Available online at www.sciencedirect.com

ScienceDirect

Transportation Research Procedia 00 (2018) 000–000

Transportation Research Procedia

www.elsevier.com/locate/procedia

World Conference on Transport Research - WCTR 2019 Mumbai 26-31 May 2019

Basics on Categorizing Travel-Time-Based Degrees of Satisfaction Using Triangular Fuzzy-Membership Functions

Akash Anand[a]*, Varghese George[b], Rohini Kanthi[c], Moduga Tagore[d], and Padmashree M. S.[e]

[a]*PhD Scholar, Deptt. of Civil Engg., National Institute of Technology Karnataka, Mangalore-575025, India*
[b]*Professor, Deptt. of Civil Engg., National Institute of Technology Karnataka, Mangalore-575025, India*
[c]*Engineer, ARCADIS Design and Consultancy, Bangalore-560017, India*
[d]*Associate Director, Systra MVA Consulting (India) Private Limited, Navi Mumbai-400614, India*
[e]*PhD Scholar, Deptt. of Civil Engg., National Institute of Technology Karnataka, Mangalore-575025, India*

Abstract

The travel desires of trip-makers in urban activity centres depend mainly on the location of residential areas, proximity to various activity centres, household characteristics, and socio-economic factors that influence the choice of travel modes. Decision-making with regard to the choice of a particular mode of travel is fuzzy in nature, and seldom follows a rigid rule-based approach. In this context, the fuzzy-logic approach was considered since it could handle inherent randomness in decision-making related to mode-choice. The present study focuses on the application of this technique making use of revealed preference survey data collected through CES and MVA Systra, later compiled and corrected in various stages at NITK. The difference between the actual travel time by a particular mode, and the theoretical travel time based on average vehicular speeds was used as an important indicator in determining the *degrees of satisfaction* of the trip-maker. This indicator was computed, and fitted using a normal distribution. It was assumed that indicator values between μ-3σ and μ could be considered for the category of *satisfied* trip-makers according to the *three sigma* rule where μ is the mean indicator value, and σ represents the standard deviation. The computed values of the indicators were used in classifying the data into 6 categories of *degrees of satisfaction* that formed the basic framework for modelling using fuzzy-logic technique. This paper aims at understanding the basic mathematical computations involved in defuzzification using the *centroid method* for triangular membership functions, and provides a comparison with results obtained using MATLAB.

Peer-review under responsibility of WORLD CONFERENCE ON TRANSPORT RESEARCH SOCIETY.

Keywords: Fuzzy-logic; revealed preference; degrees of satisfaction; normal distribution; mode-choice; modal split; travel demand

* Corresponding author. Tel.: +91-819-775-7224; +91-998-692-0093; fax: +91-824-247-4033.
E-mail address: akashcv14f12@nitk.edu.in, varghese@nitk.edu.in, varghese.goldengate@gmail.com

Peer-review under responsibility of WORLD CONFERENCE ON TRANSPORT RESEARCH SOCIETY

1. Introduction

The tendency to choose a particular mode of travel between an origin-destination pair depends on a number of factors such as travel-time, cost of travel, comfort, safety, privacy, and income. Decision-making related to the selection of a particular mode of travel in different situations is fuzzy in nature, and seldom follows a rigid rule-based algorithm. For example, if a trip-maker experiences a travel time of 31 minutes when compared to the average travel time of 30 minutes between a given origin-destination pair, it cannot be said that the trip-maker is dissatisfied with the mode of travel. On the contrary, the trip-maker is likely to consider the above travel time and hence, the travel mode, to be "acceptable". The fuzzy set theory is capable of handling and classifying such responses where the boundaries are not clearly defined. This approach can be adopted where higher degrees of randomness are experienced.

The present study focuses on the application of this technique making use of *revealed preference* (RP) survey data. However, since the *revealed preference* (RP) survey data was based on trips made for the previous day of survey, the data was considered to be insufficient. Further studies on travel desires were performed by researchers in NITK based on extended household surveys, and also based on secondary sources of data including Census (2011). The inter-zonal OD data comprising work-trips, school-trips, and shopping trips was then formulated for various origin (O) and destination (D) zones/ wards of the city.

The theoretical travel time between the centroids of the zones/ wards was computed assuming the average travel speed as 25 kmph for the city as per CSTEP (2014). The difference between the actual time taken for travel by a particular mode between an origin-destination pair, and the theoretical travel time based on average vehicular speeds was used as an important indicator in determining the *degree of satisfaction* of the trip-maker in using the selected mode of travel. This indicator was computed for 5009 trip-makers, and was fitted using a normal distribution. It was assumed that indicator values between μ-3σ and μ could be considered for the category of *satisfied trip-makers* according to the *three sigma rule* provided by Pukelsheim (1994) where μ stands for the mean indicator value, and σ represents the standard deviation. The computed values of the indicators were classified in a similar manner into 6 *categories*: *highly satisfied, satisfied, acceptable, dissatisfied, highly dissatisfied* and *extremely dissatisfied*. These *degrees of satisfaction* provided the basic framework for modelling using fuzzy logic technique. This is considered to be an important step in further modelling of vehicle ownership and mode choice studies based on input values related to variables such as travel time, travel cost, comfort, and income. Fuzzy membership functions can be developed to handle responses for each of the variable separately, or various combinations of the same.

While modelling data in fuzzy-logic, it was considered advantageous to adopt the triangular membership function as the modelling procedure was simpler and involved less complex computations as observed by Kompil and Celik (2013), and Kedia et al. (2015). This study was focussed on achieving a basic understanding of the mathematical computations involved in the fuzzy-logic approach where triangular membership functions are used.

2. Fundamental Aspects of Fuzzy Logic

Zadeh (1965) made pioneering studies on *fuzzy-sets* and *fuzzy set theory.* A fuzzy set is defined as a ‘class of objects’ with varying ‘degrees of membership’. The concept of fuzziness, and the use of *fuzzy set theory* was introduced by Zadeh (1965). A number of real-life situations related to human decision-making involves a certain degree of fuzziness or randomness. It is difficult to make decisions in every-day life in a deterministic fashion since precise facts and figures are not generally available. In complex decision-making environments, the possibility of making precise judgements decrease as the degree of fuzziness increase. The *fuzzy-logic* approach derives its strength from the fuzzy set theory. It incorporates the use of membership functions, to deal with the fuzziness in a complex database. According to Sarkar et al. (2002), this approach can be adopted in the study of fuzzy phenomena, relationships, and classification criteria. For example, to say whether a trip-maker is satisfied by the mode of travel, it is possible to use a *membership value* that varies from 0 to 1. If the *degree of satisfaction* is very high, then it may be possible to assign a value close to 1, and if the *degree of satisfaction* is very poor, a value close to 0 can be assigned. Also, if the *degree of satisfaction* is average, then a value close to 0.5 can be assigned. The values indicating the *degrees of satisfaction*, that range between 0 and 1 are called *membership values*, and these are *numerical* in nature, while descriptions such as *highly satisfied*, *average*, and *least satisfied* are *linguistic* expressions. However, if the *membership value* is 0.3, then a certain amount of fuzziness, uncertainty, elasticity, or flexibility is involved in classifying the response as *average* or *very poor*.

Wang and Mendel (1992) proposed a method for formulating a fuzzy-rule base using the characteristics of the numerical data available. The authors suggested a five-step procedure involving classification of the input and output data into different *fuzzy-regions*; formulation of fuzzy rules based on the data; specifying a degree of applicability (or membership value) to each of the fuzzy rules to address data spread over various categories; generating linguistic expressions based on the rules formulated; and mapping of the input data to the desired output using a *defuzzifying* procedure.

3. Literature Review

Teodorovic and Kikuchi (1990) pioneered the use of fuzzy-logic techniques in modelling binary route-choice problems. Kalic and Teodorovic (1996), and Teodorovic (1999) provided details of mathematical fuzzy logic models for the representation of complex traffic scenario. Fuzzy logic was used as a ‘universal approximator’ in data classification and analysis considering the inherent randomness of road traffic. These studies recommend the application of fuzzy logic in mode-split, trip-assignment, choice of routes, and also in the design of traffic signal controllers. Kalic and Teodorovic (1997; 2003) extended this technique for studies related to trip-generation and trip-distribution.

Nath et al. (2000) developed a method for the determination of income-based accessibility norms by the primary mode used by the trip-makers for analysing work trips in Delhi using the fuzzy logic approach. Information on travel-time, travel-cost and travel-distance were used in this study to assess the *degree of satisfaction* of usage of various modes using the fuzzy logic technique. In this investigation, the preference for travel modes such as, chartered buses, buses run by public-transport undertakings, motorized two-wheelers, and cars were analysed, and *nomograms* were developed to map membership values to various travel-related attributes such as travel time, travel cost and travel distance.

Mizutani and Akiyama (2001) investigated the use of logit models with fuzzy utility functions while performing travel demand modelling for Gifu City, Chukyo, Japan. In this study, the parameters of the membership functions were determined using genetic algorithm (GA). Vythoulkas and Koutsopoulos (2003) performed studies on assigning weights to simple rules in the fuzzy logic framework for mode-choice modelling. Artificial neural networks (ANNs) were also employed in this study in order to calibrate the weights applied to the rules. In this work, the de-fuzzification was performed using the center-of-gravity approach. The later part of the study proved that the traditional fuzzy logic models were more reliable and robust when compared to the logit models with fuzzy utility functions. Murugesan (2003) demonstrated the use of fuzzy-logic based approach in rating the quality of services provided by public and private modes of transport considering the level of comfort, convenience, noise and jerks, cleanliness, and maintenance of the modes of travel. In this study, the reliability of the fuzzy-based models was found to be comparable to the *numerical rating* method within a range of +/- 5%.

