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# The determination of distance based toll for urban expressway with deep learning approach

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## Abstract

Since the distance based toll system has been introduced in urban expressways, the determination of optimum toll is quite important in terms of practical traffic management of urban transport networks. The traffic assignment with variable demand is usually applied to estimate traffic flows on urban network to evaluate the social benefit of pricing. However, the huge load of network calculation would be reduced to discuss the combination of parameters in the distance based toll. The deep learning approach is introduced to learn the essential factors in the estimation of traffic flows in urban transport networks. In particular, the convolutional neural network (CNN) is created from the database of the estimation results in the original example networks. The estimation model for the values of indicators in the determination of parameters in the distance based toll. The advantages of the deep learning approach is to provide the approximate solution without traffic assignment process. It reflects on the efficient determination of shape of distance based toll corresponding to the variable OD conditions. Finally, the optimum solution for the specific condition of urban network can be determined practically. The proposed method would be applied to the determination of optimal distance based toll corresponding to the variable demand.

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*Keywords:* Traffic assignment; Deep learning approach; Urban expressway; Optimum toll

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## 1. Introduction

The distance based toll system has been applied on urban expressway in Japan. Even though the optimal toll should be determined referring to the congestion toll theory for all urban road networks, the toll system is applied only for urban expressways. Therefore, the optimization of distance based toll should be an essential issue in the real world. According to the circumstances, the method of determination of the optimal toll pattern of urban expressway would be developed in the study. In terms of advanced information techniques, many different values of toll between on and off ramps can be realized with ETC system. Recently, the dynamic determination of toll patterns for urban expressway is recommended as well corresponding to the variable demand (Akiyama, Inokuchi, Okushima, 2017).

Basically, huge amount of calculation effort should be required to compare many combinations of toll pattern for urban expressways (Akiyama, Okushima, Inokuchi, 2011). Therefore, the practical method of estimation of distance based toll impacts is recommended to reduce the above effort of calculation with the traffic assignment. As several indicators can be estimated in the evaluation of distance based toll determination, the approximation model would be recommended. The estimation model without traffic assignment process is proposed to realize instant estimation of evaluation indicators. In particular, the deep learning approach is introduced because many training data can be provided from the accumulation of calculation results of numerical example. As the deep learning approach is widely applied in the field of pattern recognition, the similar formulation is applied to the previous problems (Du, Li, Gong, 2018, Duan, Lv, Liu, 2016). The convolutional neural network (CNN) model is created to estimate the values of evaluation indicators for distance based toll. The CNN model can estimate instantly four evaluation indicators

corresponding to the OD matrix and toll function parameters. Therefore, the optimal combination of toll parameter can be easily determined even if the number of combinations is very large. It would be an advantage of application of deep learning approach.

## 2. Distance based toll of urban expressways

### 2.1. The outline of urban expressways in Japan

The outline of urban expressways and urban road networks are mentioned in this section. The real scale transport networks can be observed in Keihanshin metropolitan area in Japan. Fig. 1 shows the urban road network (streets) as well as Hanshin expressway as urban expressway in the area.

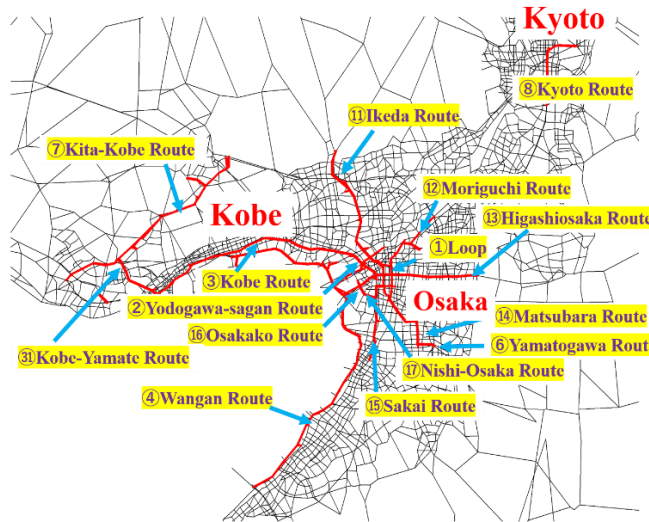


Fig. 1. Transport network with urban expressways.

The red line stands for urban expressway networks (Hanshin expressway). On the other hand, the black line demonstrates urban streets. The total length of Hanshin expressway is measured as 260.5 km in 2017. It consists of 17 routes with one loop road. The urban expressway is defined as toll roads in contrast to the free road for all urban streets. Concerning with traffic conditions, the annual average daily traffic volume of the Hanshin expressway is reported as 756,972 vehicles/day in 2017.

### 2.2. The definition of distance based toll

According to redemption of construction cost of urban expressway networks, the toll system has been applied to urban expressways in Japan. In particular, the distance based toll system has been applied on urban expressways as well as intercity expressways since 2012 (Akiyama, Inokuchi, 2013, Inokuchi, Akiyama, 2014). The definition of distance based toll of urban expressway such as Hanshin expressway has been updated to determine the common rate of toll for intercity expressway since 2017. Referring to the recent definition, the vehicle types are classified into five vehicles such as light wheel/ two wheel, ordinary, medium sized, large sized, extra-large sized. In terms of toll collection technology, the distance based toll is collected from the individual the vehicles with ETC (electric toll collection) system. On the other hand, the vehicles without the installation of ETC system are charged the upper bound toll regardless of the travel distance. It is reported in May 2018 that the ETC equipment is extended to about 95 % of existing vehicles on Hanshin expressway.

