IMPROVING TRANSIT SERVICE CONNECTIVITY: THE APPLICATION OF OPERATIONS PLANNING AND CONTROL STRATEGIES

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ABSTRACT

The continuing shift of activities from city centers to other parts of the metropolitan area is resulting in increasingly dispersed origin-destination patterns. Providing direct public transportation service for these dispersed demand patterns is very expensive if not infeasible for any transit agency. That is why most agencies rely on the willingness of their passengers to transfer between routes or services to complete many of their trips. However, transfers usually reduce the attractiveness of transit. Transfer coordination initiatives can reduce the disutility of transfers in transit networks by minimizing the expected waiting times of transferring passengers. This paper presents operations planning and control models aimed at improving connectivity on a transit corridor with multiple transfer stops.

The operations planning model involves the simultaneous application of two planning strategies: changing the terminal departure time and inserting slack time, and aims at coordinating transfers at selected transfer stops along the corridor. The operations control model determines under what conditions holding a vehicle at a transfer stop for an incoming connecting vehicle is an appropriate strategy. The results of testing both models on a hypothetical corridor are also explored, along with the sensitivity of the decision variables and model results to various exogenous factors. On the planning side, results show that there is a high threshold for introducing slack time into the schedule and that the greatest benefits from schedule coordination are attained when the variance of vehicle arrival times is small and the headway on the analysis corridor is long. On the control side, transfer demand is the driving factor behind any holding recommendation. Moreover, the greatest benefits from real-time coordination occur when the required holding time and the preceding headway of the vehicle on the destination line are short, and its following headway is long.

Keywords: transfer coordination, operations planning strategies, operations control strategies, network connectivity, schedule coordination, real-time dispatch

INTRODUCTION

The continuing shift of activities from city centers to other parts of the metropolitan area is resulting in increasingly dispersed origin-destination patterns. Providing direct transit service between all these origins and destinations is financially infeasible in any public transportation network regardless of the agency size. That is why most agencies rely on the willingness of their passengers to transfer to complete many of their trips, by connecting to other services or routes at specified transfer stops. Unfortunately, transfers have a number of attributes which make them unattractive. Among these are the physical effort associated with alighting from one vehicle and boarding a new vehicle, the additional transfer fare (if any), and the negative perception of waiting for the arrival of the destination vehicle. This paper is based on a thesis and research by Younan [2004]. It presents two computer models aimed at reducing the waiting time of transferring passengers at selected transfer stops along a transit corridor through the application of operations planning and operations control strategies.

Transfers in a Transit Network

In many large urban transit systems, 10 to 30 percent of the total daily transit trips include at least one transfer. Transfers can improve the service characteristics of any transit network by offering passengers a greater range of travel destinations, improving the transit network operational flexibility and efficiency, and concentrating passenger flows on main routes on which good service can be provided. An efficient transfer system can thus significantly improve the overall service quality offered by the transit agency, stimulating demand and increasing productivity in the network [Ting, 1997]. On the other hand, transfers also add uncertainty, discomfort, waiting time and cost to most trips, thus discouraging passengers from choosing transit.

The uncertainty associated with the transfer experience springs from the unreliability in the connecting vehicle arrival times at the transfer stop. These vehicles arrive and depart from the stop with varying levels of adherence to the schedule. This randomness in arrival time may result in transferring passengers missing their planned connection entirely and hence being forced to wait for the following vehicle on their destination line.

The transfer waiting time, the most inconvenient aspect of transfer, is therefore influenced by the reliability of the connecting routes at the transfer stop. This waiting time is also affected by the frequencies on those routes [Crockett, 2002]. When transferring between high-reliability routes, the waiting times experienced by transferring passengers are known and can be reduced substantially through the application of schedule coordination strategies. When connecting between a high-reliability and a low-reliability route, the transfer waiting times are variable, but long waiting times can be avoided by applying the appropriate strategy. Finally, for transfers between low-reliability routes, the resulting waiting times are very variable and it is often not worth attempting any coordination.

The transfer waiting time is also a function of the frequencies on the connecting routes. Connections are of particular concern when the headways on both routes are long. For such cases, the transfer waiting time can be very long if there is no coordination between the arrival and departure times of the connecting vehicles. On the other hand, when transferring between high-frequency routes, transfer waiting time is a relatively minor concern because there is an expectation that a connecting vehicle will be there shortly. This is also the case when transferring from a low-frequency to a high-frequency route.

The different aspects of the transfer experience make transfers burdensome and annoying for most passengers, but despite this, transfers cannot be avoided. The only alternative is, therefore, to minimize their disutility to transit passengers. In fact, improving the transfer experience has become a growing concern for many agencies due to its significant impact on the passengers' perception of the overall quality of transit service and hence on the agency's total ridership and revenue. Transit agencies expect that better transfers should result in travel experiences which are more satisfying to the customer and which should eventually translate into increased ridership and revenue.

Improving the Transfer Experience

There is a wide range of improvements that can be made to the typical transfer experience. If the connecting vehicles arrive and depart with more certainty, the transfer waiting time is minimized, the conditions of the transfer are made more favorable, or the cost of a transfer is eliminated, then the disutility of a transfer can be reduced. This paper focuses on strategies that minimize the transfer waiting time, which is the primary driver of customer satisfaction [Northwest Research Group, 2002]. This time is "wasted" from the perspective of the passengers.

In general, transfer waiting time can be improved through the implementation of schedule and/or real-time coordination. Schedule coordination involves modifying the service timetables in order to minimize the overall passenger waiting time in the network and to improve its transfer performance. However, no operation strictly conforms to the operations plan. Because of the inherent stochasticity in a transit network, simply synchronizing scheduled vehicle arrivals among connecting routes at transfer stops is unlikely to reduce transfer waiting time significantly. When disruptions occur, real-time control systems utilizing any available current network information (vehicle locations and passenger loads based on ITS technologies) can determine dispatch times for vehicles at transfer stops in a dynamic way to optimize the transfer performance.

SELECTION OF TRANSIT CORRIDOR FOR ANALYSIS

Since each transit agency has a limited budget available for service improvements, it is financially infeasible to apply transfer coordination to all the transit corridors in its network. Prioritizing these corridors helps the agency focus its attention on the ones that represent the greatest opportunities for improvement and that can best exemplify successful applications of transfer coordination and hence justify the associated project cost.

Crockett [2002] suggested that the total number of transferring passengers to and from a corridor should be the only determining factor in the likely benefits of a transfer coordination system along that corridor. Certainly the transferring passengers will be the major beneficiaries of such a coordination scheme since they will enjoy shorter transfer waiting times. However, there are several other factors that need to be also considered when evaluating a transfer coordination system and hence deciding where to focus the coordination initiatives [Wong, 2000]. These include: the ratio of transferring passengers to through passengers at each transfer stop, the mean and standard deviation of the expected waiting time experienced by the transferring passengers, and the compatibility of headways on the connecting routes.

THE OPERATIONS PLANNING MODEL

The first model developed to assist in transit scheduling recommends a new service timetable for the vehicles operating on the selected transit corridor. The timetable development process involves the simultaneous application of two planning strategies: changing the terminal departure time and inserting slack time. The operations planning model finds optimal values for the decision variables associated with these two strategies and outputs a schedule that should minimize the transfer disutility along the selected corridor.

Basic Elements

The first operations planning strategy, changing the terminal departure time, considers rescheduling the departure time from the terminal of all vehicles on the selected corridor during the period of analysis. Any change in terminal departure time would allow these vehicles to arrive at transfer locations as close as possible to the arrival of connecting vehicles to reduce passengers' transfer waiting time. However, due to the stochastic conditions inherent in any operation, this synchronization of vehicle arrivals may not work for all trips. That is why slack time is sometimes added to the schedule of the vehicles at certain stops, referred to as timepoints, to ensure that the connection occurs even if some of the trips run late. The addition of this slack time is meant to increase transfer reliability by absorbing some of the service randomness and hence increasing the probability of transfer connections. Adding slack time to the schedule at certain timepoints is thus the second operations planning strategy. In this model, the addition of slack time is limited so as not to increase the cycle time of vehicles on the route or alter any of the other schedule parameters of the corridor under study. It should also be noted that the planning model does not adjust the service frequency on the analysis corridor or its connecting lines because this is seldom a favorable strategy in practice. On the contrary, these headways are assumed constant for the analysis period with their values as determined by the transit agency.

Application of the Planning Model

Two restrictions have to be recognized on the analysis transfer stops when applying the operations planning model to a selected transit corridor and time period. First, since the ultimate aim of the model is to generate a new service timetable for the vehicles operating on the selected corridor, only transfer stops which are also schedule timepoints are eligible for analysis. