Available online at www.sciencedirect.com

ScienceDirect

Transportation Research Procedia 00 (2018) 000–000

Transportation Research Procedia

www.elsevier.com/locate/procedia

World Conference on Transport Research - WCTR 2019 Mumbai 26-31 May 2019

A heuristic technique for traffic assignment with variable step size and number of iteration

Alireza Mahpour[a*], Amirreza Mamdoohi[b], Ali Hakimelahi[c]

[a] *Assistant Professor, Shahid Beheshti University, Tehran, Iran.*
[b] *Associate Professor, Tarbiat Modares University (TMU), Tehran, Iran.*
[c] *Lecturer, Shiraz University, Shiraz, Iran.*

[*] *Corresponding Author email: a_mahpour@sbu.ac.ir*

Abstract

Prediction of traffic flows on network links is an important issue in urban transportation modeling and planning. Although manual assignment methods are possible for very small networks, the networks involved in practical-size problems usually require complex calculations with large amounts of data. In the traditional incremental traffic assignment technique (TITAT), the O-D matrix is divided into N equal portions and assigned to the network at each iteration. Travel times are then updated, based on which the next portion of the O-D matrix is loaded onto the network. In this paper, the assumption of equal step size in all iterations is relaxed and variable step size for the different iterations is proposed to increase the efficiency of this technique. It is shown that intelligent choice of step size can cause more efficiency. To implement this concept quantitatively for a real-size problem, Mashhad as the second largest city of Iran with about 4.5 million trips per year is used as the case study to analyze and evaluate the proposed technique and compare the results with those of TITAT. Results suggest that in regard to the reliability of the outcomes and computational efficacy, the proposed algorithm is as good as other methods. Unlike other methods, there is no additional parameter to be calibrated, and the convergence behavior of the algorithm is promising. It is observed in this particular case, that there is an inverse linear relationship between the number of iterations and the initial step size; and that for a specified number of iterations, the best results are obtained for equal step sizes for all but the first iteration. The main advantage of this technique is that it can provide a simple and economical method for traffic assignment as compared with user equilibrium for which goodness-of fit measure of predicted-observed flows on selected links equals 0.57, while this measure for the proposed incremental traffic assignment technique (PITAT) equals 0.52. The main shortcoming of the method is the fact that it is a heuristic method. Despite showing promising results, the convergence of the algorithm has yet to be practically proved. Considering the order of calculations, PITAT seems very attractive and can be applied as a substitute in situations similar to the case of this paper.

Peer-review under responsibility of WORLD CONFERENCE ON TRANSPORT RESEARCH SOCIETY.

Keywords: Incremental traffic assignment, TITAT, PITAT, user equilibrium, iteration, step size, Iran

1. Introduction

A major concern in the urban travel analysis is the prediction of link flows on transportation facilities, particularly during periods of potential congestion. The problem of finding the flow pattern over a given urban transportation network, known as traffic assignment, is the last stage in the urban transportation modeling system (UTMS) as depicted in Figure 1.

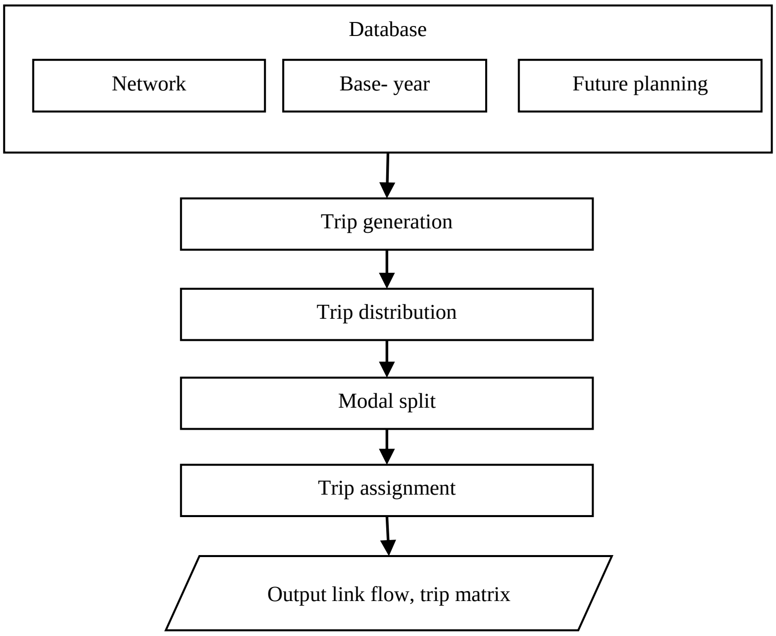

Figure 1- General form of the four-stage classical urban transportation modeling system

The traffic assignment problem has intrigued many researchers to develop models that can appropriately reflect the thought process that a driver faces while choosing a way towards his/her destination. Most basic models among these are Deterministic User Equilibrium models. These models are based on Wardrop's First Principle, which states that “The journey times in all routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route” (Wardrop 1952).

Each user non-cooperatively seeks to minimize his costs of transportation. The traffic flows which follows this principle will lead to a User Equilibrium point (commonly referred to as UE). This is called equilibrium because no user can improve upon his costs from this point by unilaterally shifting from one route to another. The simplest route choice and assignment method is All-or-Nothing which assumes that there are no congestion effects and all drivers perceive the alternatives in the same way. An improvement over this method is an Incremental Assignment Technique in which the network is loaded incrementally using All-or-Nothing and the results of this assignment are used to update the travel time on the network as an input to another All-or-Nothing assignment. This is done iteratively to distribute all the flows in the network. These methods are deterministic in nature and assume that the drivers are perfectly rational and identical and that they have complete knowledge of the network and flows (Wardrop 1952).

