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# A Vulnerability Mathematical Model for Port Transportation Road Network

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## Abstract

This paper aims to study how to accurately judge the impact of port development on the vulnerability of port transportation road network. Therefore, the port traffic flow changing with continuous port development is chosen as research variable which is an output from a simulation model. In order to obtain the comparison of road network vulnerability results with or without port traffic flow, some suitable indexes, such as the network capacity index, the delay time index, the travel efficiency index and the N-Q index, are taken to describe road network vulnerability. Then, with those indexes, a simulation-based vulnerability mathematical model is established in which the port traffic flow from simulation works as an input. Finally, this simulation-based mathematical model is apply to quantitatively analyze the impact of the increasing port traffic flow on the vulnerability of urban trunks in a new north port project, which provides some references for port traffic planning.

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*Keywords: port transportation, network vulnerability, road network, port traffic planning, mathematical model*

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## 1. Introduction

Road network in container terminals area has the different characters with urban road network (Wang X K and Lu H P, 2005; Wei L, 2015), and it also faces a series of problems such as traffic jams and traffic accidents caused by increasing traffic flow. Under such circumstances, the issue of the vulnerability of road network is constantly exposed and the impact it produced is becoming more and more overlooked. The traffic flow in the road network of container terminals area is composed of two parts: one from the city and the other from the port. Therefore, the assessment of the vulnerability of road network should not only consider the city traffic flow with the morning and evening rush hour, but also consider and accommodate the randomized peak-set terminal traffic flow that changes with port throughput. So, how to analyze vulnerability of road network with mixed traffic flow characteristics is the question.

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At present, vulnerability of transport network system has been studied both at home and abroad. Definition of road network vulnerability first proposed is incident sensitivity that caused a drastic decline in the capacity of transportation network (Berdica K, 2002). In contrast, vulnerability of transportation networks is closely related to the consequences of side-effects, but not to the probability of failures (Michael A, 2008). There are four ways to analyse the vulnerability of road network: based on a specific strategy assessment method (Alan T and Timothy C, 2008), based on a specific scene method, a mathematical model assessment method and a simulation-based assessment method, while a scenario-based approach is utilized to simulate 12 different external interfering scenes to study Stockholm road network model (Berdica K and Mattsson L, 2002). On specific evaluation indexes, many researchers study the vulnerability of the road network based on the travel time index and the network capacity index (Michael A and Glen M, 2003; Kuang A W and Tang Z Q, 2013). To sum up, we can see that there are many methods and indexes to study vulnerability of road networks, but the current researches do not consider traffic characteristics in port area.

This paper is organized as follows. Section 2 focuses on the problem description, including the reasons for the problem and the influence of the traffic flow characteristics from city and port. Section 3 focuses on modelling and solution methods. In this section, a simulation-based mathematical model is built to evaluate the vulnerability of road network around container terminals area. Study case and results will be given and discussed in Section 4, followed by conclusions and discussions in Section 5.

## 2. Problem Description

The purpose of this paper is to evaluate the vulnerability of road network around the port with mixed traffic characteristics, shown as Figure 1, which is the problem that should be focused. Therefore, when analysing this kind of traffic flow, we should not only consider the city traffic flow at both morning and evening peak, but also take into account the port traffic flow that varies with the throughput of port. And the peak time of the port traffic flow is subject to the uncertainty of the loading and unloading time on container terminals. So, how to analyse the urban traffic flow as well as to emphasize the influence of the traffic flow from port on the vulnerability of road network is the key of the whole evaluation problem. This paper builds a simulation-based mathematical model to solve this problem. It takes the port traffic flow as a variable to calculate indexes' value with or without the variable and discuss the influence level caused by the port traffic flow under this two kinds of results.