Fig. 2 illustrates the previous and the current definitions of distance based toll for ordinary vehicles with the ETC system. The both distance based toll are determined as linear functions.

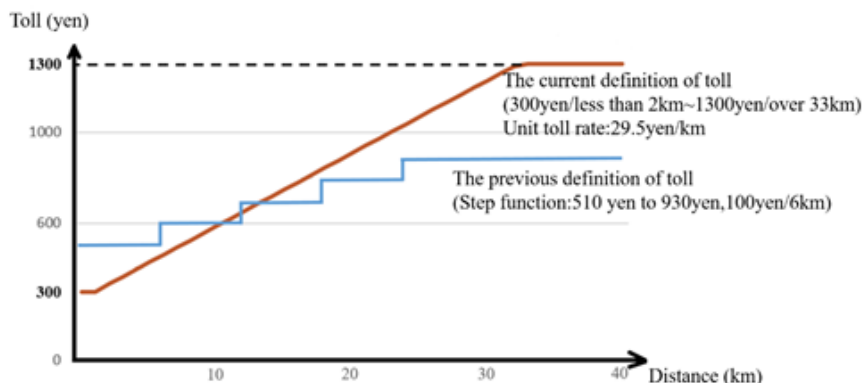


Fig. 2. The distance based toll of urban expressway.

As shown in the figure, the distance based toll consists of two elements such as terminal charge for entering and the toll rate in proportion to the travel distance. In terms of toll function, the lower bound toll with less than 1 km and the upper bound toll over 33 km are determined respectively for the range of distance based toll.

2.3. The numerical example of urban transport network

The numerical example for urban road network is created to discuss the determination of distance based toll for urban expressways in the study. Therefore, the urban expressway network and urban street networks are separately illustrated. Fig. 3 illustrates the numerical example network applied to the fundamental analysis of the optimal toll pattern with referring to the traffic flow conditions on urban transport networks.

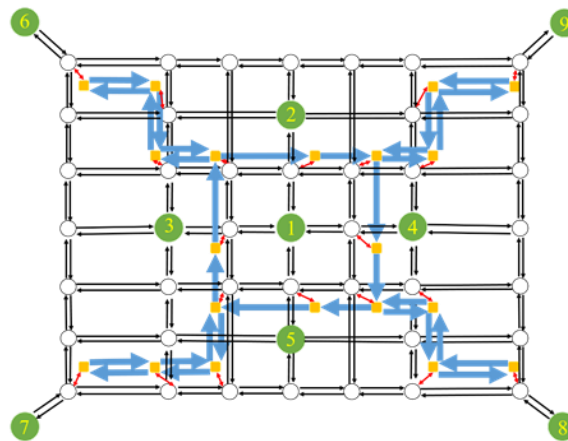


Fig. 3. The numerical example of urban transport network.

The grid network in black for urban streets is imitated to road network in Osaka. On the other hand, the urban expressway is demonstrated in blue with ramp way for entrance and exit of expressway in red. The urban expressway consists of loop road as well as several radiation routes. This is resemble to Hanshin expressway networks.

Furthermore, the centroids for traffic are indicated as green circles. The centroids are located into three areas such as city centre, surrounding area and suburban area.

Each centroid corresponds to the origin and destination of flows on the networks. Therefore, the origin destination (OD) matrix can be shown as Table 1 with nine centroids in initial condition of urban networks.

Table 1. The origin destination matrix in present condition.

O \ D	Central Area			Urban Area			Suburban Area			
	1	2	3	4	5	6	7	8	9	
Central Area	1	-	14400	7200	9000	12600	10800	10800	10800	10800
	2	14400	-	10800	12600	16200	14400	14400	14400	14400
Urban Area	3	7200	10800	-	5400	9000	7200	7200	7200	7200
	4	9000	12600	5400	-	10800	9000	9000	9000	9000
	5	12600	16200	9000	10800	-	12600	12600	12600	12600
Suburban Area	6	10800	14400	7200	9000	12600	-	10800	10800	10800
	7	10800	14400	7200	9000	12600	10800	-	10800	10800
	8	10800	14400	7200	9000	12600	10800	10800	-	10800
	9	10800	14400	7200	9000	12600	10800	10800	10800	-

The mono centric structure is assumed on the urban transport networks. The symmetric distribution of OD traffic is assumed in surrounding area as well as suburban area. The distribution of OD traffic is assumed corresponding to the observation of OD distribution in real scale networks in Keihanshin area.

The real scale transport network is introduced to analyze the relation between the distance based toll and the real traffic flows on the network in the related studies (Akiyama, Okushima, Inokuchi, 2014). The small scale numerical example is created to cover the same scale urban network. Therefore, the similarity of numerical example to real scale network should be mentioned to summarize the evaluation results for real scale networks. Table 2 summarizes the results of comparison between the real scale road network and the numerical example for urban road networks.

The number of links representing urban expressway in the numerical example is counted as about 5 percent of the number of real scale road network. On the contrary, the number of links representing urban streets in the numerical example is counted as about 1/87 of the number of real scale road network. Therefore, the numerical example seems to cover the relatively small area of urban street networks.

Table 2. Similarity of numerical example to real scale networks.

	Real scale network	Numerical example
Number of links of urban expressway	831	40
Number of links of urban street	6,963	80
Number of nodes of urban expressway	466	20
Number of nodes of urban street	4,232	49
Number of zones	786	9
Road length of urban expressway	504 km (each direction)	118 km (each direction)
Total travel time of whole network	3,608,051 veh.hour	777,809 veh.hour
Total travel time of urban expressway	107,982 veh.hour	212,333 veh. hour
Toll revenue of urban expressway	454,559,878 yen	507,257,174 yen
Traffic volume of urban expressway	907,628 veh.	850,787 veh.

The results of user equilibrium conditions given by the traffic assignment are shown in the table as well. The toll revenue and the traffic volume of urban expressway of real scale network and numerical example are relatively similar. In the real scale road network, calculation time of traffic assignment model of each case is necessary about 70 minutes (Akiyama, Inokuchi, Okushima, 2013). Therefore, it is difficult to consider many cases. The numerical example road network model is used to evaluate the distance based toll of urban expressway.

Fig. 4 shows the link traffic volume of the calculation result of user equilibrium traffic assignment.

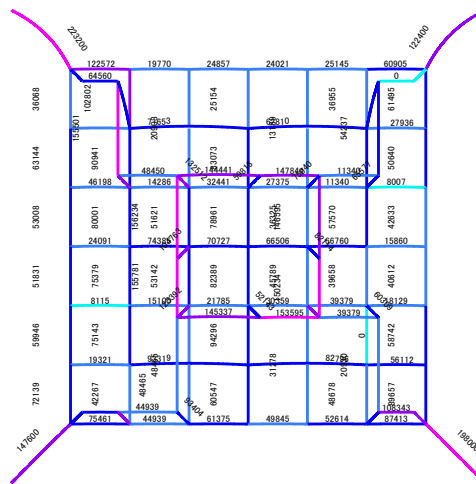


Fig. 4. Traffic volume of each road section.

Congested traffic volumes are estimated in the sections of the loop road and some radius routes of the urban expressway. The similar traffic condition can be observed on Hanshin expressway in the real scale networks.

### 3. The estimation of user equilibrium with deep learning

#### 3.1. The estimation approach of the optimum toll function

Traditional estimation approach of the optimum toll function can be illustrated in Fig. 5.

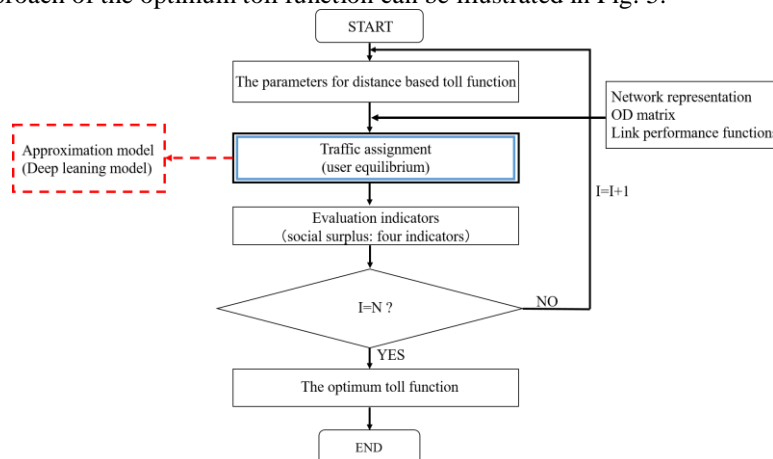


Fig. 5. The estimation of user equilibrium with deep learning.

After setting the parameters of the distance based toll, the traffic volume is calculated by the traffic assignment model. Using the calculation results of traffic assignment, four indicators of social surplus are calculated (Akiyama, Mun, Okushima, 2004, Mun, Akiyama, Okushima, 2007). This process is repeated, and the optimum toll pattern is determined based on the social surplus.

The alternative estimation model of traffic assignment is created with the CNN approach (Wu, Tan, Qin, 2018). However, the CNN model cannot provide the detail estimation values for urban networks. Therefore, the accuracy of approximation of evaluation indicators are confirmed. The values of social surplus indicators are basically estimated from the traffic assignment process. On the other hand, the CNN model is created to directly estimate values of social surplus indicators without traffic assignment process (Akiyama, Inokuchi, 2014).

### 3.2. The outline of CNN model

According to the previous analysis, the deep learning method is applied to estimate the evaluation indicators corresponding distance based toll pattern with various travel flow conditions. The deep learning method is proposed from methods of machine learning (Goodfellow, Bengio, 2016). This model is an extension of the neural network model. Many applications have been reported in the field of image recognition (Hou, Edara, 2018, Inokuchi, Akiyama, 2018, Lv, Duan, Kang, Li, Wang, 2015, Ma, Dai, He, 2017, Ma, Yu, Wang, 2015, Polson, Sokolov, 2017). In the conventional neural network model, as the number of layers increases the number of parameters increases rapidly. Therefore, it becomes difficult to proceed the learning process. Fig. 6 illustrates the outline of CNN model (Patterson, Gibson, 2017).

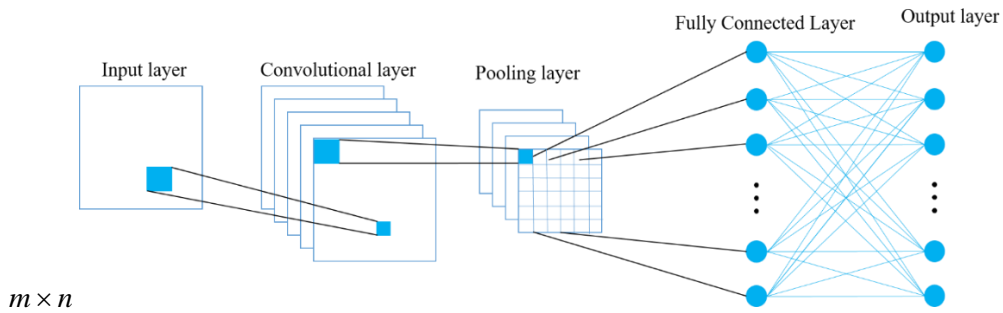


Fig. 6. Mechanism of convolutional neural network.

In the convolutional layer, a feature map of input layer is determined using some filters. The original input data is defined  $x_{ij}$ , and the  $k^{\text{th}}$  filter of  $m \times n$  size is defined as  $w_{st}^{(k)}$ . The values of element of convolutional layer are calculated by the equation (1).

$$z_{ij}^{(k)} = \sum_{s=0}^{m-1} \sum_{t=0}^{n-1} w_{st}^{(k)} x_{(i+s)(j+t)} \tag{1}$$

Since the data matrix size is still large, the data matrix should be aggregated into the pooling layer. For example, in the max pooling method, the equation (2) is used.

$$y_{ij}^{(k)} = \max \left( a_{(l_1 i+s)(l_2 j+t)}^{(k)} \right) \tag{2}$$

If  $2 \times 2$  elements are used for pooling, the maximum value of data matrix becomes is the output within this range. Therefore, the data size becomes a fourth of original data matrix. In the final stage of the CNN model, the values of output variables can be calculated with the fully connected layer same as in a normal neural network model.

This CNN model can be applied to estimate the values of evaluation indicators for urban road networks. Fig. 7 shows the outline of the CNN model proposed in the study.

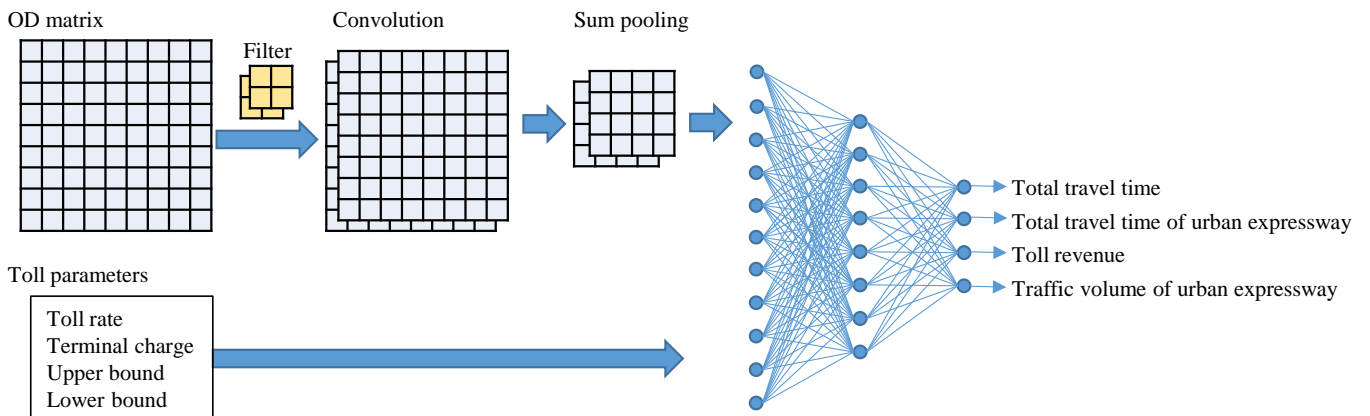


Fig. 7. The outline of CNN model in deep learning.

The input variables to the estimation model are determined as the OD traffic volume and the toll parameters for the urban networks. On the contrary, the output variables are determined as the total travel time of the overall urban road networks, the total travel time of urban expressway, the toll revenue of urban expressway and the entering traffic volume of urban expressway.

Four parameters of distance based toll are assumed such as the toll rate (yen/km), the terminal charge (yen), the upper bound and the lower bound of toll (yen). As the OD matrix is commonly large-scale data with large information, it would be aggregated with importing to the process of convolutional layers and pooling layers. On the other hand, four parameters of distance based toll cannot be complicate not needing to aggregate in convolution / pooling process. According to these analysis, the fully connected layer is designed to have two parallel inputs variables as shown in the figure.

Two types of  $2 \times 2$  matrix filters are installed to provide the convolution layer matrix. Furthermore, the pooling layer is installed with  $2 \times 2$  matrix. The max value operation is often applied to create the pooling layer. It seems to correspond to the maximum element represents to the overall information of matrix. In the study, the OD matrix is introduced as input variables. Therefore, the sum operation of all elements of matrix seems to provide the effective information in the pooling layer matrix.

The estimation model of social surplus indicators with CNN approach should be discussed. The conditions for deep learning with CNN is summarized. In particular, the filter matrix for convolution layer and connecting weights for fully connected layer are determined through deep learning process.

Basic idea of CNN model in Fig. 5 is mentioned previously. The training database would be created to provide for the deep learning process. In particular, the different traffic flow pattern is estimated from the different OD matrix. On the other hand, the different toll function produces the different traffic flow pattern as well. Therefore, the variation of OD matrix as well as variation of distance based toll function should be considered for preparing the training database of deep learning.

In terms of OD matrix, it is assumed that the magnitude of traffic is different among the locations such as central area, urban area and suburban area. Therefore, the magnitude of traffic in each location is determined with random distribution to the original OD matrix determined in Table 1. The mechanism of OD matrix production illustrated in Fig. 8. The uniform random number as multiplier of the original OD matrix is generated with the range between 0.8 to 1.2.

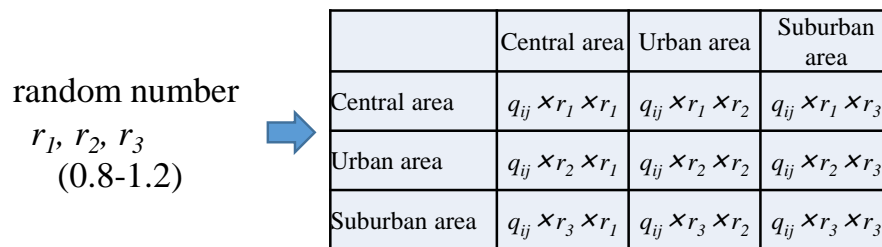


Fig. 8. The variations of OD matrix.

On the other hand, the different definition of distance based toll function is provided as well. The toll function is determined with four parameters such as terminal charge, toll rate, upper bound and lower bound. These values are determined with the combination of each value of parameter. Table 3 summarizes the distribution of parameters. Each value of parameter is determined with the particular range as shown in the table.

Table 3. The parameters for urban expressway tolls.

	Range	Number of cases
Terminal charge	200 – 400 yen	201
Toll rate	25 – 35 yen/km	11
Lower bound	200 – 400 yen	201
Upper bound	1100 – 1500 yen	401

Therefore, number of cases corresponds to the number of possible definitions of parameter. It means that all combinations of the toll parameters are counted as 178,208,811 ( $201 \times 11 \times 201 \times 401$ ).

### 3.3. The calibration of CNN model

According to the database installation, the training database is created for 10,000 samples. Therefore, the dataset consists of OD matrix and parameters for distance based toll function are determined as input variables. At the same time, the values of four evaluation indicators for social surplus are prepared as output variables which are estimated through the traffic assignment process. Furthermore, another 1,000 samples are created as house hold database for validation of the CNN model after the deep learning process.

The learning process of the CNN is performed with 500 times iterations. Back propagation technique is introduced to reduce the error to the training data. Fig. 9 shows the convergence of connecting weights for CNN with the error of output layers. The values of four variables in output layer are normalized between zero and one for learning.

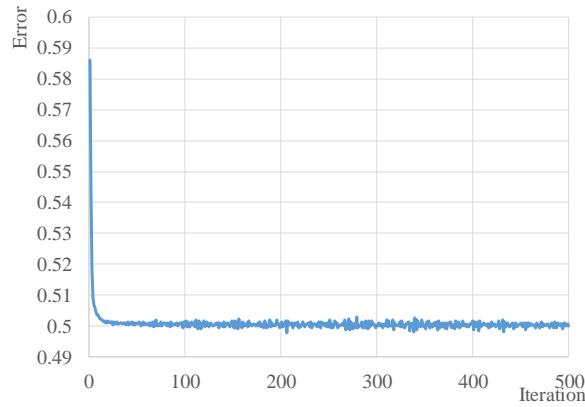


Fig. 9. The convergence of deep learning.

The rapid convergence can be observed in the figure. The value of error becomes stable with very small vibration after 100 times iterations. The convergence of CNN model can be confirmed after the iterations.

The CNN model proposed here is regarded as the estimation model for social surplus indicators corresponding to the distance based toll definition of urban expressway. Therefore, the estimation error for the indicators would be discussed for calibration of the model.

Table 4 summarizes the statistics for the estimation of values of indicators after CNN learning with 10,000 samples.

Table 4. The summary of deep learning results (learning data).

	r	RMSE
Total travel time	0.989	61,281 veh. hour
Total travel time of urban expressway	0.978	33,248 veh. hour
Toll revenue	0.993	14,984,680 Yen
Traffic volume of urban expressway	0.992	23,973 veh.

Each value of indicator is estimated with rather small value of RMSE with high coloration. It is confirmed the value of estimation of indicators for social surplus is sufficiently applied to practical evaluations.

### 3.4. The estimation results of CNN model

It is required that the estimation model with CNN can be applied to the other field. Therefore, the CNN model is applied to the house hold data for validation. The 1,000 sample data have been installed for validation as mentioned previously. The estimation process of the CNN model is performed with the connecting weights determined from the previous deep learning process.

Table 5 summarizes the statistics for estimation of validation data.

Table 5. The summary of deep learning results (validation data).

	r	RMSE
Total travel time	0.989	58,787 veh. hour
Total travel time of urban expressway	0.980	30,277 veh. hour
Toll revenue	0.994	14,186,756 Yen
Traffic volume of urban expressway	0.993	22,818 veh.

The value of RMSE in the table is almost equivalent to the value of calibration as shown in Table 4. It corresponds to the applicability of the CNN model to the estimation of social surplus indicators for urban networks. It means that the CNN model can be applied to the evaluation of distance based toll practically instead of evaluation of traffic assignment.

The alternative estimation model of traffic assignment is created with the CNN approach. However, the CNN model cannot provide the detail estimation values for urban networks. Therefore, the accuracy of approximation of evaluation indicators are confirmed. The values of social surplus indicators are basically estimated from the traffic assignment process. On the other hand, the CNN model is created to directly estimate values of social surplus indicators without traffic assignment process.

Fig. 10 shows the relation between the values of CNN estimation and the values of normal estimation with traffic assignment process for all social surplus indicators.

In the case of indicator as total travel time of urban expressway, the errors distribute rather widely in the figure. On the other hand, the values of other indicators are estimated rather correctly. Therefore, the accuracy of approximation for the CNN model is confirmed as well from the observations.

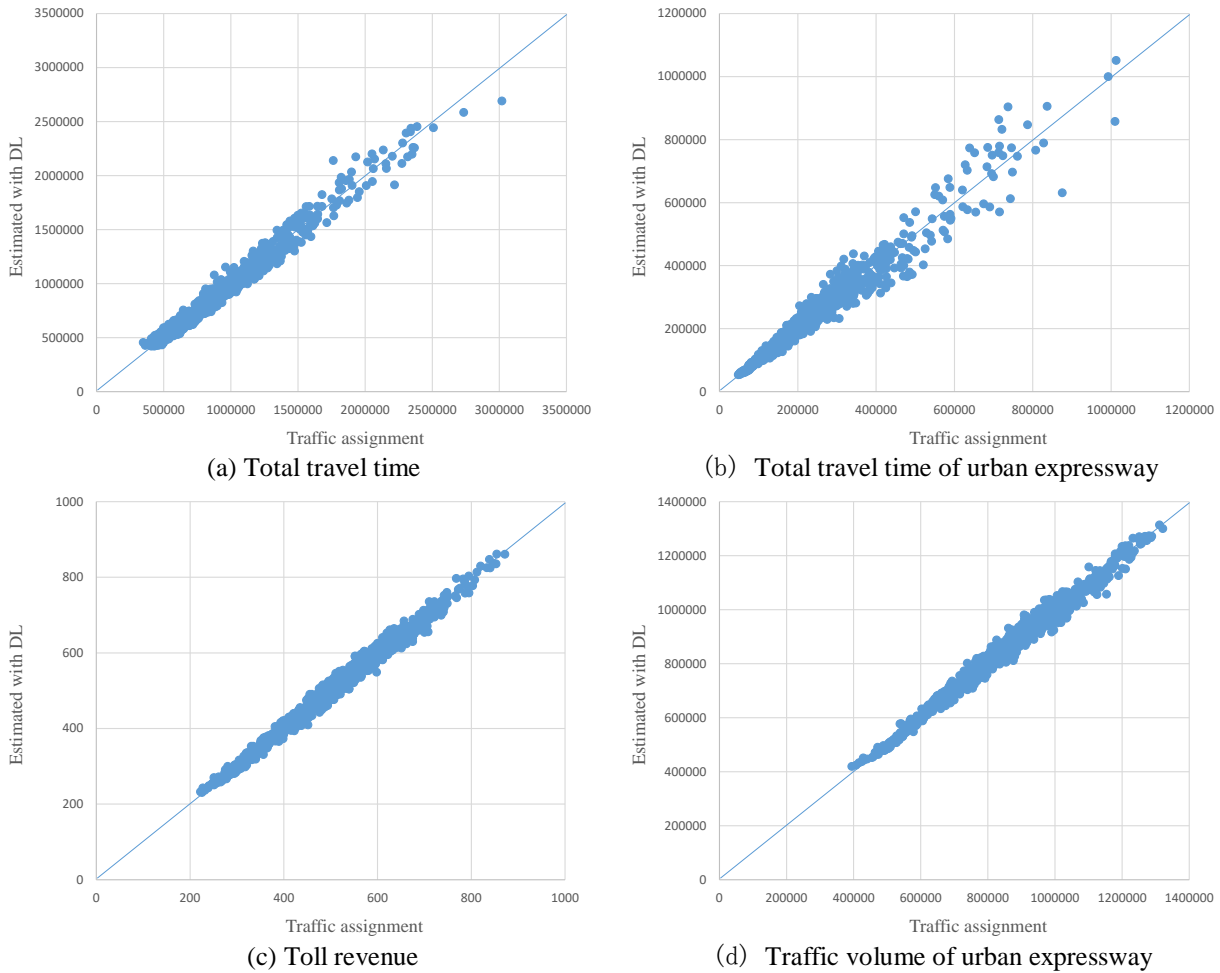


Fig. 10. Relationship between variable of traffic assignment and estimated variable.

#### 4. The combinatorial optimization for distance based toll

The estimation model of social surplus indicators with the CNN approach has been developed in the previous chapter. The model has the advantage to directly estimate the values of social surplus indicators without traffic assignment. Therefore, the optimum toll function can be easily determined through the algorithm in Fig. 5

##### 4.1. The outline of numerical example

The combinatorial optimization problem for distance based toll function is formulated in the study. The iteration of optimization is performed according to the algorithm in Fig. 5. In the study, the traffic assignment process is replaced by the CNN estimation model. The reduction of calculation time for traffic assignment seems to improve the ability of combinatorial optimization.

The numerical example is provided with assuming the different OD matrix. It is assumed that the suburbanization is proceeded. The relative ratio to the original OD matrix are determined as central area: 0.8, urban area: 1.1, and suburban area: 1.2 respectively. Therefore, the updated OD matrix is indicated in Table 6. The total traffic volume of OD matrix is increased 23 % compared to the original OD matrix.

Table 6. The origin destination matrix in suburbanization condition.