It is clear that the traditional incremental traffic assignment technique (TITAT) does not converge or produce a set of flows that is not in agreement with the user-equilibrium criterion. It may be reasonable to believe, though, that in general, as the number of increments grows, the incremental assignment algorithm may generate a flow pattern closer to the user-equilibrium condition. A very large number of increments (associated with a considerable computational effort) may be required, however, and even then the method will not always produce the user-equilibrium flow pattern. This is because the accuracy of the input data does not warrant the effort needed to obtain an extremely accurate equilibrium flow pattern. The number of iterations required for convergence is significantly affected by the congestion level on the network. In relatively uncongested networks, a single iteration may suffice since the link flows may be in the range where the performance functions are almost flat. This means that the updated travel times are very close to the initial ones, generating a set of link flows that is quite similar to the initial solution. As congestion builds up, more iteration is required to equilibrate the network. In solving the UE program over a large network, each iteration involves a significant computational cost, due primarily to the effort required to calculate the shortest paths in the direction-finding step. It is important, then, that a good answer is achieved after a relatively small number of iterations (Sheffi 1985).

The formulation of the user-equilibrium problem as a mathematical program was first developed by Beckmann et al. (1956), who proved the equivalency and the existence and uniqueness of the solution. Boyce

(1981), in an editorial note, traces the historical development of the concept of equilibrium assignment, including many algorithms and problem formulations. The incremental assignment technique was used in the DODOTRANS package developed at M.I.T. by Manheim and Ruiter (1970).

The traffic assignment problem (TAP) is used to predict network link flows under different assumptions, the most widely used being the user equilibrium (UE) principle (Wardrop, 1952), which was first formulated as a convex optimization problem with linear constraints by Beckmann et al. (1956). Since then, finding efficient algorithms to solve this problem has attracted much attention, especially following the initial success of the Frank–Wolfe algorithm (Frank and Wolfe, 1956; LeBlanc et al., 1975) and the later recognition of its painfully slow convergence behavior (LeBlanc et al., 1985; Hearn et al., 1985; Fukushima, 1985; Larsson and Patriksson, 1992; Jayakrishnan et al., 1994; Patriksson, 1994; Florian and Hearn, 1995; Jayakrishnan et al., 1995). The past decade has witnessed the development and experiments of new algorithms (Bar-Gera, 2002; Nie et al., 2005; Dial, 2006; Nie, 2010; Bar-Gera, 2010; Gentile, 2012; Nie, 2012; Inoue and Maruyama, 2012; Boyles, 2012; Xie et al., 2013; Zou et al., 2013; Asadi and Sarvi, 2015; Perederieieva et al., 2015; Ryu et al., 2016; Asadi et al., 2017). The promises of these algorithms, especially their ability to solve large-scale UE-TAPs at a high level of precision and efficiency, make them a new focus in traffic assignment research.

A recent study of a real-size problem showed that the flow pattern obtained by TITAT for the city of Mashhad is a rather good estimate of the observed flow pattern after eight iterations, based on the comparison of results with the UE assignment (Asadi and Sarvi, 2015; Mamdoohi and Mahpour, 2011).

In this paper, the assumption of equal step size in all iterations of TITAT is relaxed and variable step size for the different iterations is proposed to increase the efficiency of this technique. It is observed that intelligent choice of step size can lead to more efficiency of the technique. Considering the order of calculations in TITAT, PITAT seems promising if it can produce link flows comparable to those of TITAT, in which case it can be very attractive and can be applied as a substitute. To implement this concept quantitatively, Mashhad as the second largest city of Iran with about 4.5 million trips per year is used as the case study to analyze and evaluate the proposed technique and compare the results with those of TITAT. The structure of the paper is as follows: in the next section, the methodology including the traditional (TITAT) and the proposed incremental assignment techniques and the data of the case study are presented, followed by model results. The paper ends with conclusions and future research.

2- Methodology

Traffic assignment is the process of assignment given set of origin-destination pair to the existing suitable road Network based on specific travelers rout choice criteria. The route choice criteria is the travel impedance of the transportation network be minimized for a given origin-destination pair. Present studies show that the travel impedance of the transportation network includes travel time, travel cost, travel distance. However, due to congestion on particular route on particular time period, route choice not only depends on the shortest path, but also departure time and reliability of link of the network.

The traffic pattern is time dependent and vary with respect to time for the example OD pair and link flow vary with time (Zou et al., 2013). Whereas static traffic assignment methods assume that the O-D demand is uniformly distributed over the time to estimate the traffic pattern (Jaykishnan et al., 1995). This method also cannot describe time dependent demand and dynamic flow characteristics, so, it is very difficult to solve the congestion issue and apply traffic management policies (Saw et al., 2015; Lui et al., 2005).
In other words, it can be said that as long as there is an accurate estimate of the time of the arc travel, and if there is an accurate estimation of the paths between a source-destination pair at a macro level, then the accuracy of the assignment method will be increased.

