PUBLIC ECONOMICS FOR INFRASTRUCTURES IN PPP’S

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ABSTRACT

(topics: E3, E4, E5)

Proposed for SIG4

During the twentieth century, the trend that seemed to be emerging in many countries was towards a certain distribution of roles whereby transport operations were assigned to the private sphere and infrastructure to the public sphere. Over the past twenty years, however, growth in the use of PPPs for new infrastructures has signalled a significant change which completely redefines the issues of public economics in the field of transport policy.

This communication is based on a research programme of LET, running over several years on public economics of PPPs. It examines to what extent it is necessary to change the way that government uses socio-economic and financial analysis to solve three of the main issues of transport policy:

1) Regulation and casting between public sphere and private operators:
   Because the private operator’s charges include the remuneration of his own capital and therefore allow him to make a profit, the choice of a PPP needs more subsidies if we assume that the Internal Rate of Return (IRR) for the project will be the same for both a public and a private operator. The PPP option is only justified when this assumption is not relevant and under specific conditions.

   It would be fair to assume that private operators are capable of improving the IRR of the operation, either though better control of operating costs, lower investment costs, short construction lead times or a combination of these profitability factors. It is necessary to formalise the effect of this improvement on the subsidy rate in order to determine the specific conditions under whose the private issue is the best one.

2) Assessment and Investments planning:
   We begin by showing that when projects are financed by both users (toll revenues) and taxpayers (subsidies), it is socially beneficial to plan these investments on the basis of the net present value (NPV) provided by each unit of public money invested. The formal demonstration provides a definition of the concept of public-funding scarcity coefficient. The NPV/subsidy ratio must obviously be higher than the public-
funding scarcity coefficient or else the investment would destroy more wealth than it would produce.

3) Financing mechanisms and pricing:
One of the ways of improving this ratio is also to optimise the toll level, since increasing it lowers the subsidy but has an adverse impact on the user surplus, it is essential to set the optimal toll and thus the optimal financing mechanism. This optimisation makes clear the role of public-funding scarcity coefficient.

Key words: PPP, programming, financing, pricing.

1. INTRODUCTION

During the twentieth century, the trend that seemed to be emerging in many countries was towards a certain distribution of roles whereby transport operations were assigned to the private sphere and infrastructure to the public sphere. From the nineties, however, growth in the use of PPPs has signalled a significant change which completely redefines the issue of infrastructure funding.

This new trend has been systematically tracked in developing countries and transition economies by the World Bank. The Bank (2009) reported 1147 projects in the transport sector of 81 countries involving private operators from 1990 to 2008. These investments in transport infrastructure nonetheless amounted to 232 billion dollars. This trend is also apparent in the developed economies, although initiatives in this area remain still limited for transportation.

These trends obviously reflect economic rationales which, although they may sometimes prove controversial, can draw on an extensive literature. Among the reasons that may favour this private sector involvement, this paper addresses only one, namely the scarcity of public funds, either for the public sector operator taking on the project or the nation as a whole.

The vigorous development of various forms of public-private partnerships in the field of public infrastructure investment and operation renews interest in theoretical thinking about what had been considered as methodological givens in the field of the public economy. By definition, a PPP system must combine the rationales of government and of a private operator. The latter’s objective function is the profit of the operation, a profit that is obviously discounted and if appropriate enriched by taking uncertainty into account. The government’s objective function is the discounted variation in social welfare that takes into account, in addition to the operator’s profit, factors such as public spending, user surpluses and environmental impact. Many factors can affect both these objective
functions differently, for instance infrastructure pricing, which is in principle not the same depending on whether it optimises the operator’s profits or social welfare.

The questions raised by these renewed arrangements between the public sphere and private partners have mainly concerned one of the most fertile fields of recent decades, i.e. the theory of incentives. The studies in this field have focused most especially on the specific nature of a partnership contract and thus the principal/agent relationship, following the groundbreaking work of Jean-Jacques Lafont and Jean Tirole (1993). Many particularly useful articles on PPPs, including the most recent ones, have continued work within this theoretical tradition (such as Hart, 2003 or Maskin & Tirole, 2008). Despite the importance of the scientific production on this field, little work has been done on the contributions and changes to the public economy implied by this new development.

The purpose of this communication is to show that all the standard tools of public economics need to be reconsidered when PPPs are implemented. To that effect we will consider five levers among the government’s controls over a transport system.

Figure 1 represents in the simplest possible terms the functioning of a transport market (passenger or freight), such as a continental (or regional or urban) transport market. At the heart of the market mechanisms is the supply of transport, which determines the mobility levels of passengers and freight provided by each mode. For centuries, the two main factors of intermodal competition have been prices and speed – the latter needing to be considered broadly, encompassing, for example, frequency or reliability.

For each mode of transport, the prices and speeds on offer are clearly dependent on the quality of infrastructure networks and how they are operated. Here we see the role of government’s controls over the system – controls shown above in grey-shaded boxes. The first two means of control, which affect the relative prices of competing modes, are **financing methods** (for infrastructure and operations) and charging for the use of infrastructure. Another means of control that also affects relative prices, but speed as well, is **regulation** – a term that we use very broadly to encompass labour and safety matters as well as the general organisation of transport markets, including those for infrastructure supply and demand, in which the degree of both regulation and competition can vary.

The traffic levels resulting from this competition, reflecting the intensity of use of the transport networks, will depend on the efficiency of the corresponding modes. In sum, it is this relative efficiency that will determine the needs for new infrastructure and, in particular, a project’s
socio-economic and financial returns. As a result, the diagram shows as means of control the evaluation of investments and the investment decision itself, which should be used consistently with evaluation. It is these investment choices that over time will shape the development of competing networks, thus “closing the loop” of the system.

In this simplified diagram, transport is shaped fundamentally by market mechanisms, but mechanisms that remain in the hands of government, which exercises the five major means of control cited above. Yet if this diagram is to have any relevance, it is clear that one must factor in all of the means of control and ascertain whether there is a good strategic fit between them, in terms of coherent PPP’s implementation, insofar as they are exercised jointly by public authorities. Each means of control must therefore be examined from two standpoints: how they must be used in order to optimise the system; and how they interact with the other means of control comprising the system.

Thus we will consider successively the consequences of a PPP device on these five means on control. The first issue we lay out in section 2 concerns the preliminary regulation problem: either to implement a PPP contract or use the usual public device. The aim, in short, is to determine

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**Figure 1 : Government’s controls over a transport system**

Thus we will consider successively the consequences of a PPP device on these five means on control. The first issue we lay out in section 2 concerns the preliminary regulation problem: either to implement a PPP contract or use the usual public device. The aim, in short, is to determine
the conditions under whose the use of a private partnership can reduce the burden on public finances compared with the use of a public enterprises whose debts are guaranteed by the State. In section 3, we study the couple of means of control “evaluation” and “investments decision”. The issue is to determine if in a PPP system there is a specific projects ranking according to the welfare purpose. Except the main result on optimal ranking, this analysis allows to make clear the public-funding scarcity coefficient. This concept plays a central role in section 4 which concerns the couple of means of control “financing” and “Charging”. The aim of this paper is to formalise the role of pricing when these projects involve a public-private partnership or, more generally, joint financing by users and taxpayers and to point out some recent results on the optimisation of the pricing subject to a public funding constraint. Section 5 conclude by checking that the five levers form a comprehensive package.

2. BURDEN ON PUBLIC FINANCES AND PPP

Once again, we consider this means of control names above “regulation” in the broad sense, i.e. encompassing all of the dimensions that govern the sector’s operations, including institutional mechanisms. The competitive orientation of these mechanisms may have greater repercussions on the funding of the transport system, particularly on the public finances.

In order to present the problem clearly, we will not consider the full diversity of situations in which private investors could be involved, but just two stylised cases opposing the “public” alternative and the “private” alternative.

2.1. The need of subsidy in two stylised cases

These cases are characterised by the following restrictive hypotheses:
- In the “public” alternative, it is assumed that the operator in charge of the project is a non-profit company which nevertheless has to achieve a balance between the project's investment and operating costs (including financial charges) by using revenues from fares (and perhaps tolls or even shadow tolls). If the project's finances are not balanced, it is assumed that the deficit will be made up by subsidies from the public authority. The level of subsidies is agreed on the base of an ex ante cost-benefit analysis and is intended to guarantee a balance between future expenses and revenues.
- In the “private” alternative, as the operator in charge of the project is a private company, the mechanism is the same except that expenses must
also include the operator's profit.
- Assessments and interest rates are inflation-adjusted.
- The financial internal rate of return (IRR) is temporally assumed to be the same for both the public and private alternatives for a given project. We know this hypothesis is not really true (Dewenter and Malatesta, 2001) but it cannot be eliminated at this first stage in our analysis.

With reference to these considerations, it is assumed that a public operator will implement a project if the expected IRR covers the market interest rate plus a risk premium which takes account of the uncertainties that necessarily affect assessments of, for example, costs and future traffic and revenue. Thus, with a market interest rate of 4% and a risk premium of 4%, the minimum IRR will be 8%. If the IRR of the project is any lower than this the public operator will require an additional subsidy in order to reach 8%.

For the same project, a private operator has to cover an interest rate which is assumed to be the same, plus approximately the same risk premium (which may also include an additional amount to cover uncertainties about the stability of the country in question) and also add a profit margin - let us say 4% more. This means that an additional subsidy will be required for any IRR below 12%.

This analysis would seem to suggest that there is a range of IRRs for which the private alternative would require larger subsidies. Under real circumstances, the challenge facing the private operator is of course to achieve a higher IRR through better project management, but as stated above we shall explore this possibility later and we need beforehand to provide a formal relationship between the need for subsidies and the level of the IRR.

2.2. Relationships between IRRs and subsidies

In order to explore the effect on the internal rate of return of subsidising the capital cost, suppose a standard case (Bonnafous, 2002) in which the capital cost C is incurred at an annual rate \( c = \frac{C}{d} \) between the dates \(-d\) and 0. When the project comes into use at time 0, the annual profit rate (revenues less operating and other ongoing costs) takes the form \((a + bt)\).

We now introduce this further notation:

\[ \alpha \]

is a discount rate which may be used to calculate the Net Present Value (NPV),

\[ \alpha_0 \]

is that value of \( \alpha \) which makes the NPV (of the unsubsidised project) equal to zero – in other words, \( \alpha_0 \) is the internal rate of return.
is the subsidy rate, expressed as a percentage of \( c \).

\( \delta \) is the increase in the IRR that results from the subsidy rate \( \tau \) – that is to say, for a subsidy rate \( \tau \), the IRR becomes \( (\alpha + \delta) \). If we recognise all benefits and costs up to a terminal date \( T \), then

\[
\text{NPV} = \int_{-\infty}^{0} -c \cdot e^{-at} \, dt + \int_{0}^{T} (a + b \cdot t) \cdot e^{-at} \, dt 
\]

(1)

To simplify the calculations – with little empirical effect – now set the terminal date to infinity. Then (1) becomes

\[
\text{NPV} = \frac{1}{\alpha} \left[ c(1 - e^{\alpha t}) + a + \frac{b}{\alpha} \right] 
\]

(2)

The IRR \( \alpha_0 \) of the unsubsidised project is therefore given by:

\[
c(1 - e^{\alpha_0 t}) + a + \frac{b}{\alpha_0} = 0 
\]

(3)

Note that for this unsubsidised project, \( \alpha_0 \) is an implicit function of the four variables: \( a, b, c \) and \( d \). When the subsidy \( \tau \) is applied, equation (3) becomes

\[
(1 - \tau)c(1 - e^{(\alpha_0 + \delta)t}) + a + \frac{b}{\alpha_0 + \delta} = 0 
\]

(4)

If we think of the situation as one in which we want to find the subsidy rate \( \tau \) that yields a specified IRR of \( (\alpha_0 + \delta) \), then (4) may be written as

\[
\tau = 1 - \frac{a(\alpha_0 + \delta) + b}{c(\alpha_0 + \delta)(e^{(\alpha_0 + \delta)t} - 1)} 
\]

(5)

Here, \( \tau \) is expressed as a function of six variables. However, these are not all independent, because \( \alpha_0 \) depends on \( a, b, c \) and \( d \), according to equation (4).

