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Weighted Center of Mass based Optimal Control Scheme for Pre-timed Signalized Junctions

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Abstract

Pre-timed signalized junctions are prevalent in real world as they are inexpensive and easy to implement. However, traffic flow in these junctions exhibit large variations even for the same time intervals in each day. Hence, robust signal timings are desired to address this uncertainty in traffic flow. The existing robust signal control models for pre-timed signalized junctions with uncertain traffic use min-max approach. The limitations of these models are: (i) the optimality of the solution is not guaranteed, and (ii) inability to scale for larger ranges of traffic flow due to long computational times. In this work, we propose a weighted center of mass based optimization approach which overcomes these limitations. It finds the robust signal timings for uncertain traffic demands that minimizes the average delay per vehicle. Simulation results show that our approach performs better than the existing models for both under-saturated and over-saturated traffic flows. Also, the computational time taken by our approach is significantly less.

Keywords: Pre-timed Signal Control; Optimization; Uncertain Traffic Flow; Delay Minimization; Robust Signal Timings.

1. Introduction

Although the study of real-time traffic control systems is in recent trends, most of the signalized junctions around the world still use pre-timed traffic control. This is because the real-time traffic control needs infrastructure, such as detectors and processors to calculate the optimal signal timings quickly for each cycle, which adds to the cost and needs maintenance. Due to these reasons, it is less likely that many of the several thousands of existing signalized

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junctions will be using real-time traffic control in near future. Thus the prevalent pre-timed signals should be made robust to handle uncertain traffic demands. This work proposes a weighted center of mass based optimization approach to find the optimal green times for pre-timed signal control, in order to minimize the average delay per vehicle. These green times are robust to uncertainty in traffic demands.

In practice, most of the pre-timed systems such as TRANSYT (Vincent et al. (1980)), TRANSYT7F (Wallace et al. (1984)) and Synchro (Husch and Albeck (2003)) operate in different modes (signal timings) at different times of the day. These approaches assume that the same time interval of the day will have similar traffic flow. However, in reality, the traffic flow can significantly vary for the same interval of the day, same day of the week for a junction (Yin (2008)). Thus, the aforementioned systems may not be robust to variations in the traffic flow.

Han (1996) and Wong et al. (2002) further divided time intervals of a day into certain subintervals, and assumed that the traffic flows are constant in each sub-interval. Based on this assumption, they optimized the signal control for each sub-interval. So, these models are more suited for constant traffic patterns, which however is rarely found in reality.

Ukkusuri et al. (2010) and Yin (2008) proposed scenario based traffic control models, where each scenario refers to a traffic demand observation. Each scenario is assigned an occurrence probability, and based on these probabilities, the signal timings that minimizes the average delays per vehicle across all scenarios are calculated. These models are more suitable when a large number of scenarios with their occurrence probabilities are known.

Yin (2008) and Li (2011) proposed min-max optimization approaches to obtain signal timings using the ranges of traffic flows. It is relatively easy to estimate the potential range of uncertain traffic flows than their occurrence probabilities. These works minimize the maximum delays with respect to green times and demands over the uncertain traffic volume set. The uncertain traffic volume set, Q has been defined as

$$Q = \left\{ (q_1, q_2, \cdots q_M) \,\middle|\, \sum_{i=1}^{M} \left(\frac{q_i^{max} - q_i^{min}}{2} \right)^{-2} \cdot \left(q_i - q_i^0 \right)^2 \leq \theta^2 \right\}$$

where, M is the total number of movements, q_i is the traffic flow for movement i, q_i^{max}, q_i^{min} and q_i^0 are the maximum, the minimum and the nominal or average traffic flow for movement i respectively. θ is the total volume variation parameter that controls the total volume variation. The choice of the parameter θ, affects the robustness of the green times to the uncertainty in traffic flow. Higher the value of θ, better the robustness. But with the increase in the value of θ, the solution time increases drastically.

The general structure of the min-max approach is as follows Li (2011):

Minimize {max *delays with respect to timings and demands*

Subject to: *uncertain traffic volumes set}*

Subject to: *certain constraints on green times and cycle length*

Yin (2008) used a cutting plane algorithm to solve the min-max problem. However, cutting plane algorithm does not guarantee the global optimal solution for the min-max problem as mentioned in Yin et al. (2008). It rather gives the local optimal solution to the min-max problem.

To address the local optimality issue of the min-max problem, Li (2011) proposed discretization modeling approach, where the cycle time, green times and traffic volumes are divided into a finite number of discrete values. The values generated from this approach are used to solve the min-max problem using dynamic programming. This approach can have long computational time when larger traffic variations are considered. Also the solutions obtained are not global optimal, due to the discretization approach.

Thus, the limitations of using min-max approach are: (i) the optimality of solution is not guaranteed, and (ii) inability to scale for larger value of θ due to long computational times. In this work, we propose a weighted center of mass based optimization approach to overcome these limitations. The proposed approach first samples different flow profiles to capture the uncertainty in traffic flow and then quickly solves the proposed delay minimization model to obtain the optimal green times and corresponding delays for each sample. Then the obtained optimal green times and corresponding delays are used in a weighted center of mass based optimization model to determine the robust signal timings for uncertain traffic flows. These robust signal timings minimize the average delay per vehicle for the total volume variation.