2-1 User equilibrium assignment (UE)

Traffic equilibrium models are commonly in use for the prediction of traffic patterns in transportation networks that are subject to congestion phenomena. User equilibrium assignment is based on the Wardrop's first principle as stated as *"It* states *that the traveler time between a specified origin to destination of all used routes is equal and less than or equal or the travel time that would be experienced by a traveler on any unused route."* User equilibrium (UE) is the most common and well accepted traffic assignment technique, whose solution calls for computations and calculations, generally by a form of convex combination. The user equilibrium assignment is based on Wardrop's first principle, and formulated as follows:

$$Min \sum_a \int_0^{x_a} t_a (x_a) dx \qquad (1)$$

Subject to:

$$x_a = \sum_r \sum_s \sum_k f_k^{rs} \delta_{a,k}^{rs} \qquad \forall a$$

$\sum_k f_k^{rs} = q_{rs} \qquad \forall\, r, s$

$f_k^{rs} \geq 0 \qquad : \forall\, k, r, s$

$x_a \geq 0 \qquad : \forall\, a \in A$

Where:

x_a: Flow on arc (link) a

t_a: Travel time on arc a

f_k^{rs}: Flow on path k connecting $O - D$ pair $r - s$

q_{rs}: Trip rate between origin r and destination s

C_a: Capacity of arc a

t_0: Free-flow travel time on arc a

$\delta_{a,k}^{rs} = \begin{cases} 1 & \text{if link } a \text{ is on path } k \text{ between } O - D \text{ pair } r - s \\ 0 & \text{otherwise} \end{cases}$

The solution to the user equilibrium problem can also be found by solving the equivalent nonlinear mathematical optimization program. This problem (UE) states that no driver can unilaterally reduce travel costs by shifting to another route, which for a given O-D pair can be written as (Sheffi 1985):

$f_k (c_k - u) = 0 : \forall\, k$ (2)

$c_k - u >= 0 : \forall\, k$ (3)

Where f_k is the flow on path k, c_k is the travel cost on path k, and u is the minimum cost.

2-2- Traditional incremental assignment

TITAT is a rather simple procedure involving less calculation time whose results are only rough estimates of the equilibrium flow and travel time. In this method, each origin-destination entry is divided into N equal portions and assigned at each iteration. The travel times are then updated and an additional portion of the O-D matrix is loaded onto the network. In this manner, the general shape of the link performance functions can be ***"traced"*** with the successive assignments. In this procedure, outlined below and shown in Figure 2, w_a^nw denotes the flow on link *a* resulting from the assignment of the *n*th increment of the O-D matrix onto the network (Sheffi 1985).

Step 0: *Preliminaries*. Divide each origin-destination entry into N equal portions (i.e. Set $q_{rs}^n = q^{rs}/N$) Set iteration counter n*:* = 1 and x_a^0= 0 ,$\forall a$.

Step 1: *initial assignment*. n/N of total demand is assigned, based on all-or-nothing assignment.

Step 2: *Update*. Set $t_a^n = t_a(x_a^{n-1})$, $\forall a$.

Step 3: *Incremental loading*. Perform all-or-nothing assignment based on $\{t_a^n\}$, but using only the trip rates q_{rs}^n for each O-D pair. This yields a flow Pattern $\{ w_a^n \}$.

Step 4: *Flow summation*. Set $x_a^n = x_a^{n-1} + w_a^n, \forall a$.

Step 5: *Stopping rule*. If n = N, stop (the current set of link flows is the solution); otherwise, set n: = n + 1 and go to step 1.

In some versions of this algorithm, the incremental all-or-nothing procedure in step 2 is modified and origin-destination pairs are selected in random order, with a flow summation phase (as in step 3) and travel-time update (as in step 4) following each partial assignment (i.e., after each **O-D** entry is loaded). Incremental loading has two advantages: 1- it is very easy to program; and 2- Its results may be interpreted as the build-up of congestion for the peak period. Unfortunately, such techniques cannot be used to solve for equilibrium over networks with a large number of nodes, links, and O-D pairs (Sheffi 1985).

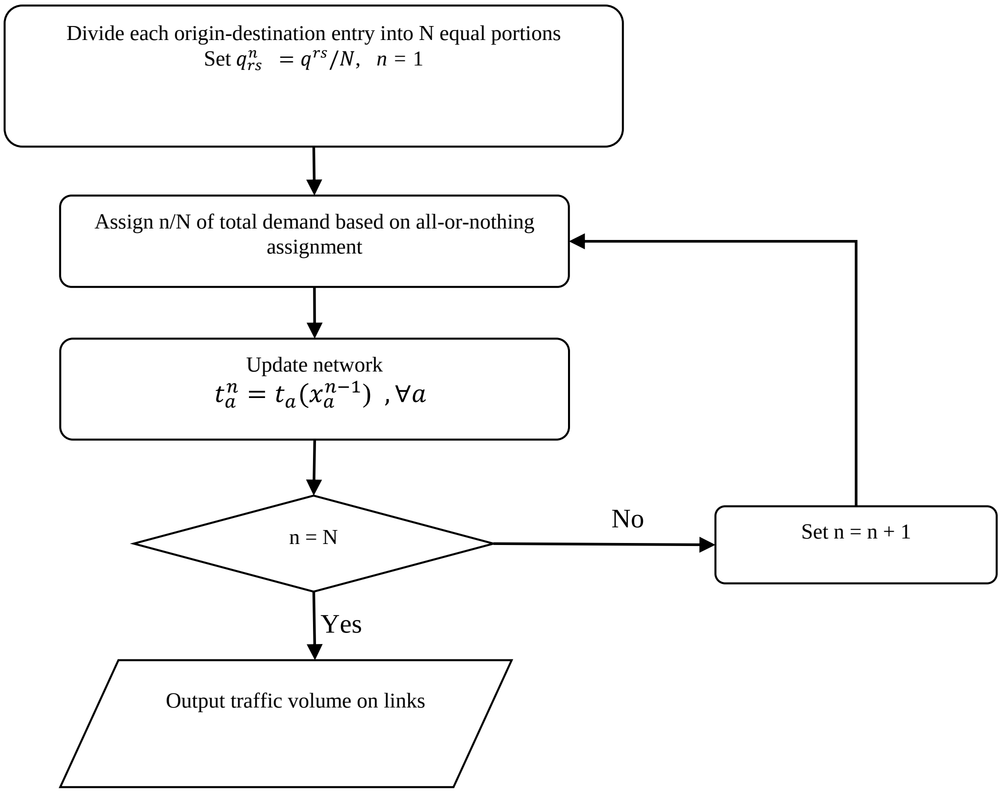

Figure 2- Flow chart of traditional incremental traffic assignment technique (TITAT)