What is of prime importance to us in this function is clearly the relationship between \( \tau \) et \( \delta \). However, equation (5) also shows that this relationship obviously depends on the values of the parameters \( c, d, a, b \) and, of course, \( \alpha_0 \), which characterize the economics of the project and which are moreover linked together by equation (4) which established the IRR of the project \( \alpha_0 \). If we wish to represent equation 5 we therefore need to keep some of these 5 parameters constant and vary just those whose role we wish to demonstrate. This is the well-known nomogram technique.
illustrated by the family of curves shown in Figure 2. It is drawn for the numerical case given by $c = 100$, $b = 1$, $d = 5$ together with alternative values of $a$ chosen so that as $a$ increases from one value to the next, $\alpha_s$ increases by 0.4 percentage points. Each curve corresponds to a particular value for $a$, and shows how the required subsidy $\tau$ increases as we increase the IRR from $\alpha_0$ to the target IRR, $(\alpha_0 + \delta)$.

It is quite natural for the need for subsidy to be an increasing function of the additional IRR which the operator must receive. However, the gradient of the curve decreases in a marked manner. This concavity has been demonstrated in a previous paper (Bonnafous & Jensen, 2004) is a counter-intuitive result: it means that the first differences between the targeted IRR and the IRR of the operation can be extremely costly.

In some cases, the choice of a private operator could be expensive for the public authorities. Thus, Figure 2 shows that a project with an initial IRR $\alpha_0$ of 8%, and which could therefore be possible for a public-sector operator without a subsidy (with the hypotheses set out in paragraph 2.1) would need a subsidy of 45% to raise its IRR to 12%.

Figure 2. The relationship between the subsidy rate and the IRR

(Nomogram based on $c = 100$, $b = 1$, $d = 5$ years and $a$ variable)
The shape of the curve, in particular its downward gradient, nevertheless has an other consequence. The larger the margin by which the targeted IRR exceeds the IRR of the project, the lower the marginal cost to the public purse of an increase in this targeted IRR: thus, in the case where the profitability of a project is 4% (shown in bold on Figure 2), increasing the targeted IRR from 8% to 12% will require the amount contributed by public finance to be increased by 13% of the cost of the project. Furthermore, an increase from 12% to 14% would require an additional subsidy of only 3%.

We therefore arrive at the following surprising paradox: the additional cost for public authorities who use a private operator is less for projects whose intrinsic profitability is lower.

This finding ties up with the observation, which is also paradoxical, that private company involvement in the development of major transport infrastructure is increasing at a time when the projects that remain to be constructed are considerably less profitable than those that are already in service (at least in European countries). The theoretical paradox does not, of course, explain the empirical paradox, as each experience of privatising public facilities takes place within a specific historical context. There is obviously a difference between the historical context of major sub-Saharan railway lines in Africa and that of a Californian toll motorway. Nevertheless, the theoretical paradox explains why the shift towards privatisation should result in fewer financing difficulties for the public authorities than might be suggested by too summary an analysis.

It obviously remains for us to add to these considerations the dimension to the public-private partnership issue that we have until now avoided, namely, the respective efficiency of public and private enterprises.

2.3. When the private company is more efficient

It would be fair to assume that private operators are capable of improving the internal rate of return of the operation, either though better control of operating costs [improvement of a and b in equation (3) which determines \( \alpha_s \)], lower investment costs (lowering of c), short construction lead times (reduction of d) or a combination of these profitability factors. By way of a simple illustration, we shall assume that the initial IRR \( \alpha_0 \) is thereby improved by 2 per cent.

As we vary the value of the IRR \( \alpha_s \), we obtain the subsidy rates that are shown on Figure 3 below, with the hypothesis described above, namely that the “target IRR” is 8% for a public sector operator and 12% for a private sector operator.
We have therefore used a set of parameters which is more specific still than that which provides the basis for the nomogram in Figure 2. We have done this by setting thresholds for the target IRRs thereby formalising, in an admittedly crude manner, the effect of efficiency. Nevertheless, the plots are merely the outcome of the concave nature of the subsidy rate function.

**Figure 3. Subsidy rate as a function of initial IRR on the assumption that the private operator is more efficient**

(Target IRR of 8% for the public operator and 12% for the private operator, Initial IRR with public operator = \( \alpha_s \), Initial IRR with private operator = \( \alpha_s + 2 \% \))

This graph shows, for the set of parameters in question, that we can identify three distinct zones of IRR values. These relate to three fairly well-contrasted choice situations:
1) On the right hand side of the graph, where the rates of return are of the same order or higher than those targeted by public sector operators, public sector finance must lose as the result of the use of a private sector operator. When the loss is limited, such use may nevertheless be justified on the grounds of the overall increase in productivity that affects the economy as a whole as a result of the difference in efficiency.
2) On the left hand side of the graph, where rates of return are very low, the effect of the difference in efficiency is considerable, but we are not far removed from the situation in which the scheme may have an insufficient social return, casting doubt on the project’s validity, at least in the form in
question. In the case of motorways, for example, it may be wiser to abandon the idea of constructing a toll motorway in favour of a four-lane dual carriageway which has less demanding and less expensive characteristics (if only because it is possible to use some or all of the existing route). However, if its construction is justified on the grounds of socio-economic profitability it will be less costly for the public purse to let a private operator run it.

3) There is a point of transition between these zones at a certain value of \( \alpha_0 \) below which the use of a private sector operator reduces public expenditure (for the reader’s information this point is located at an \( \alpha_0 \) value of 5.2% for the case we have simulated). In this case the criterion of social return dictates the best choice for society.

We must make it plain that the existence of this transition is not an inevitable consequence of the concavity of the need for subsidy function: there are obviously some values for the parameters for which the function is higher (or lower) at all points for a private sector operator. What we call the paradox of financial profitability only means that, when there is a point of transition for a subset of the possible values of \( a, b, c \) and \( d \) and of the target IRRs of the two types of operator, the interest of a PPP is higher when the financial IRR is lower and vice versa. In any way, it is clear that the decision to choose either the public issue or a PPP requires for each project a specific appraisal in order to compare the needs of subsidies.

