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A Methodology to Estimate Parameters of Critical Gap Distribution

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Abstract

Identification of critical gap plays an important role in modeling traffic stream at an unsignalized intersection. Critical gap for a vehicle to cross a road depends on various aspects which includes driver behavior. Therefore, critical gap cannot be treated as a constant. In fact, critical gap at an unsignalized intersection should be associated with a distribution function to account for the entire driver population. In this respect, this study assumes a simplified distribution for critical gap and also assumes that if a driver rejects a gap size then one accepts only a gap size greater than that. Further, the parameters of the critical gap distribution are estimated using Maximum Likelihood technique. The objective of this study is to present a methodology, which utilizes conventional optimization technique, to estimate parameters of the distribution for which the likelihood function becomes maximum.

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Keywords: Critical Gap, Unsignalized interesction, Maximum likelihood technique, Optimization.

1. Introduction

The estimation of critical gap based on the stochastic behavior of the driver is one of the most difficult tasks. Critical gap estimation is required for determining gap acceptance models which in turn is an important parameter in capacity estimation of unsignalized intersection (see Gattis and Low, 1999; Guo and Lin, 2011; Maurya et al., 2016a; Patil and Sangole, 2015a; Prasad, 2014; Tian et al., 1999a). Many methods of critical gap estimation like Sieloch's method, Lag method, and Ashworth's method consider a crisp value of critical gap. But, gap acceptance behavior is guided by the type of intersection, driver's behavior, time, type of movement and traffic movement (see Ashalatha and Chandra, 2011; Brilon et al., 1999; Maurya et al., 2016; Patil and Sangole, 2015). Therefore, a single value of critical gap doesn't incorporate such behavior of drivers in accepting a gap. In this respect, it is assumed in this study that there exists a distribution of critical gap.

It is assumed in this paper that a distribution of critical gap must have two tails. This kind of distribution ensures the fact that only few drivers need gaps of smaller sizes or larger sizes. This kind of distribution of critical gap was

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suggested by Miller (1971). A simplified version of similar distribution is considered in this paper. Then this paper develops a methodology to estimate parameters of such a distribution. Estimation of parameters of critical gap distribution is posed as an unconstrained optimization problem here. The objective function in this optimization problem is the log-likelihood function. Moreover, among different techniques, maximum likelihood technique is known to be most reliable and accurate in estimating critical gap (Brilon et al., 1999; Patil and Sangole, 2015). The optimization problem thus posed is solved using conventional optimization technique known as Newton's method.

Different methods of critical gap estimation require different data sets (for example, Ashalatha and Chandra, 2011a; Ashworth, 1970; Brilon, 1995; Hewitt, 1983; Troutbeck, 1992). Similar to many of the log-likelihood based techniques, data on accepted and the maximum size of rejected gaps for each driver are required in the estimation procedure, using the proposed methodology, also. Further, in order to illustrate the effectiveness of the methodology, it is implemented on a large sized synthetic data set of rejected and accepted gaps. The synthetic data set is developed based on the existing studies where gap acceptance data is discussed. The paper is organized into five sections out of which this is the first. The next section discusses the distribution of critical gap which is being used in this study. In Section 3 the methodology proposed for parameter estimation of the distribution used is discussed. Section 4 discusses illustration of the proposed methodology using a large sized synthetic data and the last section ends with the conclusion of this study.

2. The model

Based on the understanding that different drivers have different perception of accepting a gap, a distribution of critical gap instead of one single value is found to be more suitable. In other words, for a traffic stream single value of critical gap will not be able to capture behavior of all the drivers. Additionally, it is a fact that critical gap cannot be negative. Further, it is hypothesized here that there are less number of drivers who accept gaps of large or small sizes. Therefore, a critical gap distribution should be defined only for positive real values and it should have two tails towards either ends. Although many distribution functions satisfy these criteria but normal distribution is assumed for critical gap by Miller (1971). In this study also to start with, critical gap distribution is assumed to follow normal distribution.