3- Case Study and model results

The proposed concepts are analyzed and implemented for the city of Mashhad as a real size problem with rich information as the case study. Mashhad urban street network consists of 3830 links, 1341 nodes and 141 traffic analysis zones (Figure 3). Eleven different types of traffic facilities, according to their performance (including e.g. access, collector, arterial, expressway, and ramp) with a total length of 1447 km has been defined for Mashhad.

Figure 3. Mashhad urban street network

3-1- Numeric approach

Origin-destination demand matrix, according to the number of iterations (n) assigned to the network of roads in Mashhad. Then volumes in the 3830 links of network with travel time were estimated and calculated. Volume counts are available for 87 links. The results of the software (VISUM software package) output (estimated) for the arc plotted in a Cartesian coordinate system.

3-1-1- Modeling results for two different iteration steps

Figure 4 (a) indicates the assignment of origin - destination matrix to assignment, part of step 2 repeat Step 90 percent and 10 percent. In this diagram, the horizontal axis indicates the observed volume in the links and the vertical axis is the outputs of the assignment process in links. The estimated value is close to the observations (bisector first quarter); this illustrates the proximity of observations to estimate. Similar analysis has been done for different amounts of steps. In figures Changes in coefficient of R^2 and the results of incremental assignment with increasing in the number of iterations was plotted. Figure 4 (b) indicates the assignment origin- destination matrix to assignment, part of step 2 repeat step 80 percent and 20 percent. Other results for a different step size assignment with two repeat presents in table 1. The results indicate for incremental assignment with 2 repeats, the best R^2 was achieved when the 90% of OD matrix assigned in the first step and remained OD assigned in the second step.

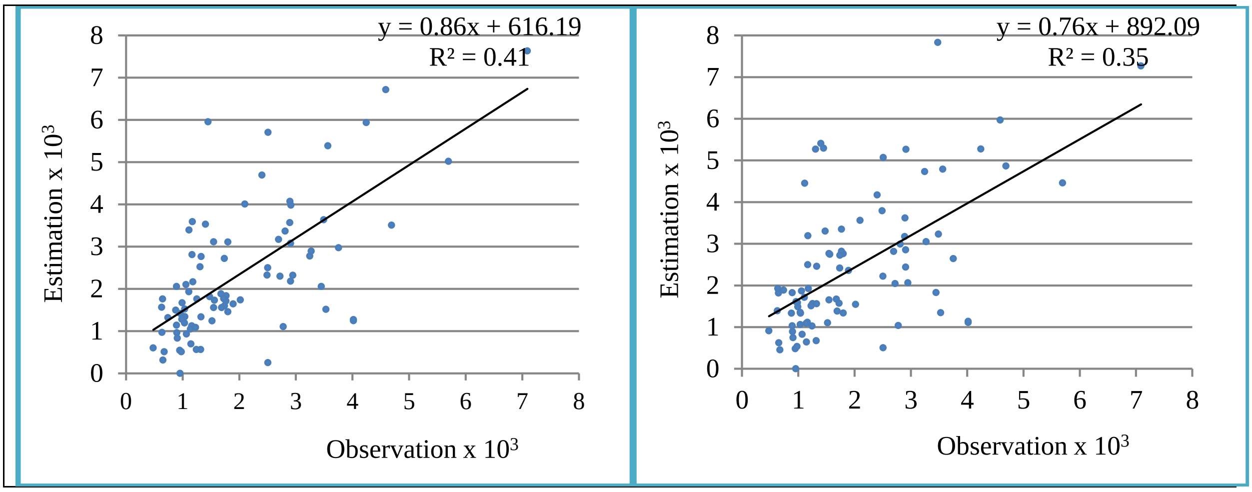

Figure 4- (a) Distribution estimates - observed for the first two iterations (90- 10). (b) Distribution estimates - observed for the second two iterations (80- 20)

Table 1- Statistical analysis of results with two repeated incremental traffic assignment

No	N=2 First Step (%)	N=2 Second Step (%)	Equation	F statistic	t(a)	t(b)	(R^2)
1*	90	10	y = 0.86x + 616.1	60.23	7.76	3.23	0.41
2	80	20	y = 0.76x + 892.0	46.04	6.78	3.34	0.35
3	70	30	y = 0.67x + 1107	28.19	5.31	3.69	0.24
4	60	40	y = 0.46x + 1595	11.67	5.03	3.41	0.12
5	50	50	y = 0.31x + 1879	4.31	5.34	2.07	0.04
6	40	60	y = 0.17x + 2044	1.34	5.9	1.61	0.01
7	30	70	y = 0.14x + 2037	0.93	5.94	0.96	0.01
8	20	80	y = 0.29x + 1695	0.45	5.47	0.20	0.05
9	10	90	y = 0.69x + 1900.7	0.32	5.7	0.33	0.02

*The best result is high lighted

3-1-2- Modeling results for three different iteration steps

A similar set of operations and analysis that was done for two iterations was performed for three iterations. Thus the results of the assignment matrix for origin - destination, method of assignment and distribution of the observed incremental traffic - estimated to be plotted and statistical parameters are calculated and analyzed. In figure 5 (a) Distribution estimates - observed for the first three iterations with 90 percent and 5 percent for second and third steps are shown. In figure 5 (b) Distribution estimates - observed for the three iterations with 50 percent and 40 percent for second and 10 percent for third steps are shown. The slope of the line (a) decreased slightly (0.5 percent), but the constant improved of about 3.5 percent. Table 2 presents statistical analysis of results with three repeated incremental traffic assignment. This process repeated for four step incremental assignment again.