The concavity of the need for subsidy function has more an important consequence when it is a question, either of a single project but a program of several projects. Indeed, projects with low profitability require a rate of subsidy which increases very rapidly with the IRR it is necessary to provide the operator, even a public sector operator. This means that under a given budgetary constraint the order in which projects are constructed which gives priority to the most profitable projects could significantly strengthens the leverage effect of unit subsidy and therefore the rate at which infrastructure will be constructed. The value of this study is not that it confirms this elementary recommendation for public economics but that it demonstrates that, because the concavity of the subsidy function, the effect of any failure to make public investment despite low profitability is far greater than straightforward economic common sense would suggest. This issue will overhang the next section.

### 3. EVALUATION AND PROGRAMMING OF THE INVESTMENTS

In the programming problem, the subsidy rate obviously plays a major role in formulating the budget constraint. For each project, this rate is simultaneously a function of the financial rate of return that an investor
may require, of the project’s intrinsic rate of return and, thus, of its economic characteristics. This function was clarified in the previous section and plays a major role in the problem of the optimal programming.

3.1. The optimal ranking of potential projects

In the specific case of France, an event renewed the problem of evaluation, and was triggered by the conclusions of the Working Group of the French Planning Authority chaired by Daniel Lebègue (Commissariat Général du Plan, 2005). Its mission was to think about the relevant value of what is conventionally known in France as “the discount rate of the Plan”, which had been set at 8% for some twenty years. In addition to the strong theoretical reasons supporting a lower rate (Gollier, 2002), the fact of the matter is that this 8% rate was ill suited to taking environmental externalities into account effectively in the economic calculation, since it resulted in giving a very low weighting to the distant future. For example, a value considered over a 30 year time horizon is virtually divided by 10 if it is discounted at an 8% rate. It is only divided by roughly three with a discount rate reduced to 4%, i.e. the rate that was recommended by the Lebègue Report¹ and that was used in official instructions (Ministry for the Environment, 2005). In this way, France aligned itself more closely with the rates used in EU countries, such as 3% in Germany.

However, this recommendation had the effect of multiplying the number of new projects considered to be cost-effective, since their socio-economic net present value (NPV) now became positive if their socio-economic internal rate of return (ERR) fell between 4% and the former rate of 8%. It also generated a growing number of “candidate” projects, i.e. for which the optimal implementation date had already passed since their immediate rate of return was higher than the official discount rate. This made it more urgent to rank the potential projects and programme them in an order that would maximise the welfare function. This optimisation not only implies the order of implementation of projects, but also the subsidies that each of them may require and, therefore, the constraint in terms of available public funding.

Before this adjustment of this discount rate, as an initial approximation, the socio-economic rate of return (ERR) was used to rank the potential projects, i.e. those whose net present value (NPV) was positive and whose optimal date of implementation had passed. When a project designated as having priority (because its ERR was very high) had a

¹. More specifically, the report recommends a decrease of the (real) discount rate to 4% and even a gradual lowering of the rate to 2% for time horizons longer than 30 years (Commissariat Général du Plan, 2005).
financial internal rate of return (IRR) that was insufficient to ensure its self-financing, additional funding was required, which might be a subsidy, as in the case of the TGV high-speed trains built after the South-East TGV line, or a disguised subsidy, as in the so-called system of “adossement” long used for toll motorways in France. This consisted of commissioning the franchisee of a motorway network to construct and operate a complementary segment that was financed partly by the cash-flows from older segments and that included, if necessary, a lengthening of their franchise.

This practice disappeared, around the turn of the last century, as it was incompatible with European legislation, but also because of the growing number of projects that do not have a sufficient IRR to be able to finance them without subsidies. All this has enhanced the rationale of joint financing by taxpayers and users and, consequently, public-private partnership in the broad sense. However, this posed the problem of the optimal programming of investments in new terms that were not immediately recognised. This problem can be stated very simply: it consists of determining, among the candidate projects, those that will be selected and their optimal implementation date so that, subject to the available public budget constraint, the net present value (NPV) of the programme thus established will be maximised.

The use of the subsidy rate function stated in section 2 has facilitated several exercises, which consist of comparing various investment programmes subject to the same public funding constraint. This exercise was conducted (Bonnafous and Jensen, 2005) for a same set of 17 candidate projects of toll motorways being considered in the early 1990s. It dealt with these 17 projects which can give rise to 17! possible permutations (roughly $10^{34}$), which requires one of the combinatorial exploration algorithms (such as the simulated annealing method) used by the experimental sciences in dealing with such complexity. The programmes explored saturate, of course, the public funding constraint. The first paradoxical result is that the overall socio-economic NPV of the programme is greater with the decreasing order of IRRs in relation to the decreasing order of ERRs: the financial ranking criterion brings back higher socio-economic gains than the socio-economic ranking criterion. The explanation for this is simple: the projects ranked according to the criterion of decreasing IRRs are implemented at a faster pace, as their strong financial rate of return implies lower subsidies. For an equivalent public expenditure, the output of socio-economic NPV is therefore higher.
These simulations have also revealed that the tighter the budget constraint, the greater the programme’s gain in overall socio-economic NPV with the decreasing order of IRRs in relation to the decreasing order of ERRs. However, first and foremost, they have revealed for us that, for standard observations of costs and benefits, the order of the ratio of NPV/public euro invested generates an even higher gain in social return and that it is in fact the criterion that designates the optimal programme, in the sense of the numerical optimisation algorithm.

A critic could be addressed to this approach because the demonstration is limited to a particular case. Fortunately, a formalised demonstration of the main result has been proposed (Roy, 2005), which shows, under relatively weak conditions, that the ratio of NPV/public euro invested is indeed the criterion for ranking projects that maximises overall social welfare. This demonstration needs to be stated here, for it makes it possible to define rigorously the concept of public-funding scarcity coefficient that we will need to address the optimal pricing in section 4.
3.2. Programming and public-funding scarcity coefficient

The demonstration established by William Roy is general and easy to interpret as it is formally close to the programme of the consumer for discrete goods. Let us assume that the decision-maker has to choose among \( n \) projects \( i \), characterised by their net present value \( \Delta U_i \) and their need for subsidies \( \text{Sub}_i \), with \( \Delta U_i > 0 \) and \( \text{Sub}_i > 0 \) \( \forall i = 1, \ldots, n \). The pricing is for the time being considered as a given, set by an exogenous rule. We then take, as the objective function of the community, the overall productivity surplus \( W \) generated by all the projects, subject to the budget constraint \( B \) capping public spending. The optimisation programme can then be written as follows:

\[
\begin{align*}
\text{Max } W(x) &= \sum_{i=1}^{n} x_i \Delta U_i \\
B - \sum_{i=1}^{n} x_i \text{Sub}_i &\geq 0 \\
\text{s.c. } x_i &\geq 0, \quad \forall i = 1, \ldots, n \\
1 - x_i &\geq 0, \quad \forall i = 1, \ldots, n
\end{align*}
\]  

The parameters \( x_i \) are continuous variables, which have a zero value when the project is not implemented and equal to one when the project is implemented fully. We shall assume that it is possible to implement partially project \( k \), with parameter \( x_k \) then ranging between 0 and 1. Obviously, possibility of implementing partially a project \( k \), while implying proportionally its characteristics (\( \Delta U_k \) and \( \text{Sub}_k \)), is a purely theoretical working hypothesis.

The solution vector \( x^* \) is therefore constituted by a value set 1 (projects to be implemented), a value set 0 (projects not to be implemented), and a value ranging between 0 and 1 for the “borderline” project \( k \) (which is very likely not to be completed if the constraint is totally inflexible). Assuming that the projects are ranked by their implementation priority, we can write this solution vector as:

\[
x^* = \begin{pmatrix}
1, & 1, & \ldots, & 1, & x_k, & 0, & \ldots, & 0
\end{pmatrix}
\]

The Lagrangian of the optimisation problem is written as follows:

\[
L(x_1, \ldots, x_n, \varphi, \alpha_1, \ldots, \alpha_n, \beta_1, \ldots, \beta_n) = \sum_{i=1}^{n} x_i \Delta U_i + \varphi \left( B - \sum_{i=1}^{n} x_i \text{Sub}_i \right) + \sum_{i=1}^{n} \alpha_i x_i + \sum_{i=1}^{n} \beta_i (1 - x_i)
\]

The Kuhn and Tucker conditions imply in particular that at optimum:
- \( \Delta U_i - \varphi \cdot \text{Sub}_i + \alpha_i - \beta_i = 0 \), \( \forall i = 1, \ldots, n \)
- \( \varphi \left( B - \sum_{i=1}^{n} x_i \cdot \text{Sub}_i \right) = 0 \)
- \( \alpha_i x_i = 0 \) et \( \beta_i (1 - x_i) = 0 \), \( \forall i = 1, \ldots, n \)

The economic interpretation of this optimisation is simple: \( \varphi \) is the variation in the community surplus generated by a loosening of the public-funding availability constraint. Being equal to the maximum surplus amount that the community can hope to obtain from an additional budgetary unit, \( \varphi \) represents the opportunity cost of public funds. It is important to distinguish this opportunity cost from the shadow cost of public funds (Ponti and Zecca, 2007), which results from the costs of collecting taxes and the price distortions associated with raising taxes by an additional unit. It is therefore not by chance that we are calling \( \varphi \) a scarcity coefficient: dual value of the budget constraint, it really is the signal “price” of the scarcity of public funding.

For the projects accepted (indexed \( j \)), the Kuhn and Tucker conditions imply:
- That the constraint “\( x_i \geq 0 \)” is not saturated, and therefore \( \alpha_j = 0 \)
- That the constraint “\( 1 - x_i \geq 0 \)” is saturated, and therefore \( \beta_j > 0 \)

Whence \( \Delta U_j - \varphi \cdot \text{Sub}_j > 0 \Leftrightarrow \frac{\Delta U_j}{\text{Sub}_j} > \varphi \)

The set of acceptable projects is therefore composed of those having a \( \Delta U/\text{Sub} \) ratio higher than the opportunity cost of public funds \( \varphi \). For the projects rejected or postponed (indexed \( i \)), the optimisation conditions imply:
- That the constraint “\( x_i \geq 0 \)” is saturated, and therefore \( \alpha_i > 0 \)
- That the constraint “\( 1 - x_i \geq 0 \)” is not saturated, and therefore \( \beta_i = 0 \)

Whence \( \Delta U_i - \varphi \cdot \text{Sub}_i > 0 \Leftrightarrow \frac{\Delta U_i}{\text{Sub}_i} < \varphi \)

In all, the projects indexed \( j \) selected and the projects indexed \( i \) not selected confirm the fundamental relationship:

\[
\frac{\Delta U_j}{\text{Sub}_j} > \varphi > \frac{\Delta U_i}{\text{Sub}_i}
\]

(7)

The projects accepted must always have a \( \Delta U/\text{Sub} \) ratio higher than that of the projects rejected. This coincides with the result of the previous paragraph, for preference is given to the projects producing the greatest net present value per public euro invested (\( \Delta U/\text{Sub}_i \)).
By varying \( \varphi \) (loosening or tightening the budget constraint), a complete ranking of priorities can be constructed, which will be based on the criterion \( \Delta U/Sub \). Thus, to optimise the overall surplus of a programme of projects subject to a budget constraint, including if the decision-maker does not know \( \varphi \), the projects selected first must be those with the highest “social welfare per public euro invested”.

We should point out that this criterion is fully consistent with the “demonstration” by numerical simulation presented in the preceding paragraph and that it was joined into the official French Guideline “on the economic evaluation methods used for major transport infrastructure projects” (French Ministry of Public Works, 2004 & 2005).

The criterion is also consistent with the standard results of microeconomics. Its significance is directly related to consumer theory, for when a consumer’s utility optimisation is at equilibrium, he equalises all relationships between marginal utility and price. In our case, the consumer becomes the community, the “Little Father of the People” in the meaning of Jacques Lesourne (1972), in his role of purchasing public goods. This modelling, which William Roy has generalised in a dynamic configuration, i.e. of successive periods, simply bypasses the difficulty constituted by the indivisibility of the investments considered.

It also generates a concept, the public-funding scarcity coefficient, which we will need later in our discussion. Lastly, it suggests to us that the hypothesis of exogenous or predetermined pricing should be excluded, as it is clear that the \( \Delta U/Sub \) ratio is dependent on the price charged to users since this has an impact both on the variation in social welfare of a project (in the sense of its socio-economic NPV) and on the level of subsidy required.

4. FINANCING AND PRICING

Infrastructure pricing is a very old issue. An abundant literature has studied this subject, particularly in the fields of transport and energy economics, and has led to a recommended approach on which there is a near consensus. It can be summarised briefly as follows: in a short-term perspective, marginal social cost pricing leads, under ultimately predictable assumptions, to a first-order optimum. When demand is subject to a capacity constraint, the need to invest to limit congestion and its social costs generates long-term incremental and marginal costs leading to higher charges, but which are not always sufficient to cover the average costs if the fixed costs are large, which is a frequent characteristic of network activities.
This being the case, this long-run marginal social cost pricing must be modified so as to increase the revenue generated. The modification that deviates the least from the first-order optimum is the one that deforms the least the structure of demand. The result, known as Ramsey-Boiteux, then consists of increasing the charges for a demand segment in inverse proportion to the price elasticity of demand in that segment (Boiteux, 1956). This is therefore a second-order optimum in the sense that the welfare function is optimised subject to a constraint on the capacity of deficit financing by public funds.

4.1. Pricing and public-funding scarcity coefficient

We faced with the same problem which consists in considering that the first-order optimal pricing leads to an overdrawn financing. If it is applied for a new infrastructure it implies a subsidy of balance. Marcel Boiteux raised the problem of the modification of the pricing which maximises the welfare function but assures the balance in the budget. We raise the problem a little bit differently by considering that, in the welfare function, public funds are to be assigned a scarcity coefficient that we shall call \( \phi \).

The precise definition of \( \phi \), as dual value of the budget constraint, and its theoretical implications have been specified in the preceding paragraph. The issue of the relevant estimate of \( \phi \) will not be addressed here, even as an implicit scarcity coefficient of government revealed by its decisions regarding tolls (Abraham, 2008). We shall confine ourselves below to varying \( \phi \) in a range of reasonable values.

We shall also assume that the non-monetary components of the marginal social cost (including the environmental components) are covered by taxes collected by government. An example, in the field of road transport, would be the domestic tax on petroleum products and the assumption that the corresponding revenues are not allocated to the transport system beyond covering the marginal user cost. This relieves us of addressing certain highly topical issues such as the allocation of the various taxes paid by users (De Palma et alii, 2007).

Consequently, the share of financing of a motorway provided by the users can, in this analysis, only be derived from toll revenues. This working hypothesis has the result of neglecting not only the issue of environmental costs, assumed to be suitably internalised by taxes, but also the issue of congestion tolls. This latter restriction is less problematic given that in many countries today, including France, the new infrastructure franchised or partially financed by tolls rarely involves congested routes. The problem of the optimal toll is therefore reduced to a financing issue or, if one
prefers, to the sole issue of the trade-off between paying users and taxpayers.

With this assumed equilibrium between external costs and taxes, the evaluation of a project can be formulated in a very simplified way: the variation in social welfare related to the project ($\Delta U$) is then only a function of the discounted subsidy (which can be defined as the difference between the discounted investment and operating costs $C$ and the discounted revenue of the project $R$) and the discounted user surplus $S$. This function is then written as:

$$\Delta U = \phi \cdot Sub + S = \phi (R - C) + S$$

(8)

Unless otherwise indicated, we shall assume that the revenue is always lower than the costs and that there is therefore always a need for subsidies. This is the case for the vast majority of the current motorway and rail projects in most countries. As the discounting calculations are in constant prices, we shall assume that the toll $p$ remains unchanged over the discounting period and that the discounted demand $d$ can be expressed as a linear function of $p$, as follows:

$$d = d_0 - \beta \cdot p$$

(9)

The result is a discounted revenue:

$$R = d_0 \cdot p - \beta \cdot p^2$$

(10)

The discounted user surplus, for a price level $p$, is expressed as follows:

$$S = \frac{\beta}{2} \left( \frac{d_0}{\beta} - p \right)^2$$

(11)

The last three equations are shown in Figure 5 below.
Figure 5. Demand function, user surplus and revenue

![Demand function, user surplus and revenue diagram](image)

Equations (10) and (11) enable to show explicitly the social welfare variation defined by equation (8) which is a second-order polynomial function of the toll:

$$\Delta U = -\varphi.C + \frac{d_0^2}{2\beta} + (\varphi - 1).d_0.p + \beta(\frac{1}{2} - \varphi).p^2$$

(12)

The welfare gain from the project will thus be greatest for a toll $p_{U\text{max}}$ that maximises this function and cancels out its derivative:

$$p_{U\text{max}} = \frac{\varphi - 1}{2\varphi - 1} \times \frac{d_0}{\beta}$$

(13)

This toll that maximises the welfare from the project is therefore zero when $\varphi$ equals 1 and from 0 to $d_0/2\beta$ when $\varphi$ is greater than 1. Both cases are shown in Figure 6, where the function $\Delta U$ has been deliberately translated, so that the function $(\Delta U + C)$ is shown as ordinate. This has the advantage of better reflecting the weight of revenue and surplus in the composition of $\Delta U$ and reminding us that for $\varphi = 1$, any increase in the toll has an adverse impact on $\Delta U$. 

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This is the mechanism clearly identified by Jules Dupuit (1849): the decrease in surplus resulting from this increase (which he called “social loss”) is always greater than the increase in revenue. According to our hypothesis that the demand function is linear and, in the case of an identical toll for all users, the maximum revenue equal to $d_0^2/4\beta$ is only half the maximum surplus accruing to users when the toll is zero. We know that only with entirely discriminatory pricing can the full user surplus be internalised.

The optimal toll, in that it maximises the net present value of a project that has been approved, is therefore a function of the public-funding scarcity coefficient, which can readily be shown to be an increasing function that tends to $P_{R\text{max}}$ when $\varphi$ increases but remains below it. The economic interpretation of this result is fairly predictable: in the case of franchised infrastructure, it is in the public interest for the franchisor to remain in charge of tolls, since the socially optimal toll is lower than the one that maximises revenue. However, the more active the public-funding scarcity constraint, the higher it will be.