For a valid critical gap distribution curve, the total area under the curve should be equal to 1.0. Normal distribution qualifies in this respect but it also involves negative quantities as its variates; and, a negative gap size is a meaningless quantity. Now, it is hypothesized here that rejection of a huge sized gap by a driver is unlikely. Therefore, the domain of the critical gap distribution needs to be looked into. It is well known that area under a normal distribution curve from μ-3σ to μ+3σ turns out to be around 0.997, where μ is mean and σ is standard deviation. A parabolic shaped distribution between μ-3σ and μ+3σ is assumed here. This means that any gap size below μ-3σ is always rejected and any gap size above μ+3σ is always accepted and in between the critical gap follows a parabolic distribution. This modification does not alter the shape of the normal distribution too much because under normal distribution curve the area is 0.997, and the area under the parabolic distribution curve is 1.0 over the same domain.

Let f(t) and F(t) be the probability density function (PDF) and cumulative distribution function (CDF), respectively of the critical gap. Figure 1(a) presents the parabolic probability density function of critical gap and Figure 1(b) presents the cumulative distribution of critical gap. Interestingly, Chakroborty and Das (2017) has also suggested similar parabolic probability density function for the critical gap. Now, if $a = \mu - 3\sigma$ and $b = \mu+3\sigma$, then the parabolic PDF can be represented mathematically by Equation 1.

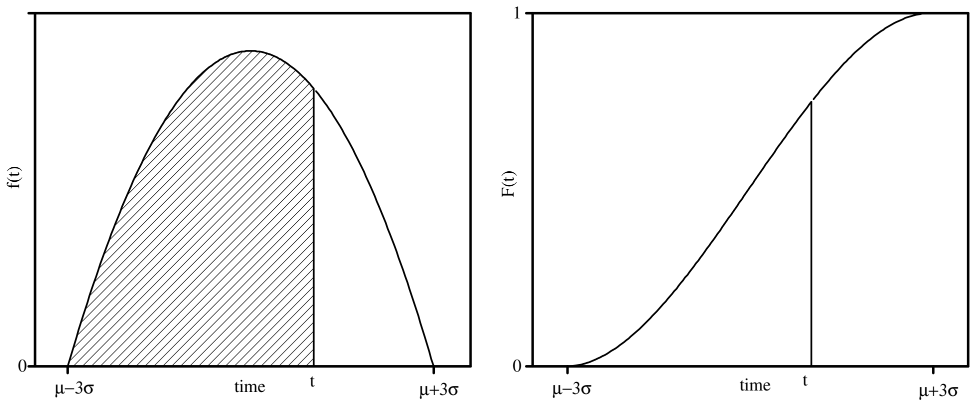

Fig. 1. (a) Probability density function of critical gap; (b) cumulative distribution of critical gap

$$f(t) = \begin{cases} 0 & 0 \le t \le a \\ \frac{6}{(b-a)^3}[t-a][b-t] & a \le t \le b \\ 0 & t \ge b \end{cases} \tag{1}$$

It can be said that the area under probability density plot between *a* to *t* represents the probability that the critical gap is smaller than or equal to *t*. Similar information is also conveyed through cumulative distribution plot; only this time the vertical separation at *t* between the *F(t)* and the horizontal axis (representing *F(t)=0*) represents the probability that the critical gap is than or equal to *t*. This idea is represented in Figure 1(a) by shaded region and in Figure 1(b) by the vertical line at *t*. Further, This relationship between PDF and CDF of critical gap can be mathematically expressed as:

$$F(t) = \int_a^t f(t)\, dt$$

3. Proposed Methodology

Using Maximum Likelihood technique one can determine the distribution of critical gap that ranges from the lowest accepted gap to the largest rejected gap (Tian et al., 1999; Troutbeck, 1992). Driver's behaviour for accepting an accessible gap varies stochastically based on time, type of intersection and traffic conditions. Therefore, to analyse this stochastic behaviour a two parameter based parabolic PDF of critical gap is assumed in this study. The two parameters of the parabolic function i.e., *a* and *b*, is determined here using MLE. In order to estimate the parameters, which maximizes the log-likelihood function, a numerical technique is required. In this study, Newton's method is used to estimate the parameters, which maximizes the log-likelihood function.