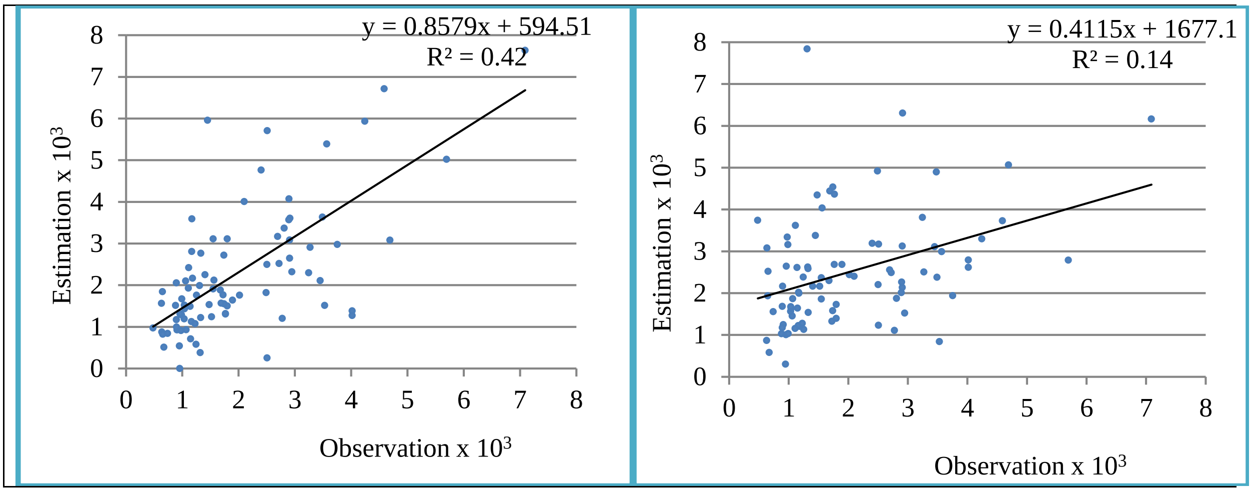

Figure 5- (a) Distribution estimates - observed for the three iterations by 90- 5- 5 percent in each step. (b) Distribution estimates - observed for the three iterations by 50- 40- 10 percent in each step.

Table 2- Statistical analysis of results with three repeated incremental traffic assignment

No	N=3 First Step	N=3 Second Step	N=3 Third Step	Equation	F statistic	t(a)	t(B)	R^2
1*	90	5	5	y = 0.85x + 594.5	61.99	7.87	2.32	0.42
2	80	10	10	y = 0.71x + 932.8	55.34	7.44	4.14	0.39
3	70	20	10	y = 0.68x + 1044.	59.11	7.78	5.02	0.41
4	70	15	15	y = 0.65x + 1093.	61.60	7.84	5.54	0.42
5	60	30	10	y = 0.51x + 1473.	28.62	5.31	6.53	0.25
6	60	20	20	y = 0.53x + 1385	30.85	5.55	6.12	0.26
7	50	40	10	y = 0.41x + 1677.	14.77	3.84	6.66	0.14
8	50	30	20	y = 0.50x + 1466.	34.23	5.85	7.25	0.28
9	50	25	25	y = 0.53x + 1358	37.96	6.16	6.69	0.30
10	40	40	20	y = 0.38x + 1697.	13.85	3.72	6.99	0.14
11	40	30	30	y = 0.45x + 1545	16.25	4.03	5.83	0.16

*The best result is high lighted

3-1-3- Modeling results for four different iteration steps

A similar set of operations and analysis was performed for four iterations.

It is observed that the greatest amount of good fitness coefficient by four steps, In exchange for the first step is obtained 80%, the second step is obtained 7%, the third step is obtained 7% and the fourth step is obtained 6%. In comparison this state with the best of three iterations, It can be said that the values of t (a) and t (b) each improved by about 7%. Checking the results shows that the best answer would be achieved if 80% of the matrix is assigned to the network in the first step. Also for the next iterations, the best result would be when the contribution of matrices from the initial matrix has the least variance (The size of the steps is equal). The estimation-observed diagram for the best assignment with four iterations is shown in Figure 6.

Table 3. Statistical analysis of results with four repeated incremental traffic assignment