However, the problem can be viewed in markedly different terms when the rationale shifts from that of an individual project to a whole programme of projects, which must be optimised subject to budget constraint. In the

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same way as we treated in section 3 the problem of the optimal ranking of a set of projects of a program, it is a question here of determining the optimal pricing for all the projects of a program with the same objective function and the same budget constraint.

4.2. From project optimisation to an “optimal programming toll” ($p_{op}$)

Figure 4, described in paragraph 3.1, suggests that ranking investments in decreasing order of financial returns, or in optimal order, produces a relative welfare gain compared with ranking by socio-economic returns, and the more active the budgetary constraint, the greater the gain will be. This raises the question of optimal pricing, not as addressed in the previous paragraph, i.e. pricing that maximises the socio-economic NPV of a specific project, but pricing that maximises the socio-economic NPV of a whole programme of projects subject to budget constraint. It is conceivable for instance that, for a given budgetary constraint, pricing that maximises revenue provides scope to make less use of subsidy and hence implement more projects than with pricing that maximises the NPV of each project.

To express the funding constraint, we recall, the level of subsidy as defined in paragraph 4.1 is as follows:

$$Sub = C - R = C - d_0.p + \beta.p^2$$  \hspace{1cm} (14)

With an available budget B, a number of projects can be subsidised and it is then a question of working out a toll that optimises the net present value of the projects implemented subject to that constraint. This amounts to optimising the NPV per euro of public investment as shown earlier and, therefore, by using equations (12) and (14), determining the toll that maximises:

$$\frac{\Delta U}{Sub} = \frac{-\varphi.C + \frac{d_0^2}{2\beta} + (\varphi-1).d_0.p + \beta(\frac{1}{2} - \varphi).p^2}{C - d_0.p + \beta.p^2}$$  \hspace{1cm} (15)

The derivative of this function is quite a complex calculation but can be considerably simplified to give:

$$\frac{d(\Delta U/\text{Sub})}{dp} = \frac{d_0^3 - 2C.\beta.d_0 + 2(C.\beta^2 - \beta.d_0^2).p + \beta^2.d_0.p^2}{(C - d_0.p + \beta.p^2)^2}$$  \hspace{1cm} (16)

Surprisingly, this equation shows that the public-funding scarcity coefficient $\varphi$ has vanished, meaning that the optimal programming toll does not depend on $\varphi$. 

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This is because the numerator, a second order polynomial in \( p \), has two real roots. One is the value \( \frac{d_0}{\beta} \) which is a trivial root and corresponds to the level of toll at which demand vanishes (like revenue and the user surplus, as shown in Figure 3). The other is a value obviously lower than \( \frac{d_0}{\beta} \) which is the “optimal programming toll”, denoted as \( p_{\text{op}} \), which optimises the social returns to the investment programme subject to a public funding constraint:

\[
p_{\text{op}} = \frac{d_0}{\beta} \left(1 - \frac{2C}{d_0^2}\right)
\]  
(17)

To verify that this toll is lower than or equal to one that maximises revenue \( \left(\frac{d_0}{2\beta}\right) \), we can put in the maximum revenue value \( \left(\frac{d_0^2}{4\beta}\right) \) denoted as \( R_{\text{max}} \) and the corresponding subsidy value denoted as \( S_{\text{min}} \), which is therefore the minimum subsidy equal to \( C - R_{\text{max}} \). Equation (17) then becomes:

\[
p_{\text{op}} = \frac{d_0}{2\beta} \left(1 - \frac{S_{\text{min}}}{R_{\text{max}}}\right)
\]  
(18)

The most significant result produced by equation (17) is of course the independence of \( p_{\text{op}} \) in relation to the public-funding scarcity coefficient, unlike \( p_{\text{Umax}} \) in equation (13). \textit{This optimal programming toll depends only on the cost of the project and the parameters characterising the demand function.}

Interpreting this \( p_{\text{op}} \) using equations (17) and (18) provides valuable insights, some of which can be developed in section 5. Previously, it can be useful to illustrate this theoretical result by concrete values.

4.3. Orders of magnitude in a case-study

To identify some orders of magnitude in a practical case-study, we shall use reference data from the SIMCALECO model developed at the LET (Chevasson, 2007) to test all the implications for the economic calculation of trade-offs used in the official evaluation methods.

This model reconstitutes all the calculations laid down in the Guideline “on the economic evaluation methods used for major transport infrastructure projects” (French Ministry of Public Works, 2004) in the case of motorway projects. It gives the calculations for the values of some 140 parameters required in such evaluations, the infrastructure characteristics in a benchmark scenario being as follows: a 90 km stretch of motorway to extend an existing 110 km-long road which, prior to the motorway opening, carries 12,000 vehicles a day; the investment amounts to €400
million while the current operating cost is €200 million, giving a total discounted cost of €600 million (for $C$ above); the initial toll is close to the average toll charged on the network franchised in 2005 (€0.066 per vehicle/km for private cars); the traffic split between road and franchised motorway is simulated with a LOGIT model.

This information gives a relatively high ERR of 21%, but a low IRR of 5.1%, which assumes a subsidy of 31% of the total discounted cost to take the IRR up to 10% for the operator. These are the orders of magnitude we are looking for in our analysis, together with some numerical values for the parameters. We accordingly need to calibrate our demand function to obtain at best the traffic and revenues produced by the SIMCALECO model when the toll is varied. As the demand function is stylised in linear form, it can be suitably adjusted to the data in the benchmark scenario by selecting:

\[
d_0 = 10000 \text{ million vehicle/kilometres (in automobile v/km equivalents)} \quad \text{and} \quad \frac{d_0}{\beta} = 0.25 \text{ euros (kilometre toll at which there is no more traffic).}
\]

These few values suffice to draw up the theoretical figures above with orders of magnitude representing specific scenarios. Thus Figure 6 becomes Figure 7 below. The social welfare variation is shown with three values for the public-funding scarcity coefficient (1, 1.4 and 1.8). The problem here is the optimal toll (the black stars on the graph) for a single project and for the three values of $\phi$ under consideration; it is clear that the toll that optimises the social welfare variation increases with this coefficient, as established with equation (13).
Moving on to the problem of the optimal programming toll, we obtain the same results from maximising the function $\Delta U / Sub$ with Figure 8, clearly confirming that variations in $\varphi$ obviously affect the level of $\Delta U$, which is perfectly consistent with the fact that the project is making a financial loss: the higher $\varphi$ is, the heavier the loss for $\Delta U$. The figure also confirms the optimal programming analysis set out in paragraph 4.2, namely that variations in the public-funding scarcity coefficient have no impact on the level of optimal programming toll, as shown in equation (17).
Moreover, we can identify on Figure 9 the three optimal tolls mentioned above as a function of $\phi$.