Errampalli et al. (2008) observed that the forecasted values based on the multinomial logit models (MNL) tended to over-estimate trips by private modes which was later rectified in a work carried out for Gifu City in Japan by applying fuzzy logic in the mode choice model on the former analysis. Murat and Uludag (2008) demonstrated that the fuzzy logic approach was more reliable in predicting route-choices compared to the logit method. Sarkar et al. (2012) performed studies on as trip generation, trip distribution, modal split, trip assignment and route choice using the fuzzy logic approach where the crisp vector inputs were mapped into crisp scalar outputs. Kumar et al. (2013) developed a fuzzy logic-based mode choice model called *FloMoChoMo* to study the correlations between the travel patterns of trip-makers, their mode-choice decisions, and the interrelationship to sensitivity to travel-policies. Sekhar (2014), and Ratrout et al. (2014) investigated the application of the fuzzy-logic approach in the study of driver-behaviour in heterogeneous traffic flow conditions.

4. Methodology

This section provides details on the methodology adopted in formulating the *fuzzy-logic* model to determine the probability of classifying a trip-maker under various *degrees of satisfaction*. The preliminary step in formulating the fuzzy-logic model is related to re-compilation of *revealed preference* (RP) data for computing the percentage of difference between the reported actual travel time, and the theoretical travel time. Subsequently, a *normal distribution* curve was fitted to the existing data spread over various classes. The various *degrees of satisfaction* were then assigned to the classified data using the *three sigma* rule.

The later stages of the work described in this paper provides details on the development of a *fuzzy logic* model using MATLAB. The overlapping of data between two consecutive fuzzy sets, the nature of fuzziness of data falling into two adjacent categories, and the probability of classifying the trip-maker under various categories of *degrees of satisfaction*, are explained using a *triangular membership function*. Similar studies can also be performed considering travel costs, and travel distances, based on which the mode-choice strategies adopted by trip-makers can be easily comprehended.

5. Selection of the Study Area, and Justifications for Fuzzy-based Analysis on Travel Time

Amritsar, is one of the busiest commercial city in India, with places of tourist attractions, and pilgrimage. The city has witnessed a tremendous growth of urban activity in the last few decades due to rapid urbanization. The travel desires of urban trip-makers depend mainly on the location of residential areas, and the proximity to various activity centers including places of work, education, shopping, and leisure. The road network is characterized by a radial cum- circumferential pattern covering an area of 139 sq. km.

CES and SYSTRA-MVA (2012) indicate that intermediate public transport systems such as auto-rickshaws and non-motorized cycle-rickshaws cater to totally 25.11% of trip-makers of the city, while bicycle and walk trips constitute 38.75% of the total trips. Also the trips performed by two-wheelers and cars constitute 26.48% and 6.78% respectively. The city does not have a public transport system per se, however, it possesses an informal public transport system comprising mini-buses operated by private owners catering to 2.80% of the total trips with an average trip length of 9km.

A closer analysis of the *revealed preference* data collected indicated that the average household monthly income of the trip-makers using private modes of travel such as cars, motorized two-wheelers, bicycles and walk, ranged between Rs. 48,000 and Rs 13,250, while the same for trip makers using IPT (intermediate modes of public transport) such as taxi/jeep, cycle rickshaw and auto-rickshaw ranged between Rs 57,500 and Rs 18,120. In the case of trip-makers using the existing informal public transport system such as the min-bus (including similar modes catering to school trips), the average household monthly income of the trip-makers was found to be approximately Rs 32,000. These statistics for the study area reveal that the monthly household income of trip-makers is generally above Rs 13,300 which indicates that these households belong to the top 20% of the country, while the median monthly household income in the country is about Rs 5000 (Desai and Vanneman, 2018). In the above scenario, it is possible to perform studies based on travel cost, travel distance, or even travel time, as the trip-makers are economically better-off, and can afford to think of traveling by alternative modes when the travel costs or travel times exceed their expectations. In this study, it was proposed to analyze the *degree of satisfaction* based on the percentage difference between the actual travel time and the theoretically computed average travel time.

Considering the heterogeneity of trip-makers belonging to various categories of income groups, and also considering the complex nature of decision-making exercised by these trip-makers, it was proposed to adopt a fuzzy-logic based approach for determining the *degrees of satisfaction* by the trip-makers.

6. Explanation of Various Stages in Data Analysis

6.1. Compilation of database and calculation of difference between the actual travel time and theoretical travel time

Data collected as part of a *revealed preference* study by CES and MVA Systra (2012), compiled by Kanthi (2012), and later corrected in various stages by researchers in NITK, Surathkal was used for this work. This database also provides information on the socio-demographic characteristics of both households and individuals. Errors in the database were corrected as part of the work performed by researchers at NITK, Surathkal, and the difference between the reported actual travel time, and the theoretical travel time were computed. The errors in the database could have been minimized if the average travel time taken between the origin and destination was considered based on travel data for 5 consecutive working days in a week in place of data for the previous day.

The information on *actual travel time* was obtained based on the trip-timings between the origin and destination zones for work trips, recreational trips, school trips, pilgrimage trips, and so on. In the case of trips by public transport and informal public transport as in Amritsar city, the time when the passenger boarded the transport mode, and the time when the passenger alighted the bus were considered to compute the *actual travel time*. In all, details of 5009 one-way trips were considered for analysis using the *fuzzy-logic* approach.

Based on previous studies performed by various researchers in NITK, the centroids of the zones of the city were computed from the location of the sub-zone centroids using the *moment area method*. Subsequently, the inter-zonal aerial distances (*Cij*) between the zone-centroids were computed using the Euclidean theory. Later, the percentage difference between the actual travel time and the theoretical travel time was computed for each trip-maker.

6.2. Grouping of 5009 data into various classes, fitting a normal distribution, and setting limits for various degrees of satisfaction

Table 1 provides details on the serial number for the data-form used in the surveys, the *start-time* and *end-time* for the trips performed, the actual travel time, the theoretical travel time, the type of travel-mode used, the percentage difference between the *actual travel time* (Actual_TT), and the *theoretical travel time* (Theoretical_TT). Based on the study of the percentage of trips for various categories of trip-purposes for the city of Amritsar, it was found that work-trips and educational trips together contributed to more than 90% of the total trips. Hence, it was decided to analyse the mode choice without considering the trip-purpose. Also, since work-trips and educational trips constitute a major share of the total trips, and since these trips are generally performed during the peak travel times, uniform travel speeds were assumed for various travel modes. The theoretical travel time between the zones (or the inter-zonal travel times), were obtained by assuming a speed of 25kmph for all motorized vehicles in Amritsar city, and a speed of 15kmph for bicycles in urban areas in general according to observations made by CSTEP (2014). The walk-speed was assumed as 5kmph according to investigations performed in Portland city by Carey (2005) based on MUTCD (1961; 2003) and Dewar (1992). Moreover, the speed for cycle-rickshaws was assumed as 8kmph based on studies conducted by Frédéric (2002) in Japan.

In the data analysed, it was seen that the percentage differences (% diff) varied between -107.88% and 94.07%. The values of the mean (μ), and the standard deviation (σ) were -8.49%, and 37.01 respectively. In order to fit a *normal distribution* curve to the data, the value of the probability mass function $f(x)$ of a *normal distribution* curve as given in equation 1 was computed, and the same is listed in Table 1.

$$f(x)=(1/\sigma\sqrt{2\pi})\ e^{-(x-\mu)^2/2\sigma^2} \tag{1}$$

where, μ = mean of the 5009 observed percentage differences between the actual travel time and the theoretical travel time ; σ = standard deviation of the observations; and x = percentage difference.

In the next step, it was required to group the 5009 observations into various classes. Table 2 provides details on the same. Fig. 1 provides details on the histogram showing the frequency distribution for observations classified under various classes. The histogram was developed using *MS Excel*. A *normal distribution* curve represented by a dotted line was fitted to the data as shown in the above mentioned figure for the computed mean and standard deviation.

The various *degrees of satisfaction* were then assigned to the classified data using the *three sigma* rule. In this approach, the normal distribution of the observed variable and the frequency of observations if first generated, and the mean (μ), and the standard deviation (σ) are observed. The limits of the observed variables that lie between μ-σ to μ+σ, μ-2σ to μ+2σ, and μ-3σ to μ+3σ are used to categorize the data into 3 classes. According to Pukelsheim (1994), 68.27%, 95.45% and 99.73% of the observed data must lie in the above mentioned three sigma ranges. A similar approach was adopted by Hsiao (1985) where grades were assigned to students. It may also be interesting to observe that Chen and Lee (1999) adopted a fuzzy-based method to classify the performance of students into 11 categories.

Table 1. Listing of trip-related details for a part of the 5009 observations compiled

Serial_No	Trip_Start_Time	Trip_End_Time	#Actual_TT	#Theoretical_TT	*Mode	#% diff (x)	$f(x)$
1	08:00:00	08:01:00	1	1.42	5	-6.21	0.01
2	17:00:00	17:01:00	1	0.87	5	-2.94	0.01
3	17:00:00	17:01:00	1	0.87	5	-2.94	0.01
4	05:00:00	05:01:00	1	1.44	4	-6.31	0.01
5	06:15:00	06:17:00	2	2.12	4	-5.25	0.01
6	17:30:00	17:35:00	5	4.34	1	13.18	0.01
7	10:00:00	10:01:00	1	1.46	2	-6.11	0.01
8	09:30:00	09:31:00	1	1.42	4	-9.58	0.01
9	09:30:00	09:32:00	2	0.87	4	57.31	0.00
10	09:30:00	09:32:00	2	0.87	4	54.55	0.00
11	06:10:00	06:14:00	4	4.99	4	-34.61	0.01
12	06:10:00	06:14:00	4	4.99	4	-33.51	0.01
-	-	-	-	-	-	-	-
5004	08:00:00	10:05:00	125	137.80	1	-10.56	0.01
5005	08:00:00	10:04:00	124	137.80	1	-10.81	0.01
5006	08:00:00	08:39:00	39	64.27	1	-65.77	0.00
5007	08:00:00	08:39:00	39	64.27	1	-65.77	0.00
5008	08:00:00	08:45:00	45	64.27	1	-42.82	0.01
5009	07:00:00	08:51:00	111	126.60	1	-14.23	0.01

Source: CES and MVA Systra (2012)
*Mode: 1-Walk; 2-Cycle; 3-Cycle Rickshaw; 4-Motorcyle (2W); 5-Auto Rickshaw-cum-Taxi; 6-Car; & 8-Mini Bus.
#Actual_TT= actual travel time (minutes), Theoretical_TT= theoretical travel time (minutes), % diff = percentage difference between the actual travel time and the theoretical travel time, $f(x)$ = probability mass function for normal distribution.