No	N=4 First Step	N=4 Second Step	N=4 Third Step	N=4 Fourth Step	Equation	F	t(a)	t(B)	R^2
1	90	3	3	4	y = 0.84x + 631.3	61.29	7.83	2.49	0.41
2	80	11	7	2	y = 0.73x + 913.7	62.38	7.89	4.19	0.42
3	80	10	5	5	y = 0.75x + 848.1	68.63	8.28	3.96	0.44
4*	80	7	7	6	y = 0.75x + 827.8	70.38	8.39	3.92	0.45
5	80	2	7	11	y = 0.74x + 828.6	66.71	8.16	3.86	0.43
6	70	20	5	5	y = 0.67x + 1096.	57.98	7.16	5.29	0.40
7	70	10	10	10	y = 0.63x + 1180	58.68	7.66	6.07	0.40
8	60	30	5	5	y = 0.52x + 1422	32.45	5.7	6.35	0.27
9	60	20	10	10	y = 0.57x + 1283	64.27	8.01	7.65	0.43
10	60	13	13	14	y = 0.55x + 1270	53.09	7.34	7.10	0.38
11	50	40	5	5	y = 0.39x + 1665.	13.05	3.68	6.52	0.13
12	50	30	10	10	y = 0.49x + 1437	39.02	6.24	7.71	0.31
13	50	20	20	10	y = 0.49x + 1348.	50.13	7.08	8.14	0.37
14	50	17	17	16	y = 0.48x + 1369.	40.92	6.39	7.65	0.32
15	40	30	20	10	y = 0.40x + 1667.	21.66	4.65	8.16	0.20
16	40	20	20	20	y = 0.39x + 1633	15.95	3.99	6.96	0.15

*The best result is highlighted

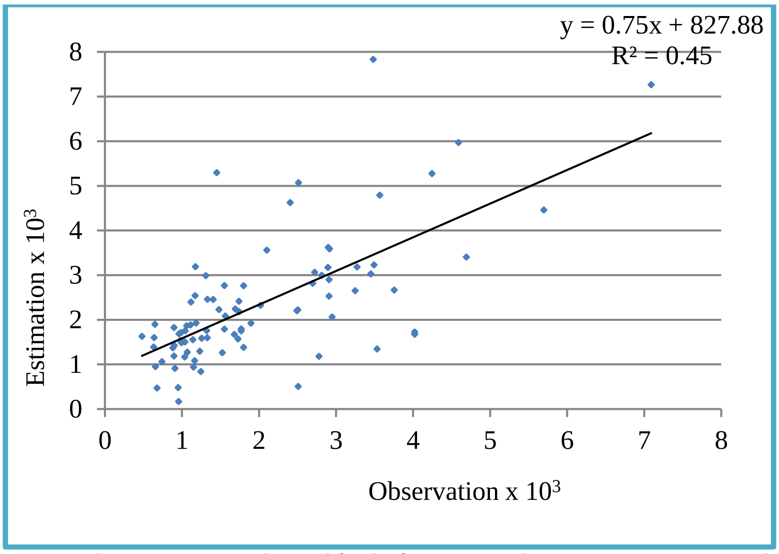

Figure 6- Distribution estimates - observed for the four iterations by 80- 7- 7- 6 percent in each step.

3-1-4- Finding number of iteration and step size

Similar calculations have also been made for the number of iterations up to 12 iterations that the results are presented in Table 4 for up to six iterations.

Table 4 indicates the combined number of steps for the various iterations with the highest R2. Can be observed for two iterations, the combined 90 percent for the first step and 10% for second step or combining of the 60% for the first step with the other steps equal to 8% for six iterations provides maximum R2 that is equal to 0.52 in comparison with user equilibrium (0.57) considering the computational simplicity, speed and ease is efficient.

Table 4: The best fit models obtained for various numbers of iterations

No	Iteration	Steps	The first successful step with the subsequent equal iterations	R^2
1	2	90- 10	90	0.414
2	3	90- 5- 5	90	0.42
3	4	80- 7- 7- 6	80	0.453
4	5	70- 7.5- 7.5- 7.5- 7.5	70	0.484
5	6	60- 8- 8- 8- 8- 8	60	0.519

* in UE method the R2 value is obtained equal to 0.57

By using the TITAT for the Mashhad Case study the best result occurred when the Number of iterations is 7. In other word when the OD matrix divided into 7 equal parts the best result obtained. In the proposed incremental traffic assignment technique (PITAT and Figure 7), based on the notion of relaxing the equal step size in all the iterations and thus allowing for step size variability to increase the efficiency of the technique Step 0 of the algorithm is hence modified as follows, and the rest of the algorithm is similar to before:

Step 0: *Preliminaries*. Calculate the size of the first step (Y_1) from: Y_1 = -10 N + 120 (3<N<13), the size of other steps ($Y_{2,...}$ Y_N) are equal and calculated from (100-Y) /(N-1), Set iteration counter n: = 1

The computational effort needed in each iteration is proportional to the number of origins (or number of destinations, if smaller) and the size of the network. The total computational costs associated with solving the UE program are, then, proportional to the product of the number of iterations, the number of nodes, and the minimum of the number of origins and number of destinations.