Except for the horizontal line representing the toll which maximises the revenue and which is a maximum in any cases, it is important to underline that the relative positions of the curves are depending on the economic characteristics of each project. This concerns either the discounted cost $C$ or the parameters of the stylised demand function $d_0$ and $\beta$. It is indeed worth noting that, while the toll that optimises a programme’s NPV is never higher than the toll that optimises revenue, as we have shown, it may be lower than the toll that optimises the NPV of a specific project.

The main advantage of this series of graphs is that they give some orders of magnitude in an investment scenario that quite closely resembles that of an “average” motorway. For instance they show that in our case study, taking a scarcity coefficient of 1.3 as now recommended in France (Commissariat Général du Plan, 2005), the optimal project toll ($p_{U_{\text{max}}}$) should be around €0.05 per v/km. However, the optimal programming toll ($p_{\text{op}}$) should be around €0.09, whereas in practice it is below €0.07. For an unsubsidised project such as those discussed below, the $p_{\text{op}}$ should be around €0.125, i.e. far higher.
5. CONCLUSION: A STRATEGIC COHERENCE OF CONTROLS

The most significant result produced by equation (17) is of course the independence of $p_{\text{op}}$ in relation to the public-funding scarcity coefficient, unlike $p_{\text{Umax}}$ (equation 13). This optimal programming toll depends only on the cost of the project and the parameters characterising the demand function.

This result, formalised according to equation (18), can be interpreted in terms of the project's financial efficiency: when this is higher, with maximum revenue covering a large share of the costs, it is socially beneficial to close the gap between the optimal programming toll and the maximum revenue toll. If maximum revenue is higher than costs, the subsidy is zero and $p_{\text{op}}$ becomes $p_{\text{Rmax}}$

In this case, a franchise contract between government and operator does not require a specific clause on tolls, even if users need to be protected from excessive charges, because it is not in the operator's interest to charge tolls exceeding $p_{\text{Rmax}}$. On the other hand, the question does arise as to how any surplus should be shared out between franchisor and franchisee.

Conversely, when the operation's financial rate of return is low, the minimum subsidy takes on greater importance and, if it reaches the maximum revenue level, the $p_{\text{op}}$ is zero. In this case, a toll does not bring any welfare gains and the scenario is typically that of a partnership contract as defined under the law voted in United Kingdom in 1992 on Private Finance initiative, or under French law (Order 2004-559 on Partnership Contracts), such as those used for toll-free stretches of motorway. Equation (18) even suggests a specific rule: toll-free infrastructure (possibly based on a partnership contract) would be the right solution whenever the maximum discounted revenue fails to cover at least half of the discounted cost.

So there are three possible scenarios, depending on the financial efficiency of the projects under consideration:

1. **When the minimum subsidy is at least equal to the maximum revenue, the optimal programming toll is zero.** Whatever the toll, the revenue cannot cover over half of the cost. If the project is nevertheless eligible for the programme, it is because its NPV/subsidy ratio is relatively high, which may stem from various factors, such as major environmental benefits or an overweighted user surplus justified by a redistribution policy. Examples include the so-called “territorial development” motorway projects, such as the work to bring up to motorway standard France's N88 highway between Toulouse and Saint Etienne. It is worth noting in this particular case that the involvement of a private partner in the building of future stretches of the motorway is indeed being envisaged on the basis of a partnership contract.
2. When a subsidy is necessary but lower than the maximum revenue, the optimal programming toll is positive but lower than the revenue-maximising toll. It should be set by government at the level defined by equation (17).

3. When the maximum revenue is sufficient to cover costs, equation (18) shows that the optimal programming toll "hits" the revenue-maximising toll. It would not be in the interests of either government or operator (statutory agency or franchisee) to go beyond that. That would mean a system in which operators were free to set tolls at their own discretion. The question arises, however, of how profits are split between franchisor and franchisee, as soon as revenue exceeds cost (which already includes interest on the franchisee’s capital). The answer does of course affect the social welfare function if a public-funding scarcity coefficient is used.

It is clear from equation (17) what makes a project fall into one of these three categories. The first scenario covers cases with relatively low demand $d_0$, or high costs. These may include upland routes such as the N88 mentioned earlier, which runs through the Auvergne region. There may also be a high $\beta$ factor, meaning demand that is highly toll-sensitive, and roads for which the alternative route is toll-free but attractive.

The third scenario is quite the opposite, particularly when the alternative is a tolled or not very attractive route, in which case the $\beta$ factor is low. This has been observed on the Annecy-Geneva motorway franchise, where the successful bid was subsidy-free: the alternative route was either a long trip on a toll motorway, or a relatively dissuasive mountain road. The second scenario obviously falls between the other two and corresponds to many of today’s potential projects.

In all the cases it is clear that the coherence between the five means of control which we distinguished in our introduction must be based on a common objective function i.e. the welfare gain brought by every spent public Euro. Resorting to a PPP or not, evaluation and ranking of projects, financing and charging rules are nothing else than the various facets of the same problem, that of the optimal transport policy.

REFERENCES


COMMISSARIAT GENERAL DU PLAN (1996), *Transports : pour un meilleur choix des investissements*, M. Boitez (prés.), Paris : La Documentation Française.

COMMISSARIAT GENERAL DU PLAN (1997), *Transports : le prix d'une stratégie*, tome 1, A. Bonnafous (prés.), Rapporteurs D. Bureau, J.P. Puig, La Documentation Française.

COMMISSARIAT GENERAL DU PLAN (2001), *Transports : choix des investissements et coût des nuisances*, M. Boitez (prés.), Rapporteur L. Baumstark, La Documentation Française.

COMMISSARIAT GENERAL DU PLAN (2005), *Révision du taux d’actualisation des investissements publics*, D. Lebègue (prés.), Paris : La Documentation Française.