O \ D	Central Area			Urban Area			Suburban Area			
	1	2	3	4	5	6	7	8	9	
Central Area	1	-	10368	10368	10368	12672	6336	7920	11088	
	2	10368	-	15552	15552	19008	9504	11880	16632	
Urban Area	3	10368	15552	-	15552	19008	9504	11880	16632	
	4	10368	15552	15552	-	19008	9504	11880	16632	
Suburban Area	5	10368	15552	15552	-	19008	9504	11880	16632	
	6	12672	19008	19008	19008	-	13068	15246	19602	
Suburban Area	7	6336	9504	9504	9504	13068	-	6534	10890	
	8	7920	11880	11880	11880	15246	6534	-	13068	
9	11088	16632	16632	16632	19602	10890	13068	-		



#### 4.2. The combinatorial optimization problems

The combinatorial optimization for distance based toll function is performed with following the algorithm in Fig.5. Furthermore, the Monte Carlo method is introduced in the iteration of optimization. Therefore, 1,000 combinations are chosen among 178 million available combinations of toll parameters in original problem with uniform random distribution. The combination of the parameters of distance based toll is updated with referring to the value of total travel time for urban road networks. After 1,000 times iteration the optimal solution for the parameters are obtained.

As the CNN model is regarded as the approximation model of social surplus indicators, the optimal solution obtained here is similar to the real optimal solution. The optimal solution for distance based toll as (terminal charge, toll rate, lower bound, upper bound)=(350, 33, 327, 1243)

Fig. 11 shows the original distance based toll function and the optimum toll function determined from the proposed method.

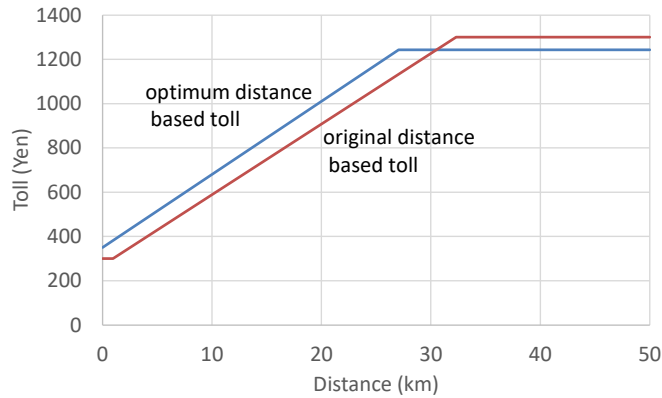


Fig. 11. The original distance based toll function and the optimum toll function.

The optimum toll function is similar to the original distance based toll function. The optimum toll function is slightly expensive compared with the original distance based toll. However, the upper bound of the optimum toll function is slightly cheap.

#### 4.3. The summary of application results

The optimal toll function is determined through the proposed method in the previous section.

Table 8 shows the values of social surplus indicators calculated by traffic assignment process with the original toll function. In the table, the values from CNN model with optimal toll function as well as the values from traffic assignment process with optimal toll function are shown.

Table 8. The summary of calculations.

	Traffic assignment (current toll)	Deep learning (optimal toll)	Traffic assignment (optimal toll)
Total travel time	1,737,324	1,694,956	1,675,382
Total travel time of urban expressway	580,736	566,457	568,530
Toll revenue	695,066,081	743,070,384	757,281,250
Traffic volume of urban expressway	1,159,774	1,094,792	1,105,358

It is known that the CNN estimation gives very closed estimation of traffic assignment. The optimum toll function provides the reduction of the total travel time by 3.6 % to the original toll function. On the contrary, the toll revenue of urban expressway is larger than the original case. Furthermore, the traffic volume of urban expressway is reduced slightly. This may corresponds to the higher value of toll.

Fig. 12 shows the link traffic volume estimated by traffic assignment with optimum toll function. Traffic volume exceeding 200 thousand vehicles is observed on the loop road. This observation corresponds to the increase of trip generation in OD matrix. Even though the combinatorial optimization is performed with the CNN model, the traffic condition of urban road network can be estimated by traffic assignment.

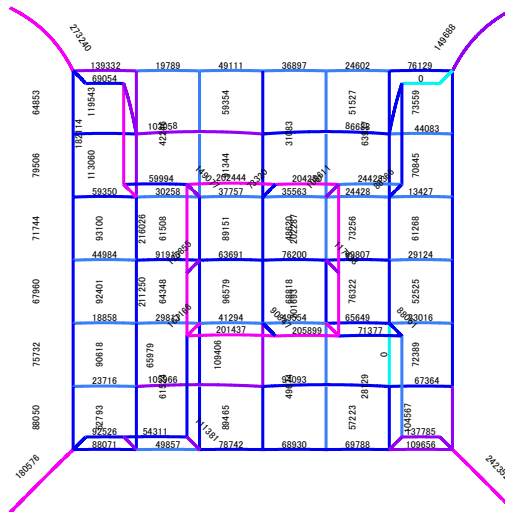


Fig. 12. Traffic volume of each road section of optimum toll case.

**5. Concluding Remarks**

The determination of optimal toll of urban expressway would be discussed in terms of effective traffic for urban transport network. Therefore, the combinatorial optimization problem of the parameters for toll function of urban expressway is introduced corresponding to the minimization of total travel time of urban streets. Furthermore, the user equilibrium traffic assignment with mathematical programming should be applied to evaluate the effectiveness on the urban transport network for solving the previous problem.

It is proposed in the study that the optimization process of toll function in urban expressway can be replaced by the deep learning model. The approximation method can be proposed to determine the parameters with high accuracy. The major findings of the study are summarized as follows:

- 1) The distance based toll of urban expressway can be discussed to realize the effective traffic on urban transport networks. The toll road as urban and intercity expressways and urban streets are determined separately in Japan. The determination of optimal toll function of urban expressway corresponds to the minimizing the total travel time for overall urban streets. The formulation of combinatorial optimization problem of parameters in toll function with user equilibrium condition is mentioned. The numerical example to evaluate the effectiveness of toll charge in urban expressway is introduced with certain similarity to the real scale transport networks.
- 2) The grate effort of traffic assignment should be required to evaluate the traffic flow on the transport network. The combinatorial optimization for the determination of toll function is formulated with the traffic assignment process. In the study, the deep learning model is proposed to replace the traffic assignment process to the approximate estimation process. The formulation of convolutional neural network (CNN) can be designed with input variables such as origin-destination matrix and output variables such as four indices for evaluation of traffic condition on the urban network. According to the basic structure of CNN with convolution layers and pooling layers, the estimation model is created. The sufficient performance of estimation is provided in terms of approximation method of traffic assignment results. After the validation of CNN model, the combinatorial optimization method for toll function determination can be established.
- 3) Since CNN model is created as deep learning result of traffic assignment, the combinatorial optimization problem can be solved for the numerical example with predicted OD matrix. The optimal parameters for toll function of urban expressway is determined though the CNN evaluation process. The total travel time of urban transport network can be minimized according to the determined parameters. Even if the approximation of CNN model is provided, the result of calculation illustrates the similar traffic condition given by the user equilibrium traffic assignment.

Following topics are recommended for further studies: (1) The proposed CNN model as deep learning approach might have much advantages for application of real scale network. (2) The combinatorial optimization problem might be solved by meta-heuristic approach. Swam intelligence such as ant colony optimization (ACO) or Genetic algorithm (GA) would be tried to apply the same problem.

**References**

Akiyama, T., Mun, S., Okushima, M., 2004. Second-Best Congestion Pricing in Urban Space: Cordon Pricing and Its Alternatives, *The Review of Network Economics*, Vol. 3, Issue 4, pp. 401-414.

Akiyama, T., Okushima, M., Inokuchi, H., 2011. Empirical Implementation of Distance based Toll for Urban Expressway, *Proceedings of the 1st Conference of Transportation Research Group of India*, pp. 1-12.

Akiyama, T., Inokuchi, H., 2013. Analysis of the model about Distance based Toll setting of the Urban Expressway, *The Japanese journal of Traffic Engineers*, pp.279-282.

- Akiyama, T., Inokuchi, H., Okushima, M., 2013. Practical Management of Distance Based Toll System for Urban Expressway, World Conference on Transport Research 2013, No. 3226.
- Akiyama, T., Inokuchi, H., 2014. Long Term Estimation of Traffic Demand on Urban Expressway by Neural Networks, 7th International Conference on Soft Computing and Intelligent Systems, No. 385.
- Akiyama, T., Inokuchi, H., Okushima, M., 2014. The Installation of Toll System on Urban Expressway Considering Diversion Traffic, The Japanese journal of transportation economics, No. 57, pp.97-104.
- Akiyama, T., Inokuchi, H., Okushima, M., 2017. Practical Management of Distance Based Toll System for Urban Expressway, Journal of Traffic and Transportation Engineering, Vol. 5, No. 2, pp. 93-103.
- Du, S., Li, T., Gong, X., et al. 2018. Traffic flow forecasting based on hybrid deep learning framework, International Conference on Intelligent Systems and Knowledge Engineering, IEEE, 1-6.
- Duan, Y., Lv, Y., Liu, Y. L., et al. 2016. An efficient realization of deep learning for traffic data imputation, Transportation Research Part C, 72, 168-181.
- Goodfellow, I., Bengio, Y., 2016. A Deep Learning, The MIT Press.
- Hou, Y., Edara, Y., 2018. Network Scale Travel Time Prediction using Deep Learning, Transportation Research Record: Journal of the Transportation Research Board.
- Inokuchi, H., Akiyama, T., 2014. Distance based Toll setting of the Urban Expressway by the Stochastic User Equilibrium with Variable Demand model, Journal of JSCE D3, Vol. 70, No. 5, pp. 1119-1125.
- Inokuchi, H., Akiyama, T., 2018. Estimation of Self-perceived Health for Welfare of Senior Citizens with Deep Learning, Proceedings of Fuzzy System Symposium, WE2-4.
- Lv, Y., Duan, Y., Kang, W., Li, Z., Wang, F. Y., 2015. Traffic flow prediction with big data: a deep learning approach, IEEE Trans. Intell. Transport. Syst., 16 (2), pp. 865-873.
- Ma, X., Dai, Z., He, Z., 2017. Learning Traffic as Images: A Deep Convolutional Neural Network for Large-Scale Transportation Network Speed Prediction, Sensors, 17(4).
- Ma, X., Yu, H., Wang, Y., et al. 2015. Large-Scale Transportation Network Congestion Evolution Prediction Using Deep Learning Theory, Plos One, 10(3), 0119044.
- Mun, S., Akiyama, T., Okushima, M., 2007. Second-best Congestion Pricing in Road Network: Cordon Pricing and Existing Toll-Roads, Journal of Applied Regional Science, No. 12, pp. 15-25.
- Polson, N. G., Sokolov, V. O., 2017. Deep learning for short-term traffic flow prediction, Transportation Research Part C Emerging Technologies, 79, 1-17.
- Patterson, J., Gibson, A., 2017. Deep Learning: A Practitioner's Approach, O'Reilly Media.
- Wu, Y., Tan, H., Qin, L., et al. 2018. A hybrid deep learning based traffic flow prediction method and its understanding, Transportation Research Part C Emerging Technologies, 90, 166-180.