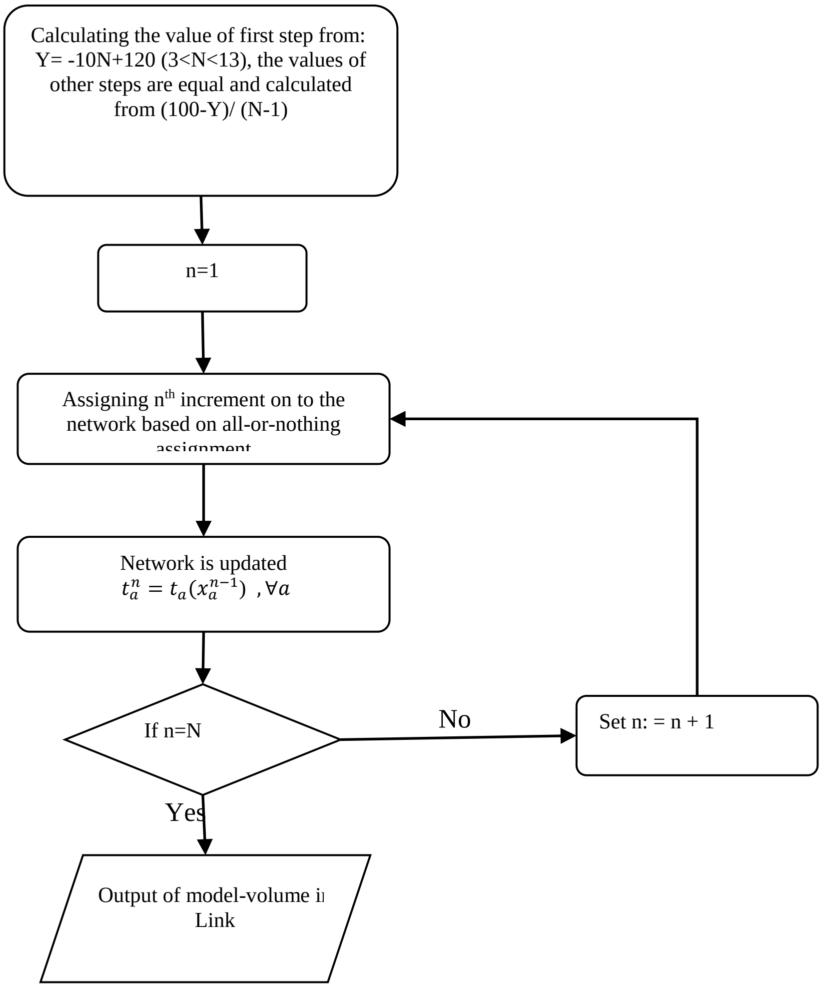

Figure 7. Flow chart of the proposed incremental traffic assignment technique (PITAT)

5- Conclusions and Future Research

A major concern in urban travel analysis is the prediction of link flows on transportation facilities, particularly during periods of potential congestion. The problem of finding the flow pattern over a given urban transportation network, known as traffic assignment, is the last stage in the urban transportation modeling system (UTMS). Although capacity constraints in traffic assignment can represent many realistic features, these constraints are largely ignored in practice because of mathematical complexities in applying the methods proposed in the literature. In solving the UE problem over a large network, each iteration involves significant computation, due primarily to the effort required to calculate gradients in the direction-finding step. The traditional incremental traffic assignment technique (TITAT) is a rather simple procedure involving less calculation time whose results are only rough estimates of the equilibrium flow and travel time. In this method, each origin-destination entry is divided into N equal portions and assigned in each iteration. The travel times are then updated and an additional portion of the O-D matrix is loaded onto the network. In this paper, the assumption of equal step size in all iterations is relaxed and variable step size for the different iterations is proposed to increase the efficiency of this technique. It was shown that intelligent choice of step size can cause more efficiency. To implement this concept quantitatively, Mashhad as the second largest city of Iran with about 4.5 million trips per year was used as the case study to analyze and evaluate the proposed technique and compare the results with TITAT. Results suggest that in regard to the reliability of the outcomes and computational efficacy, the proposed algorithm is as good as other methods. Unlike other methods, there is no additional parameter to be calibrated, and the convergence behavior of the algorithm is promising. It was observed in this particular case that: 1- there is an inverse linear relationship between the number of iterations and the initial step size; 2- for a specified number of iterations, the best results are obtained for equal step sizes for the second to the last iteration (same step size for all but the first iteration); and 3- results of the proposed technique are not very different from those of the UE.

In this study, quantitative analysis is used with relaxing the presumption of earlier methods. The main shortcoming of the method is the fact that it is a heuristic method. Despite showing promising results, the convergence of the algorithm has yet to be practically proved. It also attempts to provide a heuristic technique for measuring the step with less frequency and identical result. The results identify:

1. The result was enhanced with more iteration.
2. In a specified number of iterations for a fixed initial number, the best result is for a state in which except for the first one, the other lack many changes.
3. There is an inverse linear relation between the number of iterations and initial number.
4. With the increase of iteration, the trend of output is considerable to a specific B/C threshold and after that continue of iterations would not be economical.

Naturally, different case studies with different characteristics and datasets would not necessarily result in the same relationship functions. However, the proposed technique could be implemented to calibrate the relationship functions for different characteristics. Also, the assignment methods reviewed in this paper assume that the trip rate between every origin and every destination is fixed and known. For future research, this method can be developed to consider elastic demand assignment.

Other extensions to the TAP such as multiclass (various types or classes of vehicles), multimodal (private vehicles and transit or public transport), and consideration of no separable delay functions are left for further studies.

6- Acknowledgment

The authors would like to acknowledge with great appreciation the collaboration of Mashhad Transportation and Traffic Organization for their help in providing the valuable data and information necessary for this research which would otherwise be impossible to conduct.

7- References

Asadi, S., Sarvi, M. and Patriksson, M., 2017. A Hybrid Branch-and-Bound and Benders Decomposition Algorithm for the Network Design Problem. Computer-Aided Civil and Infrastructure Engineering, 32(4), 319-343.

Asadi S., and Sarvi M., 2015. Heuristic Approach to capacitated traffic assignment problem for large-scale transport networks, Transportation Research Record: Journal of the Transportation Research Board, 2498, 1–11.

Bar-Gera, H., 2002. Origin-based algorithm for the traffic assignment problem. Transportation Science 36 (4), 398–417.

Bar-Gera, H., 2010. Traffic assignment by paired alternative segments. Transportation Research Part B 44 (8–9), 1022–1046.

Beckmann, M., McGuire, C.B., and Winsten, C.B., 1956. Studies in the Economics of Transportation. Yale University Press, New Haven, Connecticut.

Boyles, S. D., 2012. Bush-based sensitivity analysis for approximating sub- network diversion. Transportation Research Part B 46 (1), 139–155.

