept in the estimation of sight distance to incorporate input parameter variability. Although the theoretically required minimum SSD (demand) is given by Equation 1, the Available Sight Distance (ASD) which is the supply sight distance at sight, can be expressed as,

$$ASD = 2 \times R \times cos^{-1}(1 - \frac{M}{R}) \quad (9)$$

where, R is the horizontal curve radius of centerline of lane nearer to the obstruction,

M is middle ordinate value or set-back distance or lateral distance between edge of median barriers and centerline of the adjacent traffic lane.

The minimum SSD is mandatorily recommended to be maintained all throughout the length of a highway, crucially on horizontal curves and vertical crest or summit curves. Khoury and Hobeika (2007) developed distribution for Passing Sight Distance and conducted reliability analysis. Ismail and Sayed (2010) developed a decision support tool based on reliability framework to quantify the impact of various deviations from sight distance specifications. In case of vertical sag curves, headlight sight distance is an important consideration to be made. Shine and Lee (2014) suggested another framework of reliability based optimization to report that deviations from design conditions as per current design guidelines can still assure target margins of safety against rollover and sideslip in some cases. Wood and Donell (2014) studied the reliability estimate and probability distribution of SSD and reported that the probability of inadequate visibility was lower on inside of the horizontal curve compared to that of its approach. Hussein et. al. (2014) developed reliability based design charts by calibrating middle ordinate (M) values at various probability of noncompliance levels, which can serve as a useful tool to apprehend the safety implications of adopting a particular set of radius (R) and middle ordinate (M) values on a horizontal curve. However, this methodology considered the estimation of SSD and ASD in a general perspective and visibility constrained issues such as night time driving issues were not accounted for in this study. Papadimitriou et. al. (2016) investigated the potential violation of SSD inadequacy on divided highways and identified areas of inadequacy in terms of the probability of SSD inadequacy. In the study by Essa et. al. (2016), the authors stressed on the application of multimode reliability approach considering more than one mode of Probability of Noncompliance. In another study by Rajbongshi and Kalita (2018), a probabilistic approach for estimation of SSD taking into account the variability of all input parameters and also, reliability based design charts for both plain and hill regions were presented incorporating the effect of lateral thrust. It was reported in the study that the SSD parameter follow lognormal distribution and that the SSD corresponding to 98th percentile speed is much higher compared to the 98th percentile sight distance value. This finding has an important implication in the exploration of possible scope for reduction of ASD without compromising safety in areas of environmental constraints where providing considerable ASD by deterministic methodology is not possible. Wood and Donell (2017) developed a methodology of SSD estimation accounting for lighted (i.e. daytime and lighted nighttime) and unlighted nighttime conditions. In order to make the daytime and lighted nighttime ASD models consistent with the SSD model, a new parameter $L_{front\text{-}eye}$ (the distance from the front of the car to the driver's eye) was introduced in the study and the SSD model for daytime was expressed as,

$$SSD_{daytime} = Vt_r + \frac{V^2}{2g(\frac{a}{g}+G)} + L_{front-eye} \tag{10}$$

The conceptual philosophy behind the development of the model in Equation 10 is shown in Figure 1.

In the study, Wood and Donell (2017) also conducted a reliability analysis of SSD for vertical crest curve by Monte Carlo Simulation (MCS) and reported that the relative difference in the probability of noncompliance between the traditional SSD model (Equation 8) and lighted conditions was as large as 39.5%. Therefore, this makes the model of Equation 10 recommendable to be adopted in the current design policy of highway geometric design.

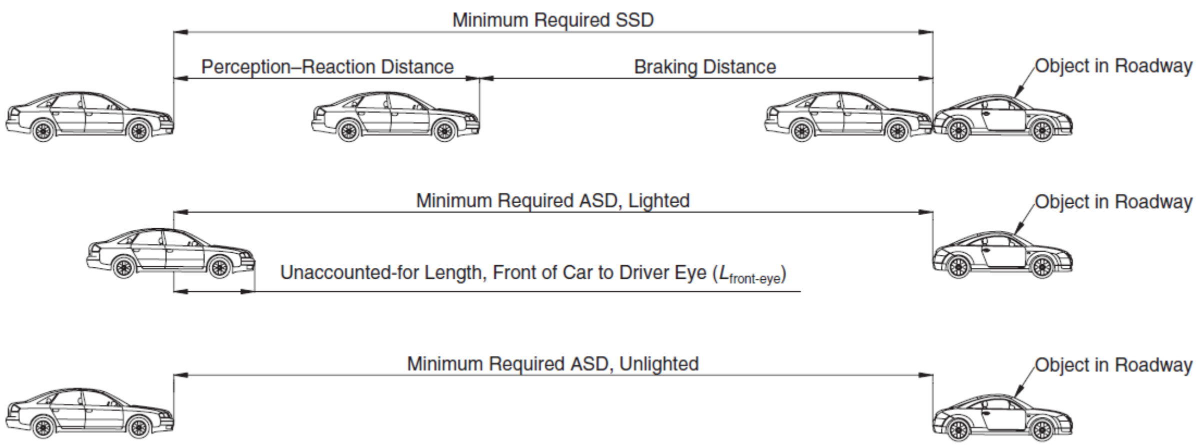

Figure 1: Minimum SSD, ASD, and unaccounted-for length in current design policy (Wood and Donell 2017)

3.2 *Vehicle Dynamic Stability and Driver Comfort Considerations*

In addition to the availability of minimum stopping sight distance, another factor of crucial importance for maintaining safety on the horizontal curve is the minimum radius, which is given by (AASHTO 2011),

$$R_{min} = \frac{V^2}{127(0.01e_{max}+f_{max})} \quad (11)$$

where, V = speed of vehicle (km/hr)
e = rate of maximum superelevation
f_{max}= maximum lateral friction factor

The provision of adequate superelevation and availability of lateral friction are crucial for the safety of a vehicle negotiating a horizontal curve, as otherwise the centrifugal force acting on a vehicle traversing a horizontal curve tends to overturn or skid the vehicle out of the curve. In this regard, as per the current design guidelines, the design of horizontal curves is based on driver comfort criterion, which is basically expressed in terms of a maximum allowable driver's comfort threshold. Further, it is also a significant fact that the parameters of crucial importance in horizontal curve design such as lateral acceleration and friction factor exhibits considerable variability depending on operating speed and vehicular type. Emmanuel (1996) and Zheng (1997) conducted reliability analysis for design of horizontal curve considering only vehicle skidding as the failure criterion without accounting for vehicle rollover.In the study by You et. al. (2012), a reliability based design concept of horizontal curves was introduced and constituted the performance functions in terms of the difference between "Supply Radius (R_s)" and "Demand Radius (R_d)". For the reliability analysis of horizontal curves, You et. al. (2012) considered three modes of failure and the performance functions can be mathematically expressed as follows:

Mode of failure by Vehicle Skidding only:

$$G_1(\mathrm{X})=R_S - \frac{V^2}{g(e+f_y)} \quad (12)$$

where f_y = lateral friction coefficient; and V = operating speed of vehicle.

Mode of failure by Vehicle Skidding involving Roll Motion:

$$G_2(\mathrm{X})=R_S - \frac{V^2}{g\left[(1-\frac{h_r}{h})e+f_y)\right]}\left[1+R_\theta\left(1-\frac{h_r}{h}\right)\right] \quad (13)$$

where h = height of center of gravity above the ground; h_r = height of the roll center above the ground at the longitudinal center of gravity location; and R_θ = roll rate (rad/gravity),

Mode of failure by Vehicle Rollover only:

$$G_3(\mathrm{X})=R_S - \frac{V^2}{g\left[(e+\frac{t}{2h})\right]}\left[1+R_\theta\left(1-\frac{h_r}{h}\right)\right] \quad (14)$$