The likelihood of acceptance of gap for the i^{th} driver having the largest rejected gap R_i and the accepted gap A_i is given by the difference between the probability of accepted gap and maximum rejected gap. Alternatively, it is the difference between cumulative distribution function for accepted and maximum rejected gap. Mathematically, it is given as

$$L_i(a,b) = F(A_i) - F(R_i) \tag{2}$$

$$F(A_i) - F(R_i) = \frac{6}{(b-a)^3}[g_1(a,b).(A_i - R_i) + g_2(a,b)(A_i^2 - R_i^2) - \frac{1}{3}(A_i^3 - R_i^3) \tag{3}$$

where, $g_1(a,b) = -ab$ and $g_2(a,b) = \frac{1}{2}(b+a)$

Let the sample size of drivers be n, where observation of n driver's largest rejected gap and accepted gap is available. Then, the likelihood function of gap acceptance of n drivers is given by,

$$L(a,b) = \prod_{i=1}^{n}(F(A_i) - F(R_i)) \tag{4}$$

Log likelihood can be defined as:

$$\ln L(a,b) = \sum_{i=1}^{n} \ln[F(A_i) - F(R_i)] \tag{5}$$

By proper substitution, log likelihood function can be expressed as:

$$\ln L(a,b) = \sum_{i=1}^{n}\{\ln [\, g_1(a,b).(A_i - R_i) + g_2(a,b)(A_i^2 - R_i^2) - \tfrac{1}{3}(A_i^3 - R_i^3)]\} - 3n\ \ln(b-a) \tag{6}$$

The problem of estimating a and b, which maximizes the log-likelihood function is actually an unconstrained multivariate optimization problem. It is tedious and difficult to find optimum value of a and b by just substituting different values. However, it can be easily determined by means of a numerical method known as Newton's method. This method is an iterative one. Let $\boldsymbol{H(x_k)}$ be Hessian and $\boldsymbol{G(x_k)}$ be gradient of log likelihood function. The update of the vector $\boldsymbol{x_{k+1}}$ at every iteration is achieved using the method proposed in Equation 7. Here, $\boldsymbol{x_0}$ represents the initial guess.

$$\boldsymbol{x_{k+1}} = \boldsymbol{x_k} - [\boldsymbol{H}(\boldsymbol{x_k})]^{-1}\boldsymbol{G}(\boldsymbol{x_k}) \tag{7}$$

where,

$$\boldsymbol{x_k} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\boldsymbol{H}(\boldsymbol{x_k}) = \begin{bmatrix} \frac{\partial G_1}{\partial a} & \frac{\partial G_1}{\partial b} \\ \frac{\partial G_2}{\partial a} & \frac{\partial G_2}{\partial b} \end{bmatrix}$$

$\boldsymbol{G}(\boldsymbol{x_k}) = \begin{bmatrix} G_1(a,b) \\ G_2(a,b) \end{bmatrix}$ where $G_1(a,b) = \frac{\partial\, ln\, L(a,b)}{\partial a}$, $G_2(a,b) = \frac{\partial\, ln\, L(a,b)}{\partial b}$

Here b should always be greater than a and the initial guess of a should always be less than the smallest of A_i and guess for b should always be greater than the largest of R_i. This could be interpreted as the minimum value for the critical gap is the smallest accepted gap and maximum value of critical gap is the greatest value of the rejected gap for the concerned data set. The stopping criterion for terminating the iteration process is presented in Equation 8.

$$\|\boldsymbol{x_{k+1}} - \boldsymbol{x_k}\| \le \boldsymbol{\varepsilon} \tag{8}$$

where, $\boldsymbol{\varepsilon}$ is very small value and in this study it is assumed as 10^{-6}. If criterion, $\|\boldsymbol{x_{k+1}} - \boldsymbol{x_k}\| \le \boldsymbol{\varepsilon}$ qualifies at the $k+1^{th}$ iteration, then the corresponding values of a and b provide optimal estimate of the parameters. Further, these values can be used to find the critical gap distribution.