The rest of the paper is organized as follows: In Section 2, we briefly discuss the background of pre-timed signal control and the proposed framework for robust signal timings to the uncertain traffic flow in details. Next, we discuss the delay minimization model and the weighted center of mass model for robust signal timings. The robustness and performance evaluation of the proposed approach is discussed in Section 3. In Section 4, we have contrasted the pre-timed approach with the real-time approach. Section 5 concludes the paper.

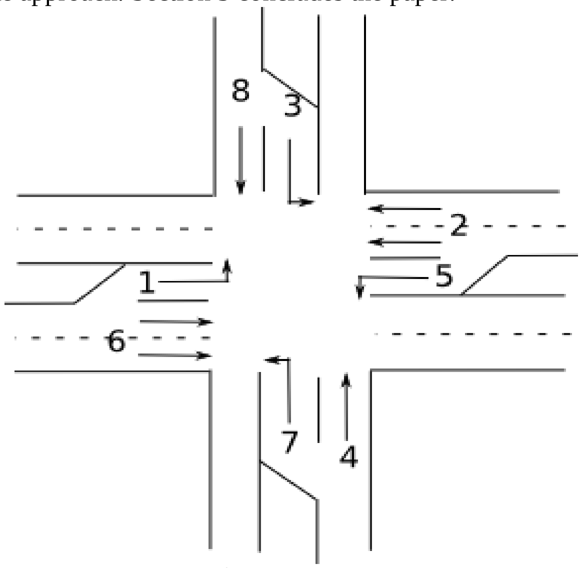

Fig. 1 NEMA (National Electrical Manufacturers Association) signal phasing [Source: Li (2011)]

2. Methodology

This work proposes a weighted center of mass based optimization approach, which finds the robust signal timings that minimize the overall average delay per vehicle for pre-timed signalized junctions with uncertainty in traffic flow. In this section, first, we give a brief background of traffic signal control. Then we propose a framework to find the robust signal timings for pre-timed signal control with uncertain traffic demands. Finally, we propose a delay minimization model and a weighted center of mass model to obtain the robust signal timings.

2.1. Background

We first provide some basic definitions in traffic signal control. Let N be the number of lane groups and M be the total number of movements. Fig. 1 depicts the standard signal phasing of NEMA (National Electrical Manufacturers Association) for a four-phased signalized junction. This junction has total eight movements (1, 2, ⋯8): movements 2,4,6,8 for the through and right movements and movements 1,3,5,7 for the left movements. Movements 1 and 5 are in lane group 1, movements 2 and 6 are in lane group 2, movements 3 and 7 are in lane group 3, and movements 4 and 8 are in lane group 4. All the movements in same lane group will have the same signal timings and will be active simultaneously. However, the saturation flow and traffic flows may be different.

2.2. Proposed Framework

In this subsection, we discuss the working principles of the proposed framework. The details of this framework is shown in Fig. 2. First, we simulate numerous flow profiles to capture the uncertainty in traffic flow. The inputs for generation of flow profiles are the minimum flow ($q_j^{\min}$) and the maximum flow ($q_j^{\max}$) for each movement j. The flow profile for every movement j is then randomly generated in the range ($q_j^{\min}$, $q_j^{\max}$). Consider a toy example where there are 8 movements {1, 2, ⋯, 8} in a signalized junction. Let, for movements 1, 3, 5 and 7, $q_j^{\min}$ be 50 and $q_j^{\max}$ be 100; and for movements 2, 4, 6 and 8, $q_j^{\min}$ be 80 and $q_j^{\max}$ be 150. Then a possible flow profile can be (63, 149, 51, 88, 85, 123, 92, 142). Several such flow profiles are generated. Optimal green times of all lane groups and corresponding delays are obtained by solving the proposed delay minimization model (OPT) for each flow profile. These obtained optimal green times and corresponding delays are then used in the proposed weighted center of mass based optimization model (WCM) to determine the robust signal timings for a pre-timed junction with uncertain traffic flows.

OPT and WCM are discussed in details in the following subsections.

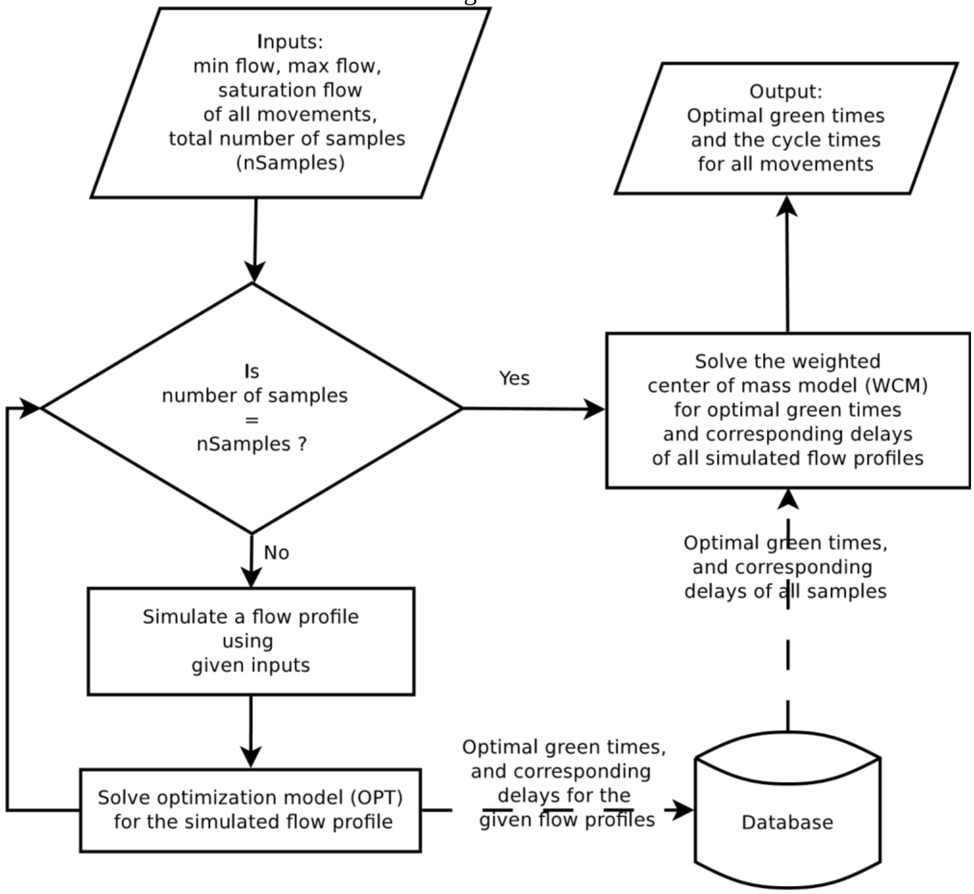

Fig. 2 Working principles of the proposed framework

2.3. The Optimization Model (OPT) Formulation

The objective of the model is to minimize the average delay per vehicle, by finding the optimal green times and the cycle time for a given flow. HCM 2000 delay equation TRB (2000) is widely used and field tested in signal optimization (Li 2011) to minimize delay, so we use the same to estimate delays. The HCM 2000 delay equation computes the average delay per vehicle as follows:

$$d = \frac{0.5C\left(1-\lambda\right)^2}{1-\left[\min\left(1,x\right)\lambda\right]} \;+\; 900T\left[\left(x-1\right)+\sqrt{\left(x-1\right)^2+\frac{4x}{cT}}\;\right] \tag{1}$$

Where, d is the average delay per vehicle *(seconds/veh*), C is the cycle length (*seconds*), λ is the effective green split, T is the duration of analysis period (*hours*), x is the degree of saturation of the lane group, and c is the capacity of the lane group (*veh/hour*).

Let g_j, q_j and s_j be the effective green time (*seconds*), traffic flow (*veh/hour*) and saturation flow (*veh/hour*) for movement j respectively. The saturation flow is the number of vehicles that could have passed through a lane group in an hour, if the traffic signal were always green for an hour (Roess et al. (2004)). Then $\lambda = g_j/C$, $x = q_j/\lambda s_j = q_j C/s_j g_j$ and $c = \lambda s_j = g_j s_j/C$.

Thus, HCM 2000 delay equation (1) for movement j can be rewritten as

$$d_j = \frac{0.5C\left(1-\frac{g_j}{C}\right)^2}{1-\left[\min\left(1,\frac{q_jC}{s_jg_j}\right)\frac{g_j}{C}\right]} + 900T\left[\left(\frac{q_jC}{s_jg_j}-1\right)+\sqrt{\left(\frac{q_jC}{s_jg_j}-1\right)^2+\frac{4q_jC^2}{s_j^2g_j^2T}}\right] \quad (2)$$

The proposed optimization model (Model 1) minimizes the average delay per vehicle with respect to bound constraints on green times and cycle time. Let M be the total number of movements, q_j be the traffic flow of the movement j, d_j be the delay per vehicle for the movement j as given in equation 2, N be the total number of phases, P be the set of phases, $g_{\min}$ be the minimum green time for any movement j, $C_{\min}$ and $C_{\max}$ be the minimum cycle time and the maximum cycle time respectively, L be the total time lost per cycle (the amount of time lost due to signal change over O'Flaherty (1997)) and T be the duration of the analysis. The decision variables are g_j, the green time for the movement j, and the cycle time C. The optimization formulation is

$$(\textbf{Model 1}) \qquad \min \sum_{j=1}^{M} \frac{q_j \cdot d_j}{\sum_{i=1}^{M} q_i} \quad (3)$$

Subject to:

$$g_j \geq g_{min}, \qquad \forall j \quad (4)$$

$$g_k = g_j, \ \forall k,j \in p \text{ and } \forall p \in P \quad (5)$$

$$C_{\min} \leq C \leq C_{\max} \quad (6)$$

$$C = \sum_{j=1}^{N} g_i + L \quad (7)$$

$$g_j \in \mathrm{Z}, \qquad \forall j \quad (8)$$

The objective function (3) minimizes the average delay per vehicle (the total delays divided by the total volumes). Constraint (4) ensures that the allotted green time is more than the minimum green time for every movement, and constraint (5) ensures that the green time of all movements in a phase are same. Constraint (6) bounds the cycle time and constraint (7) ensures that the cycle time is equal to the sum of green times of all lane groups and the total lost time per cycle. Constraint (8) is the integerabilty constraint on green times.

Model-1 is an integer non-linear model. Furthermore, its objective (3) is non-convex and nondifferentiable, since its first term has a minimum operator in the denominator. If the optimization model is convex and differentiable, then there are numerous fast mixed-integer non-linear optimization solvers that can be used to find the global optimal solutions (Horst et al. (2000), Burer and Letchford (2012), Nesterov (2004)). Hence, the optimization Model-1 is reformulated, where the term $\left[\min\left(1, \frac{q_j C}{s_j g_j}\right)\frac{g_j}{C}\right]$ in equation 2 is replaced with an auxiliary decision variable m_j. The complete mathematical formulation of the proposed optimization model is as follows:

$$(\textbf{OPT}) \quad \min \sum_{j=1}^{M} \frac{q_j \cdot \bar{d}_j}{\sum_{i=1}^{M} q_i} - M_{big} \sum_{j=1}^{M} m_j \tag{9}$$

Where, M_{big} is a very large number and

$$\bar{d}_j = \frac{0.5C\left(1 - \frac{g_j}{C}\right)^2}{1 - m_j} + 900T\left[\left(\frac{q_j C}{s_j g_j} - 1\right) + \sqrt{\left(\frac{q_j C}{s_j g_j} - 1\right)^2 + \frac{4 q_j C^2}{s_j^2 g_j^2 T}}\right], \quad \forall j \tag{10}$$

subject to:

constraints (4) to (8)

$$m_j \leq \frac{g_j}{C}, \quad \forall j \tag{11}$$

$$m_j \leq \frac{q_j}{s_j}, \quad \forall j \tag{12}$$

This optimization model (OPT) is a minimization problem. Where, M_{big} is a very large number($M_{big} \gg \sum_{j=1}^{M} \bar{d}_j$). Hence $-M_{big}$ incentivizes m_j to take as large value as possible in (9). Constraint (11) and (12) ensures that m_j equals to $\left[\min\left(1, \frac{q_j C}{s_j g_j}\right)\frac{g_j}{C}\right]$ and hence $\bar{d}_j$in (10) is the same as d_j in (2). Thus, Model-1 and OPT are equivalent. OPT has no min operator in $\bar{d}_j$ and is differentiable, which permits the model to be solved by non-linear optimization model solvers. The solution approach to solve OPT is discussed in Subsection 3.1.

2.4. Weighted Center of Mass (WCM) Model

The objective of our approach is to find the robust signal timings for pre-timed signal control when traffic demands are uncertain. We can use Weighted Center of Mass model (WCM) to obtain the robust signal timings. WCM is extensively used in facility location problem to identify the optimal facility location that will minimize the total cost, when multiple locations and associated cost with each location are available (Ross and Soland (1977), Chopra et al. (2012), Daskin (1983), Weaver and Church 1985). The same idea can be used to find the robust green times that will minimize the average delay per vehicle when optimal green times and corresponding delays for each sampled flow profiles are available.

We simulate different flow profiles to account for the traffic variations in the given ranges of flow and solve the OPT model for each flow profile to get the optimal green times and corresponding delays. The obtained optimal green times and delays from OPT for all samples are used in weighted center of mass (WCM) model to find the optimal green times, that minimize the overall delay per vehicle and are robust to the uncertainty in traffic flow.

Let nSamples be the total number of simulated flow profiles from the given ranges of flow, M be the total number of movements, N be the total number of phases, P be the set of phases, and g_j^i and d^i_j be the optimal green time and corresponding delay of the movement j, obtained from OPT for flow profile *i*. The decision variables are the green times (g_j^*) that minimize the overall delay per vehicle. Then the WCM is:

$$\sum_{i=1}^{nSamples} \sum_{j=1}^{M} d_j^i (g_j^* - g_j^i)^2 \quad (13)$$

Subject to:

$$g_j^* \geq g_{min}, \quad \forall j \quad (14)$$

$$g_k^* = g_j^*, \quad \forall k, j \in p \text{ and } \forall p \in P \quad (15)$$

$$C_{min} \leq C \leq C_{max} \quad (16)$$

$$C = \sum_{j=1}^{N} g_j^* + L \quad (17)$$

$$g_j^* \in Z, \quad \forall j \quad (18)$$

The objective function (13) minimizes the weighted average delay per vehicle and deviations in the green times. Constraint (14) ensures that the green time of each movement is more than the minimum green time, and constraint (15) ensures that the green time of all movements of a phase are same. Constraint (16) bounds the cycle time and constraint (17) ensures that the cycle time is equal to the sum of green times of all lane groups and the total lost time per cycle. Constraint (18) is the integerabilty constraint on green times. The solution approach to WCM is discussed in the Subsection 3.1.

3. Numerical Examples

The robustness and performance of the proposed approach for pre-timed signalized junctions are tested with two examples proposed by Yin (2008), and the results are compared with those of the models by Yin (2008) and Li (2011). The parameter values used in these examples are: the minimum cycle time (C_{min}) is 50 seconds, the maximum cycle time (C_{max}) is 140 seconds, the lost time (L) is 14 seconds, the minimum green time ($g_{\min}$) is 8 seconds and the analysis period (T) is 15 minutes.

Example 1 consist of four lane groups (as shown in Fig. 1) where movements 1 and 6 belong to lane group 1, movements 2 and 5 belong to lane group 2, movements 3 and 8 belong to lane group 3, and movements 4 and 7 belong to lane group 4 Yin (2008). Thus, for this example, the resulting constraints in OPT are constraints (8), (11), (12) and:

$$g_1 + g_2 + g_3 + g_4 + L = C \quad (19)$$

$$g_1 = g_6,\ g_2 = g_5,\ g_3 = g_8,\ g_4 = g_7 \quad (20)$$

$$g_j \geq 8, \quad j = 1, 2, \cdots 8 \quad (21)$$

$$50 \leq C \leq 140 \quad (22)$$

Example 2 is from a real-world junction in the City of Lynnwood, Washington (Yin 2008). Here, movements 1 and 5 belong to lane group 1, movements 2 and 6 belong to lane group 2, movements 3 and 8 belong to lane group 3, and movements 4 and 7 belong to lane group 4. Thus, the constraints for this example are same as that of example 1 except for constraint (20), which is replaced with:

$$g_1 = g_5,\ g_2 = g_6,\ g_3 = g_8,\ g_4 = g_7 \quad (23)$$

Table 1 and 2 presents the saturation flow (s_j), the average flow (q_j^{avg}), the standard deviation (SD) (q_j^{sd}) of the flows, the minimum flow ($q_j^{\min}$) and the maximum flow ($q_j^{\max}$) for all the movements used in Examples 1 and 2 respectively. These have been extracted from Table 1 and 7 of Yin (2008). Li (2011) has also used these examples to evaluate their model.

Table 1**.** Flow characteristics for example 1

Movements		**Under-saturated**				**Over-saturated**			
(j)	s_j	q_j^{avg}	q_j^{sd}	q_j^{min}	q_j^{max}	q_j^{avg}	q_j^{sd}	q_j^{min}	q_j^{max}
1	1900	225	65	100	350	275	90	100	450
2	3800	400	100	200	600	525	140	250	800
3	3800	650	125	400	900	875	160	550	1200
4	1900	275	65	150	400	275	60	150	400
5	1900	250	25	200	300	350	75	200	500
6	3800	500	100	300	700	650	175	300	1000
7	3800	650	75	500	800	900	150	600	1200
8	1900	170	25	120	220	250	65	120	380

Table 2. Flow characteristics for example 2

Movements (j)	s_j	q_j^{avg}	q_j^{sd}	q_j^{min}	q_j^{max}
1	1650	214	33	168	288
2	3200	1012	147	780	1348
3	1650	271	53	188	408
4	1700	157	27	88	208
5	1650	66	24	28	100
6	3200	1064	89	860	1252
7	1650	59	16	32	92
8	1700	423	80	296	656

3.1. Implementation

The flow profiles generation and data file creation was done using Java. The proposed optimization models (OPT and WCM) are coded in AMPL (Fourer and Gay (2007)) and solved using Bonmin solver (Bonami and Lee (2015)). AMPL (A Mathematical Programming Language), is a language for mathematical programming problems and Bonmin (Basic Open-source Nonlinear Mixed INteger programming) is a free open-source solver for solving general MINLP (Mixed Integer NonLin-ear Programming). System configurations used are: Intel core i3 dual core processor, 4 GB RAM, and 64 bits Windows 7 operating system. All the algorithms are sequential and do not leverage multiple processors.

3.2. Results and Discussions

The exact traffic flows are uncertain in the given ranges of traffic flows. The sample flow profiles for both Example 1 and Example 2 are generated using truncated Normal distribution, with flow characteristics given in Table 1 and 2 respectively, and rounded to the nearest integer. We use truncated Normal sampling as in (Yin 2008) to generate flow profile because we compare our results with those of Yin (2008) and Li (2011). Although, truncated Normal sampling has been used to show the working of our approach, our approach is equally applicable to any other flow sampling.

Table 3. The robust green times (g_j^*), cycle time and execution time in seconds for different number of samples in example-1, under-saturation (US) case

Ex-1 (US)	**green times (s)**				**cycle**	**Execution**	**Average**
nSamples	g_1^*	g_2^*	g_3^*	g_4^*	**time (s)**	**time (s)**	**delay (s)**
50	10	9	12	12	57	3.12	34.73
100	10	9	12	12	57	5.17	34.73
200	10	9	12	12	57	12.53	34.73
300	10	9	12	12	57	18.14	34.73
400	10	9	12	12	57	19.86	34.73
500	10	9	12	12	57	23.38	34.73
1000	10	9	12	12	57	56.57	34.73
2000	10	9	12	12	57	98.79	34.73
3000	10	9	12	12	57	147.57	34.73
4000	10	9	12	12	57	201.33	34.73
5000	10	9	12	12	57	285.76	34.73
10000	10	9	12	12	57	648.14	34.73

If only a few samples are considered, then the results might be under-conservative or over-conservative. To avoid this, the proposed model (OPT) is solved for different number of flow profiles. Then, the obtained optimal green times and corresponding delays are fed to the weighted center of mass model (WCM) to get the robust green times for uncertain traffic flows. The obtained optimal green times (*s*), cycle time (*s*) and execution time (*s*) from the WCM model with different number of samples are listed in Table 3, 4 and 5 for Example 1 (under-saturation case and over-saturation case) and Example 2 (real world case) respectively. For each sample, we also calculated the average delay per vehicle (seconds) for 30,000 simulated flow profiles, using the obtained optimal green times. These average delays for each case are listed in the last column of the respective tables. We found that *50* samples are sufficient for under-saturation case and *400* samples are sufficient for over-saturation case of Example 1 to get the optimal green times that are robust to the traffic variations as listed in Table 3 and 4. For the real world case (Example 2), we found that *400* samples are sufficient to get robust signal timings as listed in Table 5.

Table 4. The robust green times (g_j^*), cycle time and execution time in seconds for different number of samples in example-1, over-saturation (OS) case

Ex-1 (OS)	**green times (s)**				**cycle**	**Execution**	**Average**
nSamples	g_1^*	g_2^*	g_3^*	g_4^*	**time (s)**	**time (s)**	**delay (s)**
50	18	17	23	22	94	2.52	71.57
100	18	17	22	22	93	5.35	71.41
200	18	17	22	23	94	10.27	71.24
300	18	17	22	23	94	14.24	71.24
400	18	17	23	23	95	20.81	71.23
500	18	17	23	23	95	23.73	71.23
1000	18	17	23	23	95	47.61	71.23
2000	18	17	23	23	95	102.68	71.23
3000	18	17	23	23	95	152.06	71.23
4000	18	17	23	23	95	211.43	71.23
5000	18	17	23	23	95	250.70	71.23
10000	18	17	23	23	95	616.34	71.23

Li (2011) tabulated results of his and Yin (2008)'s model for both the examples in Table 9 of his work. In order to show the effectiveness of our model, we abstracted the proposed green times of their models. Then we simulated *30,000* flow profiles and computed the average delays per vehicle with their proposed green times and the green times obtained from WCM. The obtained results are tabulated in Table 6. The percentage reduction in average delay per vehicle obtained by the proposed approach with respect to others is given in the last column of the table. Example 1 has been solved for both under-saturation and over-saturation cases. The proposed model yields around

3% reduction in the average delay per vehicle in case of under-saturation when compared to both models and around *4%* reduction for over-saturation case.

Table 5. The robust green times (g_j^*), cycle time and execution time in seconds for different number of samples in example-2, real world junction case

Ex-2	**green times (s)**				**cycle**	**Execution**	**Average**
nSamples	g_1^*	g_2^*	g_3^*	g_4^*	**time (s)**	**time (s)**	**delay (s)**
50	12	35	23	9	93	2.31	57.58
100	12	34	24	9	93	4.77	57.11
200	12	34	24	9	93	9.49	57.11
300	12	34	24	9	93	14.39	57.11
400	12	35	24	9	94	19.26	56.65
500	12	35	24	9	94	24.87	56.65
1000	12	35	24	9	94	50.41	56.65
2000	12	35	24	9	94	98.79	56.65
3000	12	35	24	9	94	147.57	56.65
4000	12	35	24	9	94	212.57	56.65
5000	12	35	24	9	94	266.12	56.65
10000	12	35	24	9	94	580.75	56.65

In case of Example 2 where real world intersection data has been used, the proposed approach reduces around *3%* average delay per vehicle when compared to Li (2011) and is as good as Yin (2008). One this to note is that the above studies considered the value of total volume variation parameter (θ) to be 0.5, which implies that their signal time is optimized with respect to limited uncertain traffic volume. However, we solved the proposed model considering the total volume variation and yet obtained better results in terms of average delay.

As pointed out by Li (2011), when the value of total volume variation parameter (θ) increases, the computational time increases significantly. This limitation is common for the approaches of both Li (2011) and Yin (2008). For example, in over-saturation case of Example 1, around 761 CPU minutes (around 12.7 hour) are needed to obtain the optimal solution by Li (2011)'s approach, whereas in our approach, around 19.26 seconds of CPU time is needed to get the robust green times for the uncertain traffic flows. Thus, the proposed approach is computationally more efficient.

Table 6. Comparison with the results of Yin (2008) and Li (2011)

Instance	Study	C (s)	Green times (s) g_1^*	g_2^*	g_3^*	g_4^*	Avg. delay/veh (s)	%Red.
Ex-1	Yin (2008)	115	24	19	29	29	74.35	4.20
OS	Li (2011)	118	24	20	30	30	74.11	3.89
	Ours	**95**	**18**	**17**	**23**	**23**	**71.23**	
Ex-1	Yin (2008)	68	13	11	16	14	35.72	2.77
US	Li (2011)	70	13	11	17	15	35.99	3.5
	Ours	**57**	**10**	**9**	**12**	**12**	**34.73**	
Ex-2	Yin (2008)	100	12	39	26	9	56.24	0.33
	Li (2011)	99	12	37	28	8	58.27	2.78
	Ours	**94**	**12**	**35**	**24**	**9**	**56.65**	

(OS - Over-Saturation, US - Under-saturation, C - Cycle time, % Red. - Percentage reduction in average delay per vehicle)

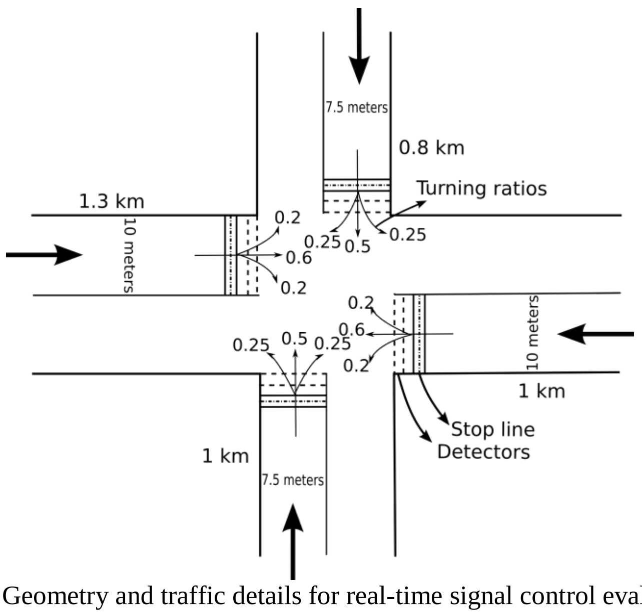

Fig. 3. Geometry and traffic details for real-time signal control evaluation

3.2.1. Impact of Flow Profiles on Green Times and Delay

The proposed approach needs flow profiles as an input. The flow profiles can be generated using the demand distribution when it is known. However, in practice, it is not always possible to capture the traffic fluctuations in a known distribution. In such cases, we can generate the flow profiles using uniform sampling from the given ranges (the minimum and the maximum) of the traffic flow. To see the effect of assuming uniform demand distribution over a known demand distribution (normal), we run the model for Example 1 (under-saturation and over-saturation). The demand distribution in Example 1 is normal distribution Li (2011). The obtained results are shown in Table 7. We found that the green times (g^*) and the cycle time (C) obtained using uniform demand distribution are marginally more than the green times and the cycle time obtained using the known demand distribution.
Also, the average delays per vehicle are similar in both the cases. Hence, even if the actual demand distribution is not known, the proposed approach can be used to find the green times, with marginal reduction in performance.

4. Pre-timed Approach vs Real-time Approach

In this section, we verify the benefits of using real-time approach (OPT) over pre-timed approach (WCM). To evaluate the real-time approach, we simulate the road geometry and phase plan as shown in Fig. 3. The traffic flows used are shown in Fig. 4. We use the values of control parameters, vehicular characteristics and traffic composition as in Patel et al. (2016). For pre-timed approach, we have considered different ranges of traffic flows (the minimum flow and the maximum flow) for different times of a day such as early morning, morning peak, etc., as shown in Table 8.

Table 7. Comparison of green times, cycle times and the average delay using normal demand distribution and uniform demand distribution

	Ex-1 Under-saturation			**Ex-1 Over-saturation**	
green times (s)	Normal	Uniform		Normal	Uniform
g_1^*	10	11		18	20
g_2^*	9	10		17	18
g_3^*	12	12		23	24
g_4^*	12	13		23	24
cycle (s)	57	60		95	100
average delay (s/veh)	34.73	35.18		71.23	71.47

The values for these times have been derived from the Fig. 4. We have generated the sample flow profiles for these times of a day using the uniform distribution. We obtain the green times for the pre-timed control using WCM as shown in Table 9.
We have compared average delays and average queue lengths of different phases using OPT and WCM. We found 47% reduction in average delay and 35% reduction in the average queue length of the junction in real-time approach (OPT) compared to the pre-timed approach (WCM). This is because the real-time approach finds optimal green times in each cycle, considering demands of each phase. But, the pre-timed approach does not re-compute the green times in each cycle. In WCM approach, we observe that there is large increase in average delays of minor phases (phase-3 and phase-4), while average delays for major phases (phase-1 and phase-2) are improved compared to OPT approach. This is because, in pre-timed control, large green times are assigned to the major phases and lesser green

times are assigned to the minor phases for longer time periods (not just a cycle). For the same reason, the average queue lengths in the major phases using WCM approach are much smaller when compared to the average queue lengths using OPT approach, and vice-versa.