Fig. 1. Mixed Traffic Characteristics in the Port Transportation Road Network

### 3. Simulation-Based Mathematical Model

In this paper, four indexes are selected for the road network in the container terminals area to establish a simulation-based mathematical model, which is used to assess the vulnerability of the road network from both the local and holistic aspects. Among them, the selection of the coefficients in the road resistance (BRP) function needs to consider road traffic characteristics of road network in the port area. The variable  $V$  in the mathematical model consists of two parts: the city traffic flow  $V_{city}$  and the port traffic flow  $V_{port}$ . It should be noted that the port traffic flow  $V_{port}$  is obtained by a simulation model. The road space conditions, traffic conditions and organization scheme in the terminal are important parts in the simulation model. And, the container trucks entering and exiting the port, the checked area and the route of the vessels should also be considered. Then, through simulating the traffic situation with the random traffic arrivals of vessels, the traffic flow at the port gates can be calculated which is defined as  $V_{port}$ . Another part of traffic flow  $V_{city}$  is obtained through the traditional traffic forecasting method, with the data of the important road intersections from the site investigation. This logical structure of simulation-based mathematical model can be shown in Figure 2.

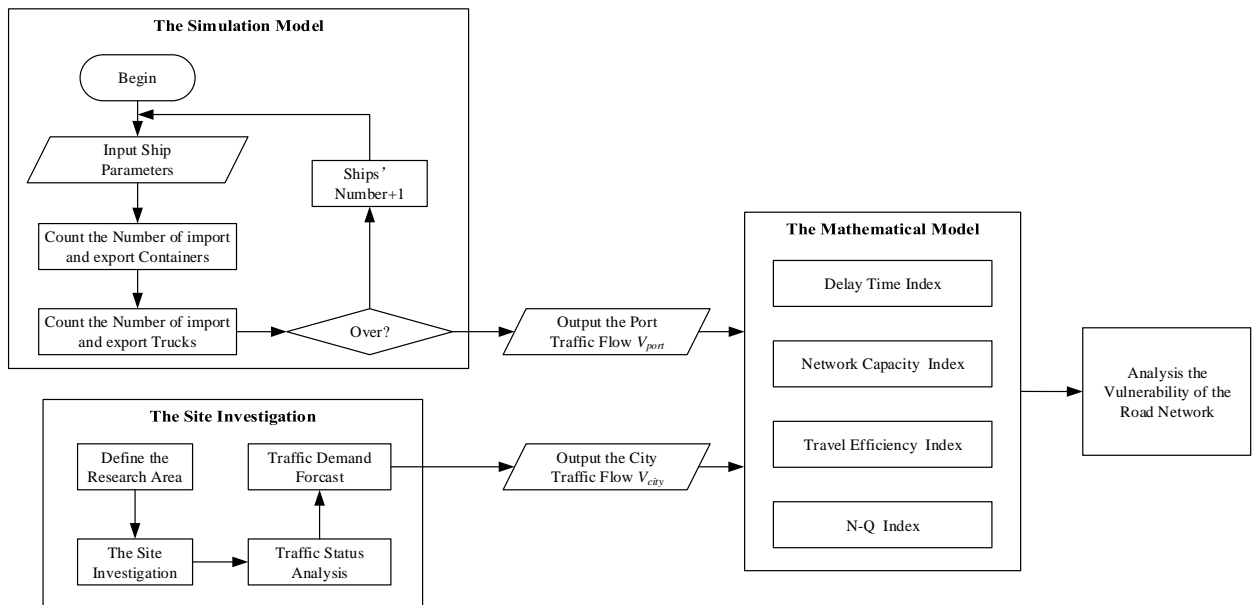


Fig. 2. Logical Structure of Simulation-Based Mathematical Model

#### 3.1. Local Indexes

##### (1) Delay Time Index

The delay time is an index to detect road traffic status, that is, the difference value between the actual time  $t$  required to pass a road and the free time  $t_0$  without any interruption. The main influencing of the delay are traffic control and traffic frictions. This paper defines the difference value between the actual travel time  $t$  and the free travel time  $t_0$  as the delay index of a road, that is:

$$d_i = t - t_0 \quad (1)$$

The actual road travel time can be calculated by BRP function:

$$t = t_0 \left[ 1 + \alpha \left( \frac{V_i}{C_i} \right)^\gamma \right] \quad (2)$$

where,  $V_i$  (pcu/h) and  $C_i$  (pcu/h) represent the traffic flow and the capacity of road  $i$  separately;  $\alpha$  and  $\gamma$  are determined by domestic research on BRP function related parameters, combined with the feature of larger proportion that the freight traffic network system has in the port area, as  $\alpha=0.6$ ,  $\gamma=1.6$  (Xia Z H, Bai G A and Zhou J B, 2009; Wang S S, Huang W and Lu Z B, 2006; Zhang Q, 2013).

### (2) Network Capacity Index

The network capacity  $\kappa$  is an index in reliability theory, and is intended to evaluate the service level of road section by calculating whether the network capacity meets the probability of traffic flow on a certain road.

The definition of this index is as follows:

$$\kappa = \frac{V_i}{C_i} \quad (3)$$

where,  $\kappa$  represents the network capacity index;  $C_i$  is the capacity of road  $i$ ;  $V$  represents the actual traffic flow assigned to each road section based on the port gate flow.

## 3.2. Holistic Indexes

### (1) Travel Efficiency Index

One of the parameters that must be mentioned when considering travel efficiency is the “delay”  $d_i$ . This concept is an important factor in assessing traffic congestion and is also the most fundamental factor affecting the efficiency of road network operation. However, in order to avoid the shortcoming that the travel efficiency is reduced too fast when the delay index is small, or the travel efficiency is reduced too slow when the delay index is large. The definition of a

road  $i$  of travel efficiency is  $\eta = \sqrt{1/d_i}$ , the collection of roads is  $I$ ,  $n_I$  represents the number of roads,  $d_i$  represents the “delay” of a road. And the travel efficiency  $\eta(G)$  is the average of each sub-road of the road network  $G$ , that is:

$$\eta(G) = \frac{1}{n_I} \sum_{i \in I} \eta_i = \frac{1}{n_I} \sum_{i \in I} \sqrt{\frac{1}{d_i}} \quad (4)$$

### (2) N-Q Index

The N-Q index is proposed based on the travel cost in equilibrium state. Taking the average of the efficiency of OD pairs as a road network performance index, it is an index reflecting the overall operation efficiency of road network. In the road network  $G$ , the demand vectors  $c_w$  and the number of OD pairs which is  $N_w$  are known, the travel time between OD pairs is represented by  $t_w$ , and so the network performance  $\varepsilon(G, C)$  (Lei L, 2012) can be defined like this:

$$\varepsilon(G, C) = \left( \sum_{w \in W} \frac{c_w}{t_w} \right) / N_w \quad (\text{pcu/h}) \quad (5)$$

## 4. Case Study

### 4.1. Study Area and Data Source

The road network outside of a new northern container terminals is taken as the case study, shown in Figure 3. According to road grade and geographical location, some important road sections are selected from the road network which constitute a sub-road network. This paper takes this sub-network as the research object, and explores the impact of the port traffic flow both on the sub-network vulnerabilities of those road sections and their components. The location of the subnet in the affected area is shown in Figure 4.

In this study, traffic status of important road sections and road intersections within the traffic impact of container terminals area is investigated, and the traffic flow at the gate of the port area was predicted by the simulation model. Based on the city traffic flow investigated, the average growth rate method is used to predict the traffic flow for each level year. There is the direct relationship between traffic growth and motor vehicle traffic growth rate. To investigate the data of motor vehicle ownership in the region from 2002 to 2013, trends of the rate of change in growth rate are shown in Figure 5. Therefore, the average growth rate of the city traffic flow investigated is 13.01%. The traffic flow allocated by port area is predicted from the simulation model at the port gate. The statistical results of the port traffic flow in each level year are shown in Table 1.



Fig. 3. Location of the Container Harbour in Port Transport Network



Fig. 4. Distribution of Subnets in Affected Road Network

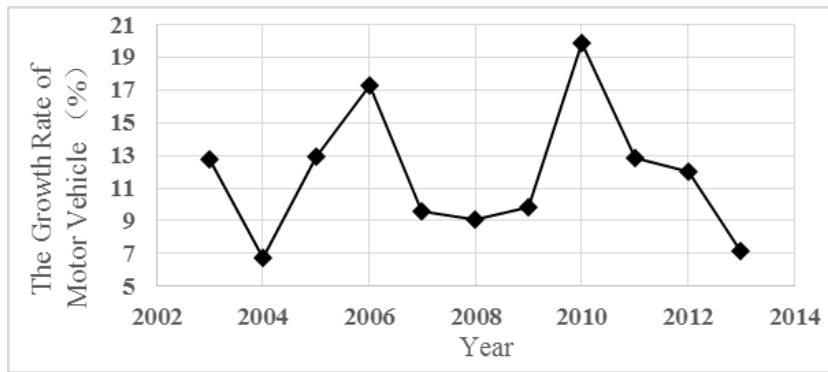


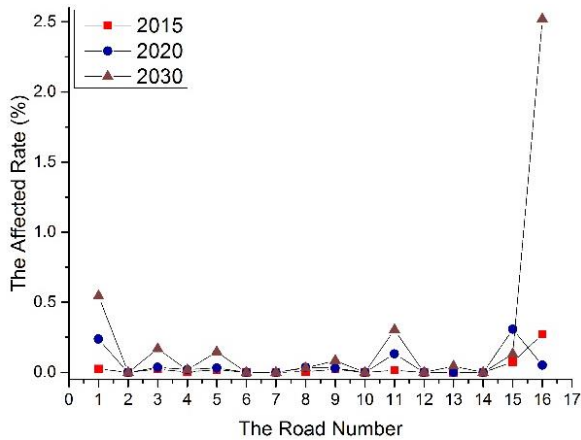
Fig. 5. Motor Vehicle Growth Rate

Table 1. Port area traffic flow in each level years

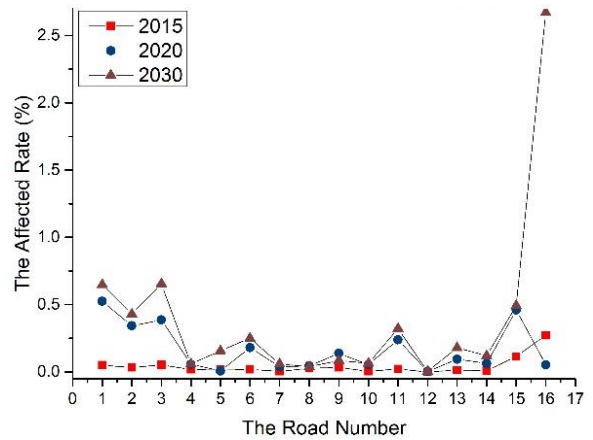
The Level Year	The Maximum Traffic Flow (pcu/h)	The Average Traffic Flow (pcu/h)	The Traffic Flow Ratio in Peak Time (%)
2015 (Gate#1)	132	74	7.46
2020 (Gate#1)	242	160	6.29
(Gate#1)	586	360	6.79
2030 (Gate#2)	534	373	5.96
(Gate#3)	422	280	6.29

4.2. Result Analysis

(1) Analysis of Local Index of Road Network



(a) The Delay Time Index at the Morning Peak



(b) The Delay Time Index at the Evening Peak

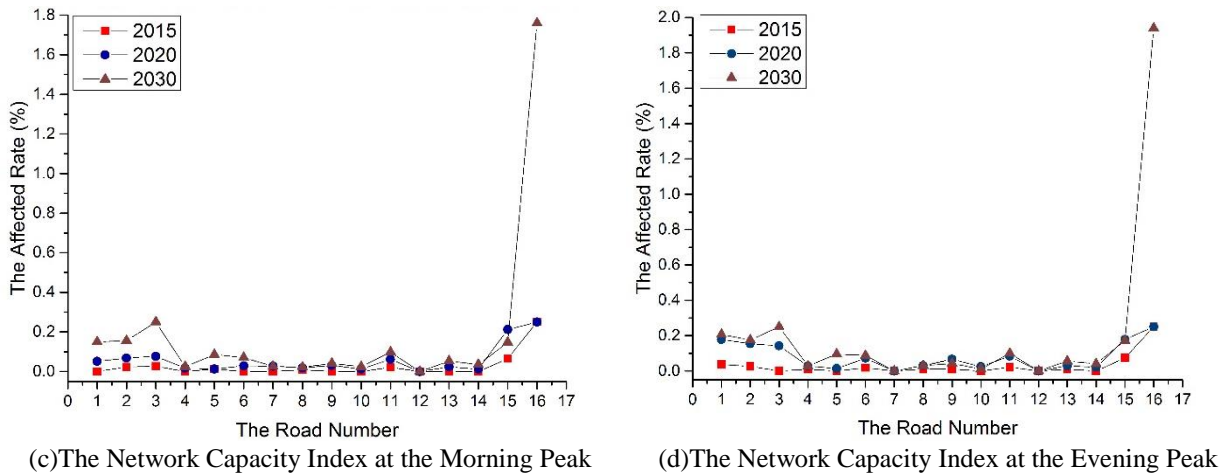


Fig. 6. Influential rate of port traffic flow to each road under different conditions

In order to explore the influence of traffic flow in port area on the vulnerability of road network, this paper divides the traffic flow into two types: the road network only with the city traffic flow and the road network with the mixed traffic flow. In both cases, comparing the vulnerability index parameters of each road in road network, the following formula is used to calculate the influence rate  $\eta$  of traffic flow in road network from port on each road in different level years.

$$\eta = \frac{R_{city+port} - R_{city}}{R_{city}} \times 100\% \quad (6)$$

where,  $R$  represents the index of vulnerability of road sections under different flow combinations in each level year. At the same time, combining with characteristics of city traffic flow, this paper examines the influence of the port traffic flow on the road network from the perspective of two indexes: the network capacity and the delay time, both at the morning and evening peak shown in Figure 6.

It can be seen from Figure 6 that the influence of the port traffic flow on the road network increases as time goes on and its fluctuation range becomes more and more obvious. In particular, the traffic flow generated by the port area in 2030 will have the strongest impact on the road network. From the point of view of possible failure of road sections, the road sections with higher vulnerability are greatly affected by the port traffic flow. As can be seen from Figure 6 (a) and Figure 6 (b), the influence of the port traffic flow on roads 1, 2, 3, 11, 15 and 16 are greater. As shown in Figure 6 (c) and Figure 6 (d), the impact of the port traffic flow on roads 1, 2, 3, 6, 11, 15 and 16 are relatively large. Among them, the highway for the port represented by road 16 is subject to the greatest influence by the port traffic flow. During each level year, the greatly influenced roads by the port traffic flow are marked in the road network shown in Figure 7 and Figure 8. It can be seen that with the continuous development of the port area, the influential area shows characteristics of spreading radiation to main urban roads around the city taking the highway of the port as the centre.

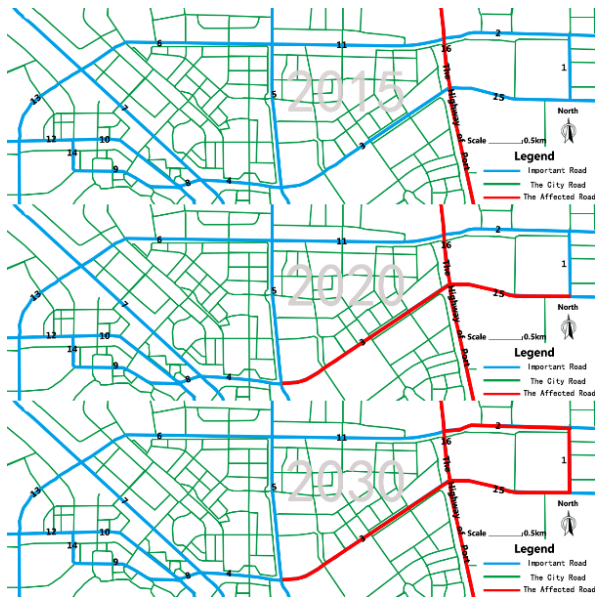


Fig. 7. Influential Areas under the Index of the Delayed Time of Each Road in 2015, 2020 and 2030

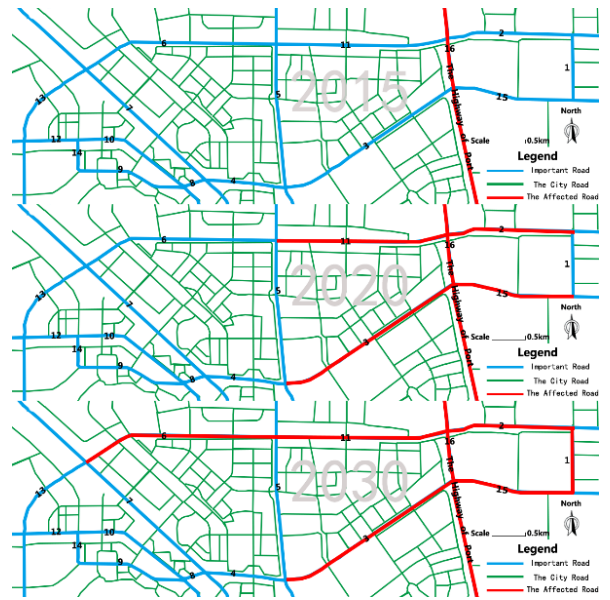


Fig. 8. Influential Areas under the Index of the Network Capacity of Each Road in 2015, 2020 and 2030

(2) Analysis of the Holistic Index of Road Network

The travel efficiency index and the N-Q index reflect running status of the road network from the overall point of view, in which the travel efficiency is the basis of the vehicle traveling convenience in the road network. The closer the value is to 1, the higher the network capacity is, and the smoother the vehicle travels in road network. The closer the value is to 0, the more traffic congestion in road network increase, and the longer the time delayed is, the vulnerabilities of road network increases. According to Table 2, we can see that, the travel efficiency reduces influenced by the traffic flow from ports as time goes on. And with the increase of time, this impact is also gradually increasing. The N-Q index is considered based on travel cost when a road network is operating at a balanced state. The higher the value is, the greater the required travel cost of the vehicle is, the more traffic network is congested, and more vulnerable it is. On the other state, the better the state is, the lower the vulnerability is. According to Table 3, with the impact of the port traffic flow, the travel cost increases with time going on at both the morning and evening peak , and the rate of such increase is accelerating.

Table 2. Travel Efficiency Index of Port Road Network

The Level Year	The Morning Peak		The Evening Peak	
	The City Traffic Flow	The City and Port Traffic Flow	The City Traffic Flow	The City and Port Traffic Flow
2015	0.27	0.26	0.30	0.29
2020	0.26	0.25	0.29	0.26
2030	0.26	0.24	0.28	0.25



Table 3. N-Q index of port transportation network

The Level Year	The Peak Time	The City Traffic Flow	Ratio	The City and Port Traffic Flow	Ratio
2015	The	31.48	/	31.73	/
2020	Morning	31.76	0.01	32.38	0.02
2030	Peak	32.14	0.01	33.42	0.03
2015	The	29.70	/	30.08	/
2020	Evening	30.04	0.01	31.17	0.04
2030	Peak	30.40	0.01	32.72	0.05

## 5. Conclusions

The paper use the simulation-based mathematical model to analysis the vulnerability of the road network from a new container terminals area. The impact of the port traffic flow on the road network can be analyzed quantitatively, and the rules affecting the port traffic flow on the vulnerability of road network can be confirmed to some extent. Moreover, it was verified that this model can analysis the characteristics of port transportation road network feasibly.

Some results can be concluded from this study:

(1) In the early period of port area development, the city traffic flow is the main factor that affects the vulnerability of road network. In the latter part, the port traffic flow becomes the key to restrain the vulnerability of the road network.

(2) Compared with the impact of the port traffic flow on the overall operation of the road network, the impact on some specific roads is more significant.

## 6. Acknowledgements

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