Construction, calibration and validation of traffic assignment models, 2011. Transportation Master Plan Update Studies of Mashhad, Report No. 300-39-02, Tarh-e-Haftom Consulting Engineers, Tehran.

Dial, R. B., 2006. A path-based user-equilibrium traffic assignment algorithm that obviates path storage and enumeration. Transportation Research Part B 40 (10), 917–936.

Florian, M., and Hearn, D., 1995. Network routing. Handbooks in OR & MS, vol. 8. Elsevier Science, Oxford, UK (Chapter Network Equilibrium Models and Algorithms, pp. 485–549).

Frank, M., and Wolfe, P., 1956. An algorithm for quadratic programming. Naval Research Logistics Quarterly 3 (1–2), 95–110.

Fukushima, M., 1985. A modified Frank–Wolfe algorithm for solving the traffic assignment problem. Transportation Research Part B 18 (2), 169–177.

Gentile, G., 2012. Local User Cost Equilibrium: A Bush-Based Algorithm for Traffic Assignment. Transportmetrica Available On-Line.

Hearn, D. W., Lawphongranich, S., and Ventura, J. A., 1985. Finiteness in restricted simplicial decomposition. Operations Research Letters 4 (3), 125–130.

Inoue, S., and Maruyama, T., 2012. Computational experience on advanced algorithms for user equilibrium traffic assignment problem and its convergence error. In: 8th International Conference on Traffic and Transportation Studies Changsha, China, 43, 445–456.

Jayakrishnan, R., Tsai, W. K. and Chen, A., 1995. A dynamic traffic assignment model with traffic flow relationships, Transportation research part C, 3 (1), 51-72.

Jayakrishnan, R., Tsai, W. K., Prashker, J.N., and Rajadhyaksha, S., 1994. A faster path-based algorithm for traffic assignment. Transportation Research Record 1443, 75–83.

Kanafani, A., 1983, Transportation Demand Analysis, McGraw-Hill, New York, N.Y.

Larsson, T., and Patriksson, M., 1992. Simplicial decomposition with disaggregated representation for the traffic assignment problem. Transportation Science 26 (1), 4–17.

LeBlanc, L. J., Helgason, R., and Boyce, D. E., 1985. Improved efficiency of the Frank–Wolfe algorithm for convex network problems. Transportation Science 19 (4), 445–462.

LeBlanc, L. J., Morlok, E.K., and Pierskalla, W. P., 1975. An efficient approach to solving the road network equilibrium traffic assignment problem. Transportation Research 9 (5), 309–318.

Mahpour, A. R., 2012. Development of incremental traffic assignment model and its application for Mashhad city, M.Sc. thesis, TMU University.

Mamdoohi, A. R., and Mahpour, A. R., 2011. Analyzing the effect of Incremental Traffic Assignment iteration number on the accuracy of results: the case of Mashhad city, Journal of Traffic Engineering, 11 (44), 45-52 (in Persian).

Mamdoohi, A. R., Mahpour, A. R., and Mahpour, A. R., 2012. An extension to the incremental traffic assignment algorithm based on variable step sizes: case study of Mashhad City, Journal of traffic Engineering, 12 (47), 31-37 (in Persian).

Nie, Y., 2010. A class of bush-based algorithms for the traffic assignment problem. Transportation Research Part B 44 (1), 73–89.

Nie, Y., 2012. A note on Bar-Gera's algorithm for the origin-based traffic assignment problem. Transportation Science 46 (1), 27–38.

Nie, Y., Zhang H., and Lee D. H., 2004, Models and Algorithms for the Traffic Assignment Problem with Link Capacity Constraints, Transportation Research Part B, 38 (4), 285–312.

Origin-destination survey results, 2010. Transportation Master Plan Update Studies of Mashhad, Report No. 300-28-02, Tarh-e-Haftom Consulting Engineers, Tehran.

Ortuzar, J. D., and Willumsen, L. G., 2011. Modeling Transport, Fourth Edition, John Willy and Sons.

Patriksson, M., 1994. The Traffic Assignment Problem: Models and Methods. VSP, Utrecht, The Netherlands.
Perederieieva O., Ehrgott M., Raith A., and Wang J. Y. T., 2015. A Framework for and Empirical Study of Algorithms for Traffic Assignment, Computers & Operations Research, 54, 90-107.
Ryu, S., Chen, A. and Choi, K., 2016. Solving the stochastic multi-class traffic assignment problem with asymmetric interactions, route overlapping, and vehicle restrictions, Journal of Advanced Transportation, 50,255-70.
Saw K., Katti B. K., Joshi G., 2015. Review Paper: Literature review of traffic assignment: static and dynamic, International Journal of Transportation Engineering, 2 (4), 339- 347.
Sheffi, Y., 1985, Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Method. Prentice-Hall, Englewood Cliffs NJ.
Studies to determine the control business district of Mashhad, 2010. Mashhad Traffic Restricted Area Revision and Extension Studies, Report No. 500082/02/PR001 05, Duffill Watts Consulting Engineers.
Wardrop, J. G., 1952. Some theoretical aspects of road traffic research. Proceedings of the Institution of Civil Engineers, Part II 1 (3), 325–378.
Xie J., Nie Y., and Yang X., 2013. Quadratic approximation and convergence of some bush-based algorithms for the traffic assignment problem, Transportation Research Part B, 56, 15–30.
Zou, M., Chen, X., Yu, H., Tong, Y., Huang, Z. and Li, M., 2013. Dynamic transportation planning and operations: Concept, framework and applications in China, Social and Behavioral Sciences 96, 2332 – 2343.