Note:

Since the share of taxis was insignificant, the count was merged with that of auto-rickshaws (classified under mode 5) as shown in the above table.

For the above data, mean = μ = -8.49%; and standard deviation = σ=37.01; minimum % diff = -107.88%; and maximum % diff = 94.07%.

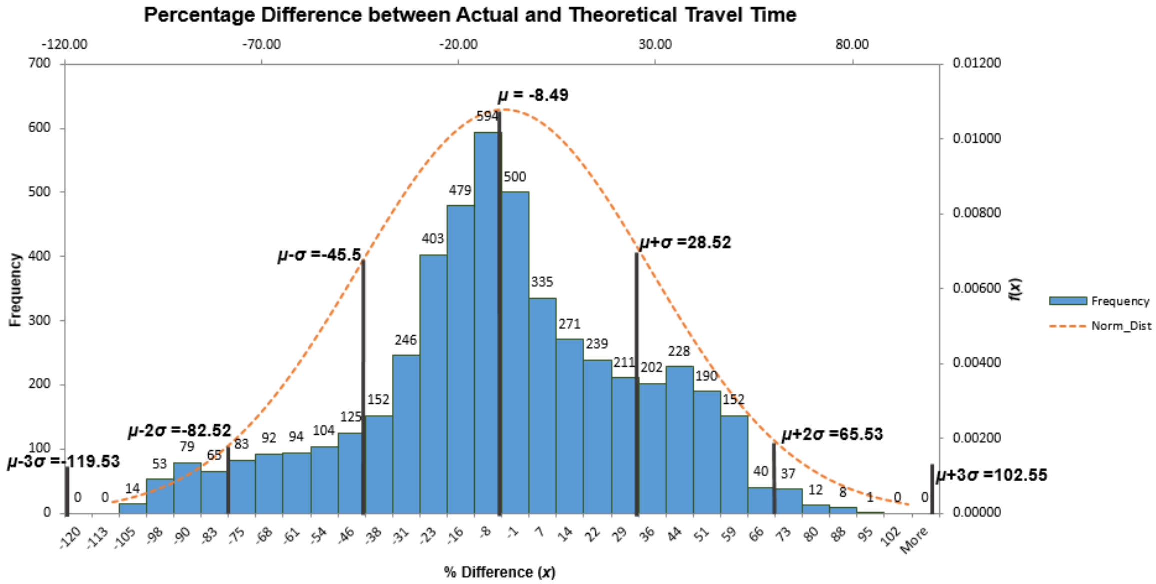

Fig. 1. Histogram showing the frequency distribution for observations classified under various classes and the Normal Distribution Curve

Table 2. Grouping of 5009 observations into various classes, and assigning of categories for degrees of satisfaction

Class	Frequency	% of observations	Category assigned
<-120	0	0	<μ-3σ
-120 to -113	0	0	μ-3σ to μ-2σ
-113 to -105	14	0.28	
-105 to -98	53	1.06	
-98 to -90	79	1.58	
-90 to -83	65	1.30	
-83 to -75	83	1.66	μ-2σ to μ-σ
-75 to -68	92	1.84	
-68 to -61	94	1.88	
-61 to -54	104	2.08	
-54 to -46	125	2.5	
-46 to -38	152	3.03	μ-σ to μ
-38 to -31	246	4.9	
-31 to -23	403	8.05	
-23 to -16	479	9.56	
-16 to -8	594	11.86	
-8 to -1	500	9.98	μ to μ+σ
-1 to 7	335	6.69	
7 to 14	271	5.41	
14 to 22	239	4.77	
22 to 29	211	4.21	
29 to 36	202	4.03	μ+σ to μ+2σ
36 to 44	228	4.55	

44 to 51	190	3.79	
51 to 59	152	3.03	
59 to 66	40	0.8	
66 to 73	37	0.74	$\mu+2\sigma$ to $\mu+3\sigma$
73 to 80	12	0.24	
80 to 88	8	0.16	
88 to 95	1	0.02	
95 to 102	0	0	
>102	0	0	$>\mu+3\sigma$
TOTAL	5009	100	

6.3. Classification of observations under six crisp categories, assigning of highest membership function value, and specifying limits for the fuzzy-set categories

In the above sub-section, it was mentioned that the data in hand was classified under six main crisp categories based on the *three sigma rule* as explained by Pukelsheim (1994). Based on the classification of the observations as provided under Table 2, it can be seen that for the first category of observations lying between -120% and -83%, the sub-category with observations lying between -98% and -90% has the highest number of observations. The mid-point (having a value of -94%) for this sub-category can be assigned a *membership function value* or *membership value* of 1 represented by the point M_1 in Fig. 2.

Similarly, for the second category of observations lying between -83% and -46%, the sub-category with observations lying between -54% and -46% has the highest number of observations represented by the point M_2. The mid-point (having a value of -94%) for this category can be assigned a *membership function value* or *membership value* of 1. In a similar manner, for each of the main categories, the sub-category with the highest number of observations is assigned a membership value of 1 as represented by points M_3, M_4, M_5, and M_6 in the above mentioned figure. The corresponding values on the abscissa for the points M_1, M_2, M_3, M_4, M_5, and M_6 are denoted as X_1, X_2, X_3, X_4, X_5, and X_6. This figure represents the classification of observations under six main crisp sets. The fuzzy-set classifications representing the six *degrees of satisfaction* are then denoted by the triangles OM_1X_2, $X_1M_2X_3$, $X_2M_3X_4$, $X_3M_4X_5$, $X_4M_5X_6$, and X_5M_6F.

The overlapping regions can be visualized in this figure. If the above 5009 datasets were classified based on crisp data ranges in place of a fuzzy-based classification approach, then the categorization of a number of datasets close to the boundaries of each class-range would have been unrealistic.

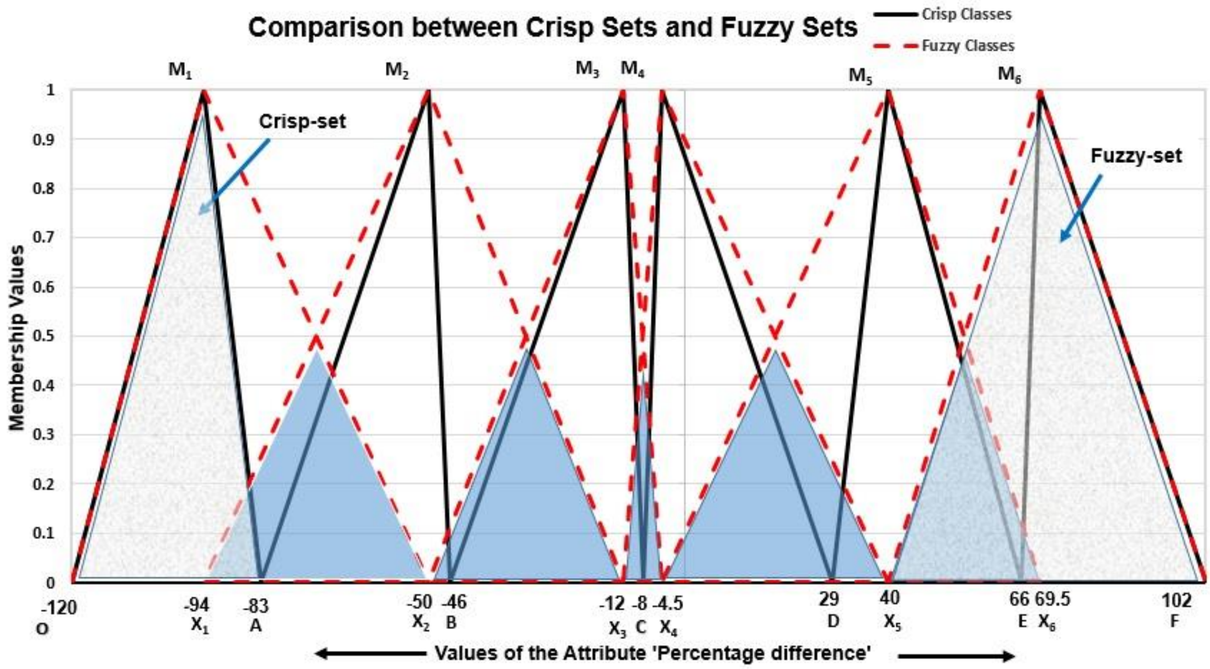

Fig. 2. A Representation of *Crisp Sets* and *Fuzzy sets* with overlapping regions

6.4. Preferred overlapping between fuzzy sets, and the overlaps adopted in this study

Considering the convenience in explanation, let us consider an example of two fuzzy-sets 1, and 2 with overlaps as shown in **Fig. 3.** Here, the *overlap scope* refers to the base of the region between the two intersecting sets where fuzziness is observed. The *adjacent MF scop*e (or *adjacent membership function scope*) refers to the base of the entire region encompassed by the two sets. The formula for computing the *overlap-ratio* is provided in equation 2 as proposed by Kalpana (2015):

$$\text{Overlap ratio} = \text{Overlap Scope}/\text{Adjacent MF Scope} \tag{2}$$

According to Helm and Hahsler (2007), the overlaps between two fuzzy sets should preferably be lesser than 50% due to the above mentioned reason. Kalpana (2015) suggested that the overlap may range between 25% and 50% for two triangular fuzzy sets. However, in a number of studies, Tang (2017) observes that the nature of overlap could also be more than 50%, or even lesser than 25% based on the decomposition of data, and the needs for ranking, and that a specific limit could not be prescribed. Smithson (2012) observes that the effectiveness of the fuzzy-logic based approach will also depend to a significant extent on the nature of data. For the present study, the *overlap ratios* were computed for the six main categories of *degrees of satisfaction* as, 0.407, 0.425, 0.083, 0.546, and 0.277 based on Fig. 2.

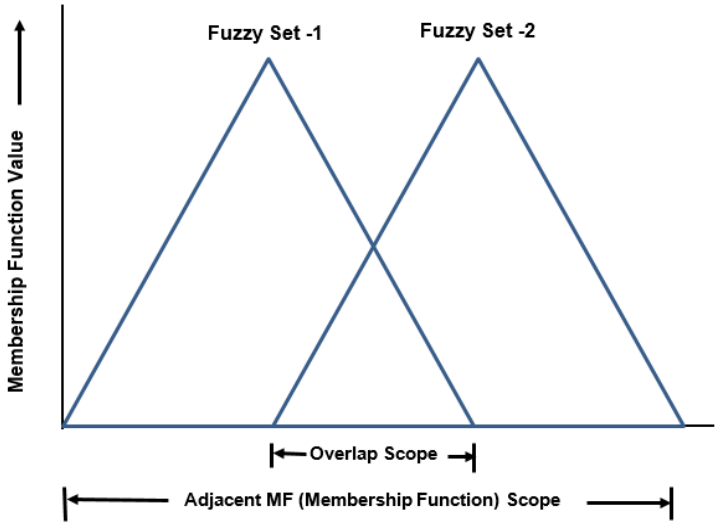

Fig. 3. A Representation of *Overlap* and *Adjacent MF scope* (Kalpana, 2015)

6.5. Development of a fuzzy logic model in MATLAB

The fuzzy logic model was developed in MATLAB by using the *Fuzzy Logic Designer* tool available under the *APPS* option. Using the *fuzzy-logic designer* toolbox in MATLAB, the *.fis* file comprising the input and output parameters was developed. In this exercise, the percentage differences between the *actual travel time* and the *theoretical travel time* were provided as the input. The upper and lower limits considered for classifying the data under various categories were also provided. The algorithm comprising the steps to be followed in the analysis procedure for MATLAB was then formulated. The membership function values for the data points were then computed using MATLAB. Fig. 4 provides a screenshot of the *Fuzzy Logic Designer* interface. The box at the left-hand side represents the *input* for percentage differences in travel time, the box in the middle represents the *.fis* file that comprises the set of rules to be applied, and the box at the right-hand side represents the *output* that comprises details on various categories of *degrees of satisfaction*. The *degrees of satisfaction* were categorised into 6 levels namely, *highly satisfied, satisfied, acceptable, dissatisfied, highly dissatisfied* and *extremely dissatisfied*. *Membership function* values ranging between 0 and 1 were assigned to these categories.

A similar approach was adopted by Nath et al. (2000), where nomograms were used for the determination of membership function values (for various input parameters such as travel time, travel distance and travel cost). However, in the present study, each of the six major categories identified based on the *three-sigma rule* were further sub-divided into 5 sub-categories, and the use of triangular membership functions was demonstrated to derive the *degrees of satisfactions* in place of nomograms.

6.6. Assigning input and output parameters to MATLAB for fuzzy based analysis, and computation of membership function values

From Table 2, it can be seen that the observations of the percentage differences between the *actual travel time* and the *theoretical travel time* range between -120% and 102%. Also, the 5009 observations were planned to be classified under six fuzzy-sets as explained in Section 4.4. Hence, the values for the *input range* and the *output range* may be specified to vary between -120% and +102%, and between 0 and 6 respectively.

For each of the six triangular fuzzy sets, it is also required to provide details on the two extreme base points (represented by the abscissa values *a* and *c*) for the fuzzy triangles, and the abscissas representing the membership values of 1 (indicated by the abscissa value b). Thus the values to be provided as input for a fuzzy triangle for a particular category can be represented as [*a b c*].

Hence, for fuzzy triangles representing trip-makers who are *highly satisfied* (*HS*), the values assigned are [-120 -94 -50]. In a similar manner, for trip-makers who are *satisfied* (*S*), the values assigned are [-94 -50 -12]; for trip-makers who find the travel times *acceptable* (*A*), the values assigned are [-50 -12 -4.5]; for trip-makers who are *dissatisfied* (*D*), the values assigned are [-12 -4.5 40]; for trip-makers who are *highly dissatisfied* (*HiDis*), the values assigned are [-4.5 40 69.5], and for trip-makers who are *extremely dissatisfied* (*ExDis*), the values assigned are [40 69.5 102]. See Fig. 5 for a screenshot providing details on input parameters for the triangular membership functions for various categories.

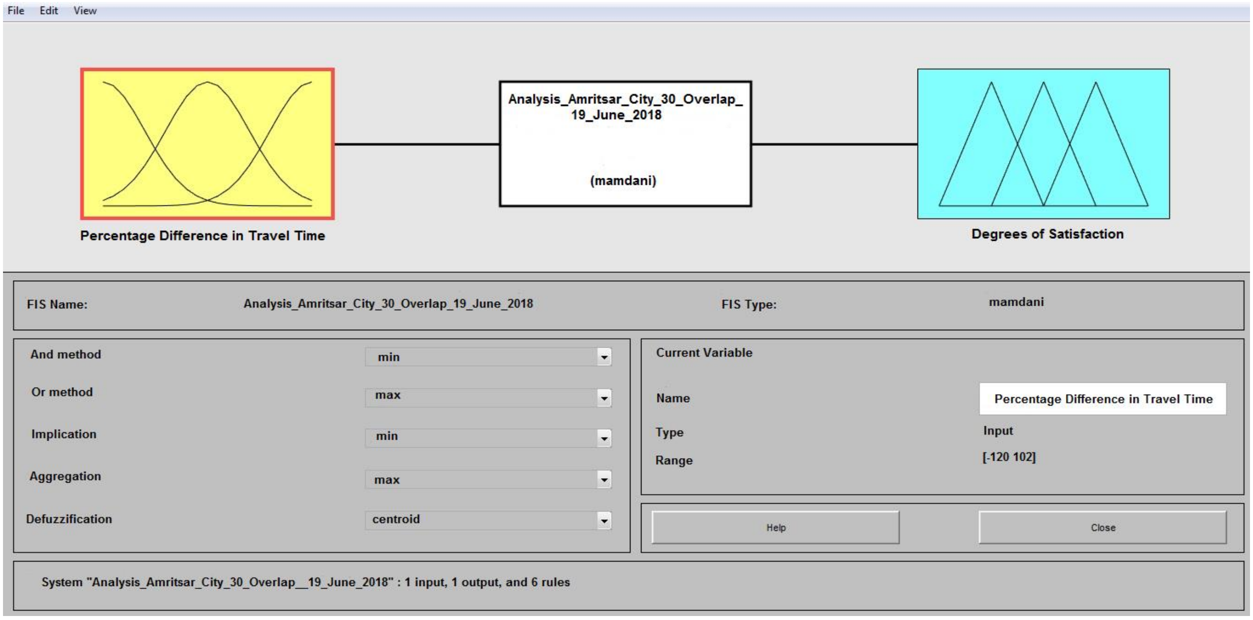

Fig. 4. Mamdani type fuzzy logic toolbox

In a similar manner, the output values also need to be scaled between 0 and 6 for percentage values ranging between -120 and +102. Thus, for output fuzzy triangles representing trip-makers who are *highly satisfied* (*HS*), the values assigned are [0 0.702, 1.89]. In a similar manner, for trip-makers who are *satisfied* (*S*), the output values assigned are [0.702 1.89 2.92]; for trip-makers who find the travel times *acceptable* (*A*), the output values assigned are [1.89 2.92 3.12]; for trip-makers who are *dissatisfied* (*D*), the output values assigned are [2.92 3.12 4.33]; for trip-makers who are *highly dissatisfied* (*HiDis*), the output values assigned are [3.12 4.33 5.52], and for trip-makers who are *extremely dissatisfied* (*ExDis*), the output values assigned are [4.33 5.52 6]. See Fig. 6 for a screenshot providing details on output parameters for the triangular membership functions for various categories.

It is also required to compute the membership function values of each of the 5009 observations. Barai (2000) and Viertl (2011), suggest that the membership function values for each of the observations in a triangular fuzzy set may be computed using the following formula:

$$\text{if } x \le a, \text{ then } y = 0 \tag{3}$$
$$\text{if } a \le x \le b, \text{ then } y = x-a/b-a \tag{4}$$
$$\text{if } b \le x \le c, \text{ then } y = c-x/c-b \tag{5}$$
$$\text{Alternatively, } y' = 1-y \tag{6}$$

In the case of observations that are found to lie in the overlapping regions between two fuzzy sets, the above mentioned equations (3) to (6) can be represented as:

$$\text{Triangle } (x; a, b, c) = \max\{\min[(x-a)/(b-a), (c-x)/(c-b)], 0\} \tag{7}$$

where *x* is the intermediate value (percentage difference in travel time); *a*, *b*, *c* are the parameters of the triangular membership function, *y* is the degree of membership function, and *y'* is the alternative value of degree of membership function of the same intermediate value *x* in case of overlapping.

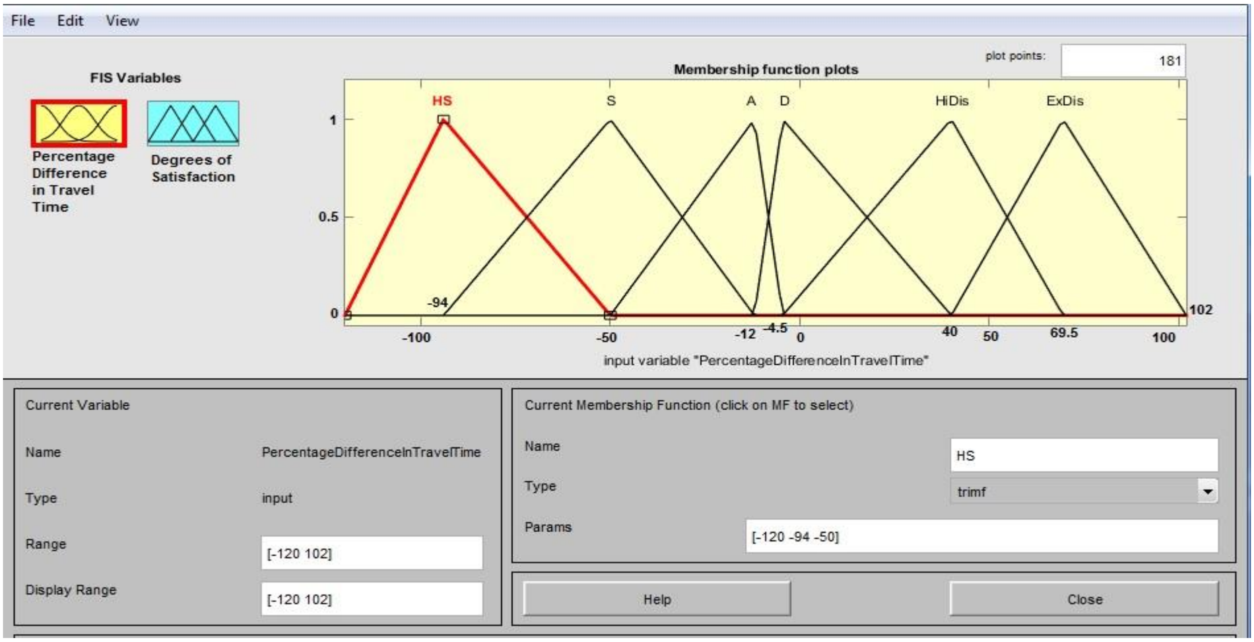

Fig. 5. Input triangular membership functions showing the range of values and the overlap between the fuzzy sets

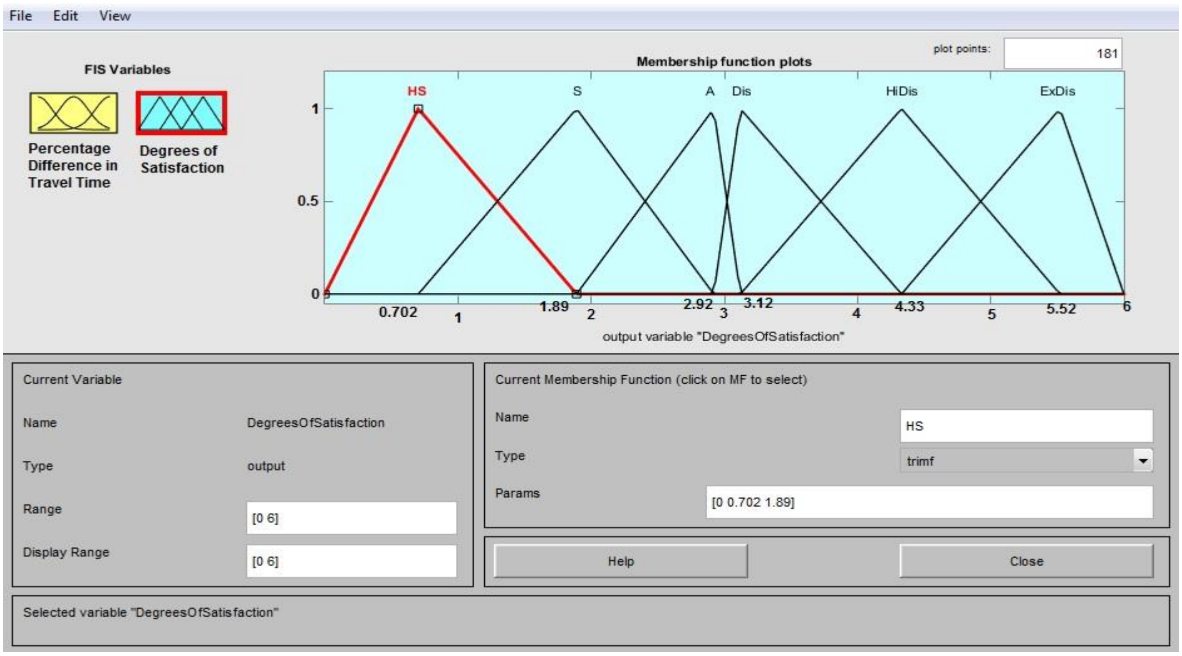

Fig. 6. Output specification for triangular membership functions with details on the scaled range and overlap

6.7. Structure of the fuzzy inference system, and formulation of fuzzy-rules

Fig. 7 shows the structure of a fuzzy inference system (FIS) adapted and modified based on the structure proposed by Kumar et al. (2013). The FIS comprises 2 sections: 1) the user interface where the input and output parameters and the corresponding rules connecting them are provided; and, 2) a fuzzy inference processor within which the FIS inputs are fuzzified, acted upon by the rules, and the fuzzified outputs are generated and converted into crisp values by defuzzification.

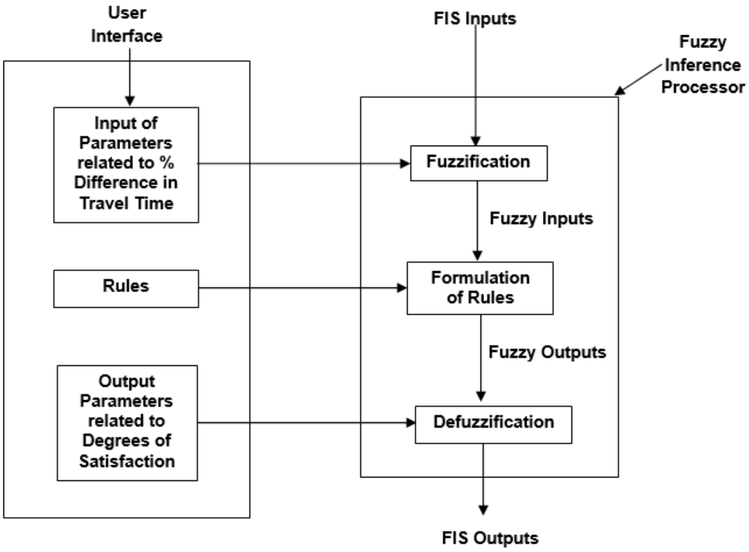

Fig. 7. The Structure of a Fuzzy Inference System (FIS)
Adapted and modified based on Kumar et al. (2013)

It may be observed that based on the input and output parameters provided through the *Fuzzy Logic Designer Toolbox*, MATLAB generates the defuzzified output for each of the 5009 observations based on the centroid method as explained by many researchers including Bourke (1988) and Samanta and Panchal (2016). The set of rules for obtaining the defuzzified (or *crisp values*) based on the centroid method was formulated according to the procedure adopted by Teodorovic (1999) and Hrehova and Mizakova (2013). In this approach, the defuzzified value is obtained based on the distance of the centroid of the subject value in the overlapping region to the y-axis. The nearness of the centroid to a particular category can be compared based on the *membership function value*. The following six rules as also displayed in the *rule editor window* shown in Fig. 8 were formulated for the present study:

If (Percentage Difference in Travel Time is HS) then (Degrees of Satisfaction is HS) (1)
If (Percentage Difference in Travel Time is S) then (Degrees of Satisfaction is S) (1)
If (Percentage Difference in Travel Time is A) then (Degrees of Satisfaction is A) (1)
If (Percentage Difference in Travel Time is D) then (Degrees of Satisfaction is D) (1)
If (Percentage Difference in Travel Time is HiDis) then (Degrees of Satisfaction is HiDiS) (1)
If (Percentage Difference in Travel Time is ExDis) then (Degrees of Satisfaction is ExDis) (1)

An algorithm was developed in MATLAB using the *editor* window as part of this study to compute the *membership function value* for each of the 5009 observations such that these can be classified under specific linguistic categories.