4. Illustration of methodology

First of all, a data set consisting of maximum size of rejected gap and the size of accepted gap for 100 drivers is developed. It is ensured that the maximum size of rejected is always less than the size of accepted one for each driver. The data is synthesized such that $0 \le R_i \le 6\ and\ A_i \ge 4$. This is done based on the observation by some studies. The studies suggest that generally gaps of greater than 6 s are always accepted (Ashalatha and Chandra, 2011; Maurya et al., 2016; Patil and Sangole, 2015). Similarly, the minimum value of accepted gap is assumed to be more than 3s as the minimum perception reaction time is generally taken as 2.5 s. The data set developed is presented in Appendix A. Further, initial guess of *a* and *b* are taken as 0.5s and 100s, respectively. This generalization is due to the fact that accepted gap size cannot be less than 0.5s and a gap of 100s cannot be rejected. At first, gradient and Hessian is computed for each iteration. Further, in this study the inverse of Hessian at each iteration is achieved using LU decomposition. Now by making use of Equation 7, new estimate of parameters *a* and *b* are computed. This way iteration goes on till the stopping criterion is met.

The proposed numerical technique along with the data set was run on a routine desktop computer with 4GB RAM and 3.3 GHz processor. It took almost 7s for estimating the parameters *a* and *b*. Total number of iterations required for convergence was 253. Some of the iterations are shown here for better understanding of methodology.

$$\begin{bmatrix} a_{k+1} \\ b_{k+1} \end{bmatrix} = \begin{bmatrix} a_k \\ b_k \end{bmatrix} - \begin{bmatrix} IH_{11} & IH_{12} \\ IH_{21} & IH_{22} \end{bmatrix} \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}$$

$$\begin{bmatrix} a_{k+1} \\ b_{k+1} \end{bmatrix} = \begin{bmatrix} 0.5 \\ 100 \end{bmatrix} - \begin{bmatrix} -0.20995 & -0.15855 \\ -0.15855 & -24.2088 \end{bmatrix} \begin{bmatrix} -18.0433 \\ -1.96173 \end{bmatrix} = \begin{bmatrix} -3.59913 \\ 49.64814 \end{bmatrix}$$

1st iteration

$$\begin{bmatrix} a_{k+1} \\ b_{k+1} \end{bmatrix} = \begin{bmatrix} -3.59913 \\ 49.64814 \end{bmatrix} - \begin{bmatrix} -0.76319 & -0.52043 \\ -0.52043 & -6.70798 \end{bmatrix} \begin{bmatrix} -5.51303 \\ -3.37525 \end{bmatrix} = \begin{bmatrix} -9.56319 \\ 24.13788 \end{bmatrix}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\begin{bmatrix} a_{k+1} \\ b_{k+1} \end{bmatrix} = \begin{bmatrix} 3.921614 \\ 6.589323 \end{bmatrix} - \begin{bmatrix} -0.00621 & -0.00211 \\ -0.00211 & -0.00553 \end{bmatrix} \begin{bmatrix} 0.00000155306 \\ -0.00000172172 \end{bmatrix} = \begin{bmatrix} 3.931614 \\ 6.589323 \end{bmatrix}$$

253rd iteration

$$\begin{bmatrix} a_{k+1} \\ b_{k+1} \end{bmatrix} = \begin{bmatrix} 3.921614 \\ 6.589323 \end{bmatrix} - \begin{bmatrix} -0.00621 & -0.00211 \\ -0.00211 & -0.00553 \end{bmatrix} \begin{bmatrix} 0.00000144412 \\ -0.00000160095 \end{bmatrix} = \begin{bmatrix} 3.931614 \\ 6.589323 \end{bmatrix}$$

$$\|\boldsymbol{x_{k+1}} - \boldsymbol{x_k}\| \le 10^{-6}$$

The optimal value of parameters *b* and *a* for the critical gap distribution using the data set of 100 drivers are found to be 6.6s and 3.9s, respectively. Thus the range for critical gap is obtained and it could be interpreted that a time gap of less than 3.9s would always be rejected and a gap of more than 6.6 seconds would always be accepted. Further, using the optimal values of *a* and *b*, the parameters μ and σ can be calculated to find out the distribution of critical gap.