With the performance functions shown in Equation 12-14 and considering vehicle speed, friction coefficient and supply radius as random variables but superelevation and vehicle parameters as deterministic, the authors recommended a methodology for iteratively calculating the P_{nc} of horizontal curve with different supply radius by First Order Reliability Method (FORM) unless the desired level of reliability is attained. This work was a pioneering study to develop a methodology for probabilistic horizontal curve design with vehicle rollover criteria. However, more studies are required to develop the methodology to incorporate the different types of vehicular suspension systems to account for the vehicle rollover effect while designing a horizontal curve. Himes (2013) conducted field studies and reported the distribution parameters of available lateral friction on horizontal curves for various speed levels. A methodology for reliability based design of horizontal curves was developed by Dhahir and Hassan (2016). In that study, the authors introduced the criterion of vehicle dynamic stability in addition to the driver comfort criterion. To carry out the reliability analysis for horizontal curve, they developed performance functions in terms of the difference between (i) available lateral friction and lateral friction demand, and (ii) driver comfort threshold and actual lateral acceleration for vehicle dynamic stability and driver comfort criteria respectively. They calibrated the performance functions considering vehicle dynamic stability and driver comfort criteria as shown in Equation 15-18 for Sedan and SUV vehicles separately and recommended an iterative process for reliability based design of horizontal curves till the estimated Probability of Failure / Noncompliance meet the target levels of probabilities by FORM.

By vehicle dynamic stability criterion:

For Sedan,

$$G_s = f_s - \left(\frac{0.014V^{1.698}}{R^{0.872}} - 0.881e^{0.642} + 0.075G\right) \quad (15)$$

For SUV,

$$G_s = f_s - \left(\frac{0.014V^{1.788}}{R^{0.872}} - 0.991e^{0.939} + 0.110G\right) \quad (16)$$

where,

f_S = available lateral friction
V = vehicle speed (km/h),
R = curve radius (m),
e = superelevation rate, and
G = vertical grade.

By driver comfort criterion:
For Sedan,

$$G_s = a_L - \left(\frac{0.007V^{2.048}}{R^{1.109}} - 0.023e + 0.002G - e\right) \quad (17)$$

For SUV,

$$G_s = a_L - \left(\frac{0.007V^{1.997}}{R^{0.953}} - 0.021e^{0.408} + 0.028G - e\right) \quad (18)$$

where, a_L = driver comfort threshold,

Himes and Donell (2014) studied the application of probabilistic approach in the design of horizontal curves and accounted for the influence of wet pavements and tire characteristics for passenger cars and heavy trucks. It was reported by them that a target reliability index of 3.0 is suitable for consideration in the design of horizontal curves. Mollashahi et. al. (2016) studied the uncertainty associated with the input parameters for superelevation design and calibrated the P_{nc} for different superelevation rates at various operating speed levels by Monte Carlo Simulation (MCS). Calibration of superelevation rates against operating speed instead of design speed can be considered to be a better methodology for horizontal curve design in comparison to the current design practice and also, the P_{nc} values at operating speeds can serve as a useful tool for understanding the measure of safety provided by a particular rate of superelevation design.

3.3 *Probability of Noncompliance (P_{nc}) and Safety implications*

Although it has been established by various researchers that reliability based design is a necessary methodology to ensure safety in comparison to the deterministic design procedure, it is also important to study the correlation of P_{nc} with accident or crash data to assess the safety implication of reliability based design in the reduction of collision frequency. Moreover, investigation in this direction would also help in performing benefit-cost analysis which in turn would facilitate an enhanced application of the probabilistic approach in highway geometric design. Ibrahim and Sayed (2011) developed two Negative Binomial Safety Performance Functions using an extensive database of collision and geometric design of two lane rural highways followed with reliability analysis by FORM and finally, developed correlations between P_{nc} and collision with very good goodness of fit considering (i) only property damage (PDO), (ii) total (tot), and (iii) injury and fatality (I+F) as shown in Figure 2. The Negative Binomial Safety Performance Functions adopted in the study are as follows in Equation 19-21.

$$\mu_{tot} = e^{-14.931} L\, V^{0.895} e^{1.461 P_{nc}} \quad (19)$$

$$\mu_{I+F} = e^{-15.624} L\, V^{0.900} e^{1.012 P_{nc}} \quad (20)$$

$$\mu_{PDO} = e^{-16.516} L\, V^{0.974} e^{1.793 P_{nc}} \quad (21)$$

where, V is the annual average daily traffic and L is the segment length.