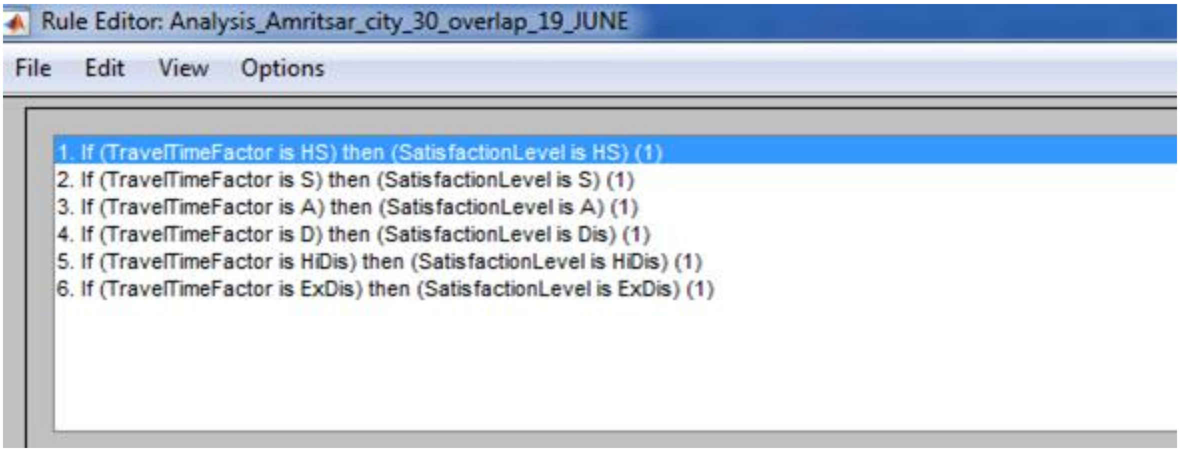

Fig. 8. A screen shot of the *rule editor window* showing the set of rules

7. Explanation of mathematical computations for determination of *membership function values*, computation of corresponding defuzzified values, and verification with output obtained using MATLAB

This section provides explanations on the mathematical computations involved in determination of membership function values and the computation of the corresponding defuzzified values for a specific observation under two sub-sections.

7.1. Computation of membership function values y_1, and y_2 for point P based from triangles $X_1M_2X_3$ and $X_2M_3X_4$

Let us consider an observation where the percentage difference between the *actual travel time* and the *theoretical travel time* is -17.245, represented in Fig. 9 as point P. This point falls within the crisp set represented by triangle BM_3C. This point also falls within the fuzzy regions defined by triangles $X_1M_2X_3$ and $X_2M_3X_4$. The projection of point P meets the fuzzy triangle $X_1M_2X_3$ at P_1 and meets the fuzzy triangle $X_2M_3X_4$ at P_2. The membership values at P_1 and P_2 are 0.138 and 0.862 respectively.

7.2. Computation of corresponding defuzzified value for point P

It was mentioned in the previous sub-section that for the values for the *input range* that varies between -120% and +102%, MATLAB provides a modified output rescaled to vary between 0 and 6. This was implemented through MATLAB as shown vide Fig. 6. In reality MATLAB attempts to scale the information provided in Fig. 10 in the rescaled output form as shown in Fig. 10. This section provides details on the basic mathematical computations involved in computing the rescaled values, and attempts to verify the result with the rescaled output obtained using MATLAB.

It may be also observed that points P, P_1, P_2, P_3, P_4, and P_5 of Fig. 9, can now be referred to points P', P'_1, P'_2, P'_3, P'_4, and P'_5 in the rescaled diagram shown in Fig. 10. The point P of -17.245% has now been represented as P' with a rescaled value of 2.35. Additionally, it may be observed that the point P' is part of the fuzzy triangles $X'_1M'_2P'$ and $X'_2M'_3X'_4$. The corresponding areas enclosed by the rescaled *membership function values* are shown using the shaded portions as in Fig. 11. The MATLAB window too provides a similar scaled output as shown in Fig. 12. This figure shows that the rescaled point P' lies between the fuzzy sets representing *satisfied* and *acceptable* degrees of satisfaction.

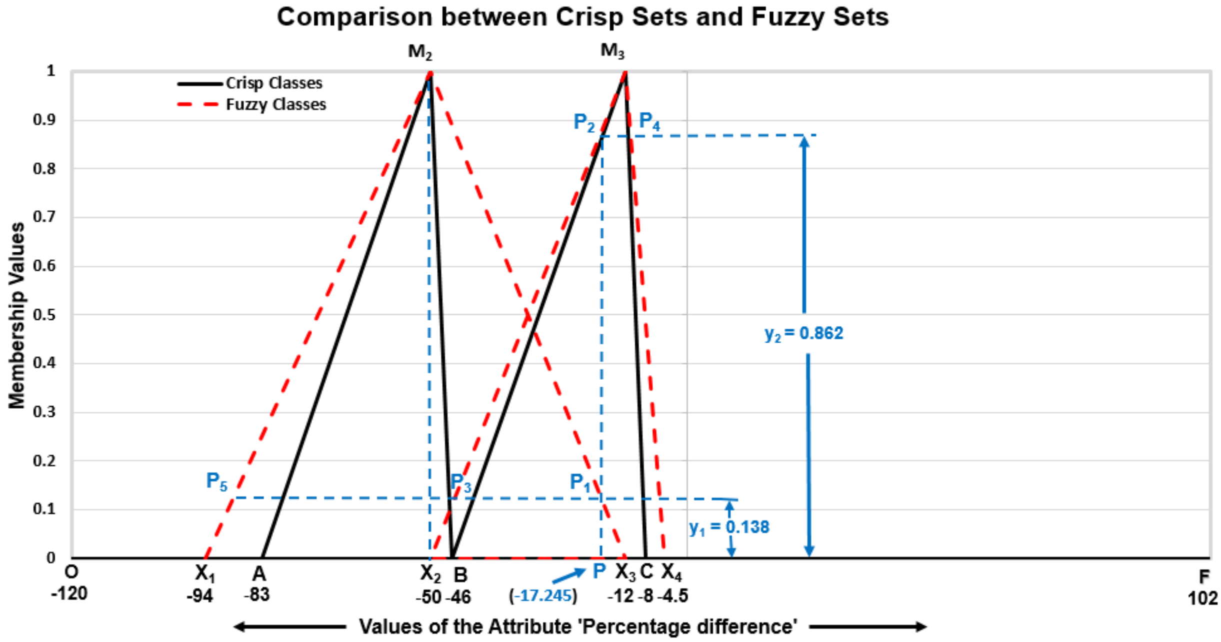

Fig. 9. Membership function values for the percentage difference -17.245 scaled between -120 and +102

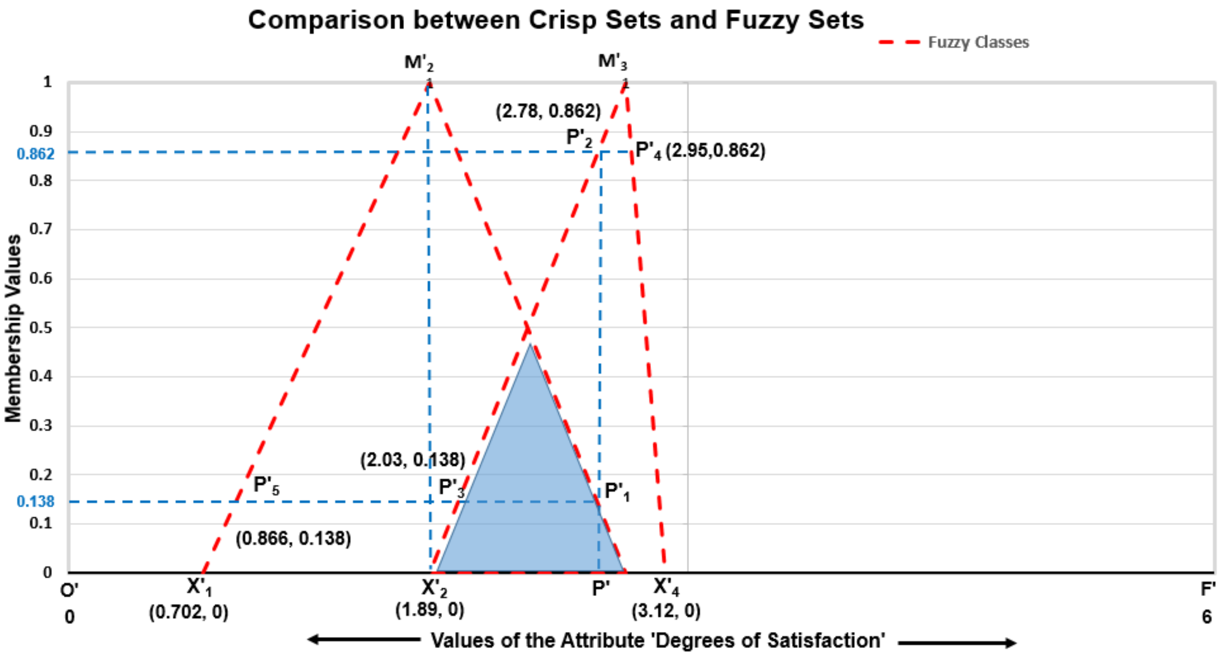

Fig. 10. Membership function values for the percentage difference -17.245 scaled between 0 and 6

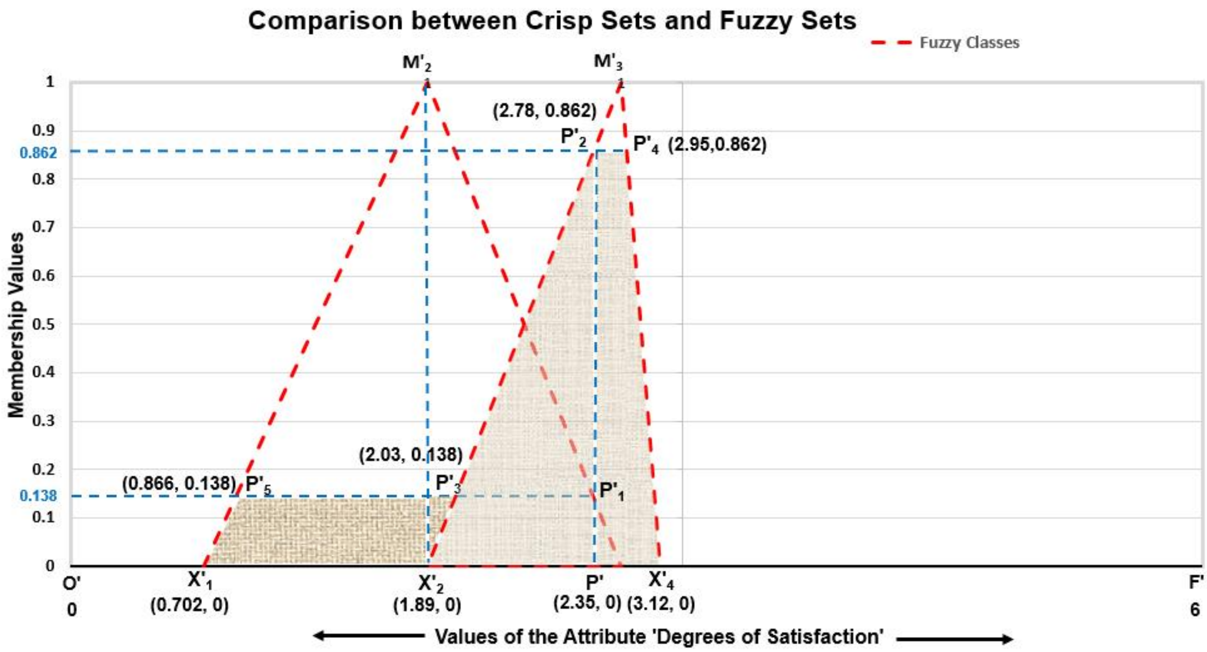

Fig. 11. Polygonal Area enclosed by the scaled *membership function values* for Point P’