5. Conclusion

For modelling traffic stream at an unsignalized intersection, gap acceptance study proves to be an important tool. Since different drivers have different behaviour, a single value of critical gap is not considered to be suitable approach. In place of that, considering a distribution of critical gap seems to be a better approach in order to capture the behaviour of driver population. A simplified distribution of critical gap is considered in this study; the assumed distribution is not significantly different from the situation where gap acceptance behaviour by driver population is represented using normal distribution. Maximum Likelihood technique is used to estimate the parameters of the simplified distribution of critical gap. This parameter estimation problem is posed as an unconstrained optimization problem. Because of the nature of the optimization problem, only numerical solution of the problem is possible. Therefore, Newton's method is applied to solve the optimization problem. Stepwise solution to a large sized problem is also illustrated in one of the sections. One drawback of the proposed methodology lies in the initial guess of the parameters. But, considering physical scenario one can always overcome this problem. In Section 4, the way to overcome this drawback is also discussed.

Appendix A. Synthetic data set

Table 1: Data on rejected and accepted gaps

Vehicle number(i)	Rejected gaps (R)	Maximum value of the rejected gap(R_i)	Accepted gap(A_i)
1	4,3,4	4	5
2	3,5	5	6
3	2,3	3	5
4	2,4	4	7
5	2,6	6	8
6	5,4	5	6
7	2,3	3	9
8	2,2	2	5
9	3,2,3	3	6
10	4,6	6	8
11	3,4	4	6
12	4,2	4	7
13	2,4	4	6
14	4,2	4	8
15	2,3	3	7
16	0	0	12
17	2	2	8
18	3	3	7
19	4	4	6
20	2,4,2	4	5
21	2,3	3	10
22	0	0	15
23	2	2	6
24	3	3	11
25	5,2,4	5	8
26	3,2	3	6
27	4,4	4	8
28	2,5	5	9
29	2,3,2	3	5
30	2,2	2	6
31	3,4	4	7
32	4,2,5	5	10
33	2,3	3	6
34	4	4	7
35	2	2	5
36	2,3	3	7
37	3,2,5	5	9
38	2	2	6
39	3,4	4	10
40	2,3	3	8
41	3,2,5	5	7
42	3	3	6
43	3	3	5

44	4,3,5	5	8
45	2	2	10
46	2,4	4	9
47	2	2	11
48	2,3	3	8
49	2	2	6
50	4,3,6	6	7
51	2,4	4	5
52	3	3	7
53	4	4	9
54	2,3	3	10
55	2	2	7
56	3	3	6
57	4	4	7
58	3,5	5	8
59	3	3	5
60	2	2	8
61	0	0	12
62	4,5	5	13
63	2,3	3	6
64	0	0	7
65	2	2	6
66	3	3	7
67	2,4	4	8
68	3	3	6
69	0	0	8
70	0	0	9
71	2	2	6
72	2	2	10
73	0	0	7
74	4,5	5	6
75	2,3	3	5
76	2	2	8
77	2	2	9
78	0	0	6
79	3	3	9
80	2	2	7
81	3	3	8
82	2	2	6
83	2	2	10
84	3	3	13
85	2	2	12
86	2	2	15
87	3	3	8
88	3	3	7
89	4	4	9
90	2,4	4	10
91	2	2	11
92	3	3	14
93	3	3	8
94	2,4,5	5	6
95	2,6	6	10
96	3	3	5
97	4	4	12
98	2	2	8
99	2,6	6	8
100	3	3	9

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