The findings of this study result in the quantification of safety based on field crash data in terms of the reliability measure of the geometric design of a highway. Oh and Mun (2011) introduced a reliability index model by combining the design speed, observed speed and their variability for identifying the risky expressway segments. Ibrahim et. al. (2012) introduced a methodology for reliability based optimization of cross-section elements of highways and inferred that the application of reliability component in the optimization of cross section elements with restricted sight distance resulted in balancing the risk along both the directions of travel and also, overall reduction of collisions. The same Safety Performance Functions as shown in Equations 19-21 are used in the study by Ibrahim et. al. (2011). Thus, incorporating the concept of reliability in the geometric design of highways can also contribute in maintaining the design consistency, which is considered as a vital parameter for ensuring safety on highways. This is because if the optimization process of risk minimization across all the cross sections of a highway is coupled with reliability based approach to account for the input parameter variability, a higher level of improvement in design consistency can therefore be attained by adopting this methodology.

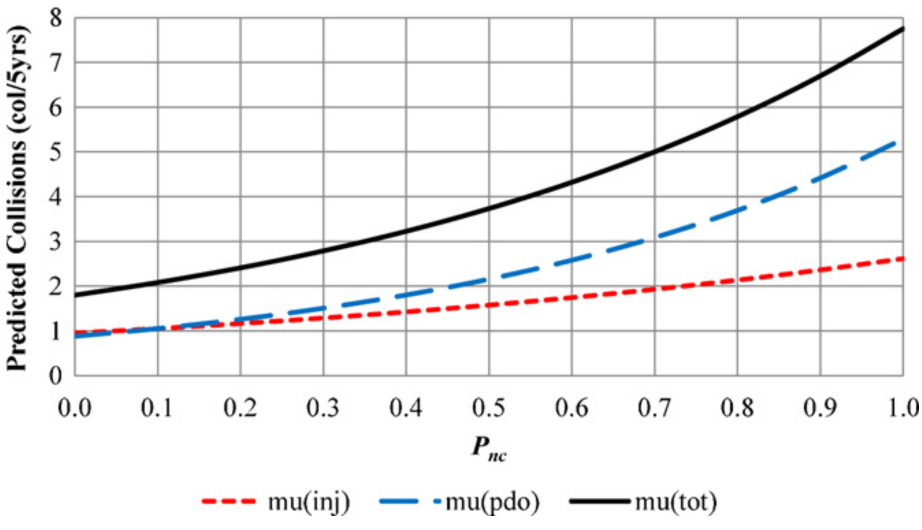

Figure 2: Predicted Collisions and Probability of Noncompliance, P_{nc} : A relation (Ibrahim and Sayed 2011).

4. Conclusions and Future Scope

The present study is based on conducting an explicit literature review of the various research works on introducing reliability based geometric design of highway elements. Literature dealing with quantification of the uncertainty of input parameters have also been discussed to apprehend the significance of incorporating probabilistic approach in the geometric design of highways. Thus, introduction of the reliability based design methodology in place of the current deterministic design procedure is recommended to arrive at the design of safer roads. Prominently, in the present study, the process of probabilistic or reliability based design in the contexts of sight distance considerations and horizontal alignment have been reviewed, and thereby, the following framework of parameters and considerations reported by various researchers, are recommended to be introduced in the reliability based design practice:
(i) Calculation of Sight Distance by the modified model to bring in consistency between the daytime and lighted night-time ASD with the inclusion of the parameter $L_{front\text{-}eye}$ (Wood and Donell 2017),
(ii) Introducing the effect of only vehicular rollover and vehicular rollover combined with skidding in the design of risk based superelevation (You et. al. (2012),
(iii) Assessment of the risk analysis of horizontal curve by estimating the Safety Perfomance Function considering the criteria of vehicle dynamic stability in addition to driver comfort (Dhahir and Hassan (2016).
Further, it is recommended in line with the reliability based design practice to study and establish the link between P_{nc} and collision frequency obtained from accident data from roads of similar topography and other factors in accordance with the methodology suggested by Ibrahim and Sayed (2011). Adopting such a methodology to correlate P_{nc} and collision frequency would provide the designers with a realistic safety implication of their target design P_{nc} in terms of the predicted number of crashes on the road. As such, it is needful to conduct further investigation to develop improved methodologies with higher precision levels to bridge the connect between reliability based design measure, i.e. P_{nc} and collision frequency. In the next step, in case of the requirement for reduction of collision frequency and balancing the level of safety in both direction of travel across the cross section, a reliability based optimization technique as reported by Ibrahim et. al. (2012) has been discussed and recommended as a viable tool for ensuring uniform level of safety on a reliability based designed road.

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