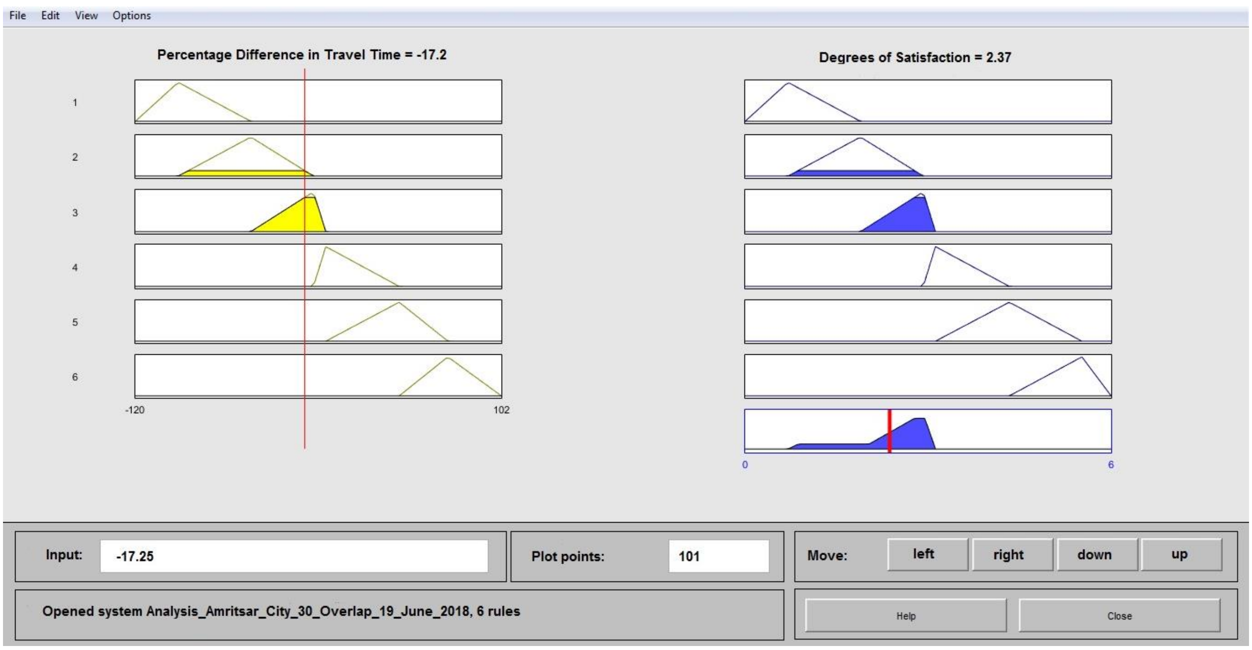

Fig. 12. The MATLAB *rule viewer window* that provides the defuzzified rescaled output value of 2.37 for Point P’ based on the value of -17.25 for point P

The background computation performed in MATLAB is based on the determination of centroids using the moment of area method as explained by by Bourke (1988) and Samanta and Panchal (2016) based on the following formula:

$$A = (1/2)\sum_{i=1}^{n}(x_i\, y_{i+1} - x_{i+1}\, y_i) \tag{8}$$

$$Cx = (1/6A)\sum_{i=1}^{n-1}(x_i + x_{i+1})(x_i\, y_{i+1} - x_{i+1}\, y_i) \tag{9}$$

where, x is the abscissa and the y is the ordinate of the vertices of the irregular polygon $X'_1P'_5P'_3P'_2P'_4X'_4$; A is the area of the polygon $X'_1P'_5P'_3P'_2P'_4X'_4$; and Cx is the centroid distance of the percentage difference -17.245 (at point P) from the y-axis.

8. Further computations performed for a selected set of observations and comparisons with MATLAB output

Table 3. Linguistic output obtained using MATLAB for the set of selected percentage differences between the actual and theoretical travel times

Data No	Actual %	Out_defuzz	Mem_Func_value_for _former_fuzzy_set	Mem_Func_value_for _later_fuzzy_set	Classification of response	
					Former fuzzy class	Later fuzzy class
1	-6.20652	3.229438	0.227535624	0.772464376	Acceptable	Dissatisfied
2	-2.94413	3.53638	0.965036627	0.034963373	Dissatisfied	Highly Dissatisfied
3	-2.94413	3.53638	0.965036627	0.034963373	Dissatisfied	Highly Dissatisfied
4	-6.30649	3.220833	0.240864776	0.759135224	Acceptable	Dissatisfied
5	-5.2521	3.332936	0.100279338	0.899720662	Acceptable	Dissatisfied
6	13.1776	4.023473	0.602750562	0.397249438	Dissatisfied	Highly Dissatisfied
7	-6.10736	3.238113	0.21431479	0.78568521	Acceptable	Dissatisfied
-	-	-	-	-	-	-
2125	-8.25446	3.089195	0.500594005	0.499405995	Acceptable	Dissatisfied
-	-	-	-	-	-	-
4892	42.462	4.370829	0.91654226	0.08345774	Highly Dissatisfied	Extremely Dissatisfied
-	-	-	-	-	-	-
5007	-65.7696	1.537316	0.35840098	0.64159902	Highly satisfied	Satisfied
5008	-42.8224	1.88601	0.811115789	0.188884211	Satisfied	Acceptable
5009	-14.2314	2.50705	0.058720096	0.941279904	Satisfied	Acceptable

Actual % = Percentage difference between actual and theoretical travel times; Out-defuzz = defuzzified output obtained from MATLAB; Mem_Func_value_for _former_fuzzy_set = y as explained in Section 4.6; Mem_Func_value_for _later_fuzzy_set = $y' = 1-y$; Classification = categorization in linguistic terms for two adjacent fuzzy sets

This section provides details on further computations performed for a selected set of observations of differences in percentages between the *actual travel time* and the *theoretical travel time* based on the approach mentioned above as shown in Table 3. The values of a series of selected observations were verified and found to be the same as the values for *Out_defuzz* listed in the above mentioned table. This table also provides details on the membership function values for the former and later fuzzy sets express the probability of the observation falling within the fuzzy sets.

9. Conclusions of the Study

In the initial part of the study, the percentage difference between the actual time taken for travel, and the theoretical travel time based on average vehicular speeds was computed for 5009 trip-makers, and it was found that the percentage difference varied from -107.88% to 94.07%. Based on these values, the probability mass function $f(x)$

was determined and a normal distribution curve was fitted. Subsequently, the data was further regrouped under six crisp categories representing various *degrees of satisfaction* based on the *three sigma* rule. In this approach, the percentage of values that lie within a boundary around the mean in a normal distribution with a width of two (μ-σ to μ+σ`), four (μ-2σ to μ+2σ) and six (μ-3σ to μ+3σ) standard deviations are 68.27%, 95.45% and 99.73%, respectively, as observed by Pukelsheim (1994). For the data presented in this study, the limits μ-3σ to μ+3σ, μ-2σ to μ+2σ, and μ-σ to μ+σ included 100% or 5009 observations, 94.63% or 4740 observations, and 68.48% or 3430 observations, respectively. This indicated that the *three sigma* rule adopted in classifying the data into six categories of *degrees of satisfaction* was reasonably correct. Later, the 5009 observations were grouped into 30 classes.

Further studies were made on the preferred overlaps to be maintained for the fuzzy sets. Investigations conducted by Helm and Hahsler (2007), and Kalpana (2015) indicated that the overlaps maintained could vary between 25 and 50%. However, Tang (2017) concluded that the overlap could also be more than 50%, or even lesser than 25%. Smithson (2012) observed that the effectiveness of the fuzzy-logic based approach will also depend to a significant extent on the nature of data. For the present study, the *overlap ratios* were computed for the six main categories of *degrees of satisfaction* as, 0.407, 0.425, 0.083, 0.546, and 0.277.

The fuzzy logic model was then developed in MATLAB where the percentage differences between the *actual travel time* and the *theoretical travel time* were provided as the input. The algorithm comprising the steps to be followed in the analysis procedure for MATLAB was then developed. The membership function values for the data points were then computed using MATLAB. The *degrees of satisfaction* were categorised into six linguistic levels namely, *highly satisfied, satisfied, acceptable, dissatisfied*, *highly dissatisfied* and *extremely dissatisfied*. A similar approach was adopted by Nath et al. (2000).

Subsequently, this paper provides explanations on the mathematical computations involved in the determination of membership function values, computation of the corresponding defuzzified values for fuzzy sets, and verification with output obtained from MATLAB for a specific observation. In a similar manner, a series of selected observations were verified and found to be the same as the output values obtained from MATLAB as listed under *Out_defuzz* in Table 3. The membership function values for the former and later fuzzy sets in this table indicate the probability of the observation falling within the two adjacent fuzzy sets.

The above study was focussed on achieving a basic understanding of the mathematical computations involved in defuzzification using the *centroid method* for triangular membership functions in the fuzzy-logic approach, and provides a comparison to results obtained using MATLAB. This study will provide the basic framework for analysis of the entire set of observations and classification of the responses under various categories of *degrees of satisfaction* in order to make policy decisions related to mode-choice based on travel time characteristics, and other attributes.

Acknowledgements

The authors would like to express their gratitude to Mr. Kaustubh Saysardar, Mr. Hithesh Kumar, and Ms. Shilpa R. Nair, M.Tech scholars of NITK, Surathkal for their valuable help and support in the completion of the initial computations related to this work.

References

Barai, S.V., 2000. Fuzzy Inference Based Procedure for Visual Inspection of Steel Bridges. *IRC Journal*, *61*(3), pp.525-545.

Bourke, P., 1988. Calculating the area and centroid of a polygon. *Swinburne Univ. of Technology*.

Carey, N., 2005. Establishing pedestrian walking speeds. *Portland State University*.

Census, 2011. Census of India (2011) Punjab: District Census Handbook: Amritsar Series 04, Part 12B Published by Directorate of Census Operations Punjab. Available at https://www.censusindia.gov.in/2011census/DCHP/0315_PART_B_DCHB%20_AMRITSAR.pdf.

CES and Systra-MVA, 2012. Comprehensive Mobility Plan for Various Cities of Punjab State (Package-1): Amritsar City, Draft Final Report Document No. 2010112; March 2012, Prepared by Consulting Engineering Services Pvt. Ltd. New Delhi (CES), and SYSTRA-MVA Consulting (India) Pvt. Ltd. (SYSTRA-MVA) for Punjab Municipal Infrastructure Development Company, Dept. of Local Government of Punjab. Available at: http://wricitieshub.org/sites/default/files/Comprehensive%20Mobility%20Plan%20for %20Amritsar%20City.pdf.

Chen, Shyi-Ming and Lee, Chia-Hoang, 1999. New Methods for Students' Evaluation using Fuzzy Sets. *Fuzzy Sets and Systems, 104 (2)*, pp.209-218.

CSTEP (2014). Review of Urban Transportation India prepared by *Centre for the Study of Science, Technology & Policy for Institute of Urban Transport* (*India*), pp.11. Available at http://www.indiaenvironmentportal.org.in/files/file/Review%20of%20Urban%20Transport%20in%20India.pdf. Accessed on 21 June 2017.

Desai, S., and Vanneman, R., 2018. India Human Development Survey-II (IHDS-II): 2011-12, ICPSR36151, *Inter-university Consortium for Political and Social Research* [*distributor*], *Ann Arbor*, *Michigan*. Document uploaded on 2018-08-08. Available at https://doi.org/10.3886/ICPSR36151.v6. Accessed on 9th Dec 2018.

Dewar, R.E., 1992. Driver and Pedestrian Characteristics. *In Traffic Engineering Handbook* (*4th Edition, J. Pline, ed*), *Published by Prentice Hall*, *Englewood Cliffs*, *N.J.*, pp.1-28.

Errampalli, M., Okushima, M. and Akiyama, T., 2008. Fuzzy Logic Based Lane Change Model for Microscopic Traffic Flow Simulation. *JACIII*, *12*(2), pp.172-181.

Frédéric, L., 2002. Japan Encyclopaedia. *Translated by* Käthe Roth. *Harvard University Press*. *Cambridge, Massachusetts*. *London, England*. ISBN 0674017536, p.424.

Helm, B.L. and Hahsler, P.D.D.M., 2007. Fuzzy Association Rules. *Vienna University of Economics and Business Administration*.

Hrehova, S. and Mizakova, J., 2013. Evaluation a Process using Fuzzy Principles and Tools of MATLAB. *Proceedings of the 2013 International Conference on Systems, Control and Informatics*.

Hsiao, F.S.T., 1985. A Statistical Method of Grading-Theory and Practice, *Comput. Educ*., *9 (4)*, pp.227-233.

Kalic, Â. M., and Teodorovic, Â. D., 1997. "A soft computing approach to trip generation modeling." *9th Mini EURO Conf. Fuzzy Sets in Traffic and Transport Systems, EURO, Univ. of Belgrade, Belgrade, Serbia*.

Kalic, Â. M., and Teodorovic, Â. D., 2003. "Trip distribution modelling using fuzzy logic and genetic algorithm." *Transp. Plann. Technol., 26(3)*, pp.213–238.

Kalic, M. and Teodorovic, D., 1996. Solving the trip distribution problem by fuzzy rules generated by learning from examples. *In Proceedings of the XXIII Yugoslav Symposium on Operations Research*. pp.777-780.

Kalpana, M., 2015. Fuzzy Expert System for Diabetes using Reinforced Fuzzy Assessment Mechanisms.

Kanthi, R.V., 2012. Multinomial Logit and Nested Logit Models for Integrated Studies on Vehicle ownership and Mode choice for Amritsar City using BIOGEME and NLOGIT based RP data. M.Tech Thesis, NITK.

Kedia, A., Saw, K., and Katti, B.K., 2015. Fuzzy Logic Approach in Mode Choice Modelling For Education Trips: A Case Study of Indian Metropolitan City, *Special Issue on Smart and Sustainable Transport, 30(3)*, pp.286–293. doi:10.3846/16484142.2015.1081279

Kompil, M. and Celik, H. M., 2013. Modelling trip distribution with fuzzy and genetic fuzzy systems, *Transportation Planning and Technology 36(2)*, pp.170–200. Available at http://dx.doi.org/10.1080/03081060.2013.770946. Accessed on 9th Dec 2018.

Kumar, M., Sarkar, P. and Madhu, E., 2013. Development of Fuzzy Logic Based Mode Choice Model considering various Public Transport Policy Options. IJTTE 2013.3(4).05, pp.408–425.

Mizutani, K. and Akiyama, T., 2001. Construction of modal choice model with a descriptive utility function using fuzzy reasoning. In *IFSA World Congress and 20th NAFIPS International Conference, 2001. Joint 9th* (Vol. 2, pp.852-856). IEEE.

Murat, Y. Z., and Uludag, N., 2008. Route choice modelling in urban transport network using fuzzy logic and logistic regression methods. *J. Sci. Ind. Res*., *67(1)*, pp.19–27.

Murugesan, R., 2003. Comparison of Level of Public Transport Service of Different Ownerships by Numerical Rating and Fuzzy Set Approaches: TRB 2003 Annual Meeting CD-ROM.

MUTCD, 1961. Manual on Uniform Traffic Control Devices for Streets and Highways. Washington, DC: *Bureau of Public Roads*.

MUTCD, 2003. Manual on Uniform Traffic Control Devices for Streets and Highways. Washington, DC: *FHWA*.

Nath, B., Sarkar, P.K., Saha, S.K. and Deb, S.K., 2000. An Application of Fuzzy Set Theory in Context of Choice of Predominant Travel Modes for Journey to Work in Delhi. HRB-IRC, No. 63, pp.73-82.

Pukelsheim, F., 1994. The Three Sigma Rule: American Statistician. 48, pp.88-91.

Ratrout, N.T., Gazder, U. and Al-Madani, H.M., 2014. A review of mode choice modelling techniques for intra-city and border transport. *World Review of Intermodal Transportation Research*, *5*(1), pp.39-58.

Samanta, D. and Panchal, G., 2016. Advances in Soft Computing. *Soft Computing Applications in Sensor Networks*, p.21.

Sarkar, A., Sahoo, G. and Sahoo, U.C., 2012. Application of fuzzy logic in transport planning. *International Journal on Soft Computing*, *3*(2), p.1.

Sarkar, P.K., Saha, S.K. and Deb, S.K., 2002. Development of Accessibility Standards by Using Fuzzy Set Theory in Context of Journey to Work in Delhi Urban Area, India. *Urban Mobility for All: La Mobilité Urbaine pour Tous*, p.95.

Sekhar, C., 2014. Mode Choice Analysis: The Data, the Models and Future Ahead. *International Journal for Traffic & Transport Engineering*, *4*(3).

Smithson, M., 2012. Fuzzy set analysis for behavioral and social sciences. *Springer Science & Business Media*.

Tang, H.C., 2017. Decomposition and Intersection of Two Fuzzy Numbers for Fuzzy Preference Relations. *Symmetry*, *9*(10), p.228.

Teodorovic, D. and Kikuchi, S., 1990, December. Transportation route choice model using fuzzy inference technique. In *Uncertainty Modelling and Analysis, 1990. Proceedings. First International Symposium on* (pp. 140-145). IEEE.

Teodorovic, D., 1999. Fuzzy Logic Systems for Transportation Engineering: The State of the Art. Transportation Res. Part A 33 (1999), pp.337-364.

Viertl, R., 2011. Statistical Methods for Fuzzy Data: 2011 John Wiley & Sons, Ltd.

Vythoulkas, P.C. and Koutsopoulos, H.N., 2003. Modelling discrete choice behavior using concepts from fuzzy set theory, approximate reasoning and neural networks. *Transportation Research Part C: Emerging Technologies*, *11*(1), pp.51-73.

Wang, L.X. and Mendel, J.M., 1992. Generating fuzzy rules by learning from examples. *IEEE Transactions on systems, man, and cybernetics*, *22*(6), pp.1414-1427.

Zadeh, L.A., 1965. Fuzzy Sets. Department of Electrical Engineering and Electronics Research Laboratory, University of California, Berkeley, California. (Information and Control 8), pp.338-353.