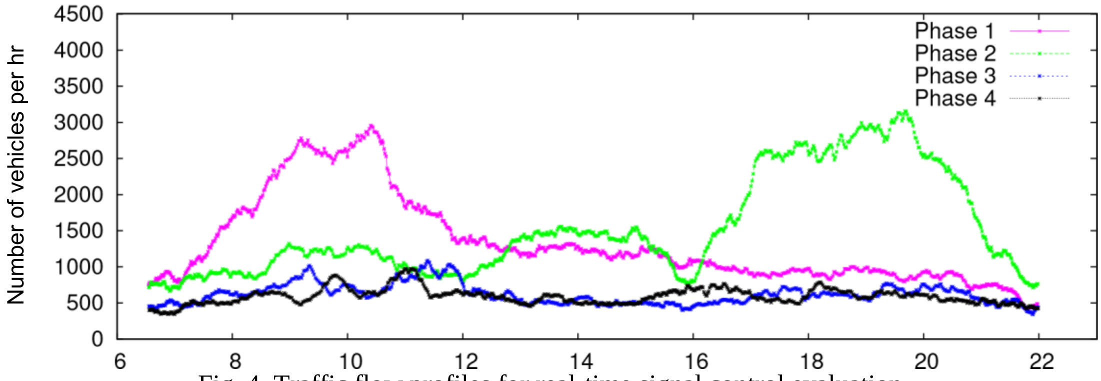

Fig. 4. Traffic flow profiles for real-time signal control evaluation

Table 8. Input flow ranges for different times of a day for pre-timed control

	Time (hr)		**Phase-1**		**Phase-2**		**Phase-3**		**Phase-4**	
			Min	Max	Min	Max	Min	Max	Min	Max
Early Morning	6:00	8:30	400	1600	300	1000	300	800	200	700
Morning Peak	8:30	12:00	1400	3000	800	1600	700	1100	500	1000
Day	12:00	17:00	800	1500	1100	1900	300	700	300	900
Evening Peak	17:00	21:00	700	1300	1600	3200	400	1100	350	1100
Night	21:00	22:00	300	700	400	1200	150	650	200	800

5. Conclusion

In this paper, we consider the robust traffic signal optimization problem for pre-timed signalized junctions with uncertain traffic flow. The existing approaches for robust signal timings to the uncertainty in traffic flow use min-max formulation. The limitations of the min-max approach are that the optimality of the solutions are not guaranteed and it takes long computational time for larger ranges of uncertain traffic flow.

We have proposed a delay optimization model (OPT) to minimize the average delay per vehicle for a given flow profile. We generate sufficient number of flow profiles to capture the traffic variations and for each flow profile, the OPT model is solved to get the optimal green times and corresponding delays. Then, we propose a weighted center of mass based optimization model (WCM), that uses the obtained optimal green times and corresponding delays of all flow profiles from the OPT model to find the robust green times for the uncertain traffic flows.

Table 9. Timing plans for pre-timed control using WCM approach

	Time (hr)		Timing Plans			
			g_1^*	g_2^*	g_3^*	g_4^*
Early Morning	6:00	8:30	26	17	14	12
Morning Peak	8:30	12:00	29	16	12	10
Day	12:00	17:00	23	29	11	13
Evening Peak	17:00	21:00	16	36	12	12
Night	21:00	22:00	14	22	12	15

Table 10. Comparison of the average delays and the average queue length obtained using OPT and WCM

Control	Average delays (s/veh)					
	Phase-1	Phase-2	Phase-3	Phase-4	Junction	*% reduction*
Real-time (OPT)	32.1	26.0	85.5	71.6	43.6	47.2
Pre-timed (OPT+WCM)	27.19	25.80	246.06	184.81	82.57	–
	Average queue length (m)					
Real-time (OPT)	30.6	21.3	50.4	38.5	31.2	35.0
Pre-timed (OPT+WCM)	24.06	22.15	124.13	90.06	48.03	–

The robustness of the proposed approach is tested for a four-phased signalized junction with under-saturation and over-saturation traffic flows. It is also tested for a real world junction data. Results show that the proposed approach reduces the average delay per vehicle significantly for all the test cases as compared to existing approaches. Simulation results also show that *500* samples are enough to capture the total variations in traffic flows for the considered test cases. Our approach can be solved in about 25 CPU seconds to obtain the robust green times for uncertain traffic flow. Whereas, the existing robust signal control approaches take hours of CPU time to get solutions.

We have also shown that, when the demand distribution is unknown, we can assume uniform distribution to generate sample flow profiles from the given ranges (the minimum and the maximum) of the traffic flow. These flow profiles can be used to find the green times, using the pre-timed approach, with marginal reduction in the average delay when compared to the known demand distribution.

Although, the proposed approach (OPT+WCM) is for pre-timed signal control. The test results show that the optimization model (OPT) outperforms (OPT+WCM) when real-time vehicle count is available. Thus OPT can also be used for real-time signalized intersection control. Furthermore, the proposed approach (OPT+WCM) can be extended for an arterial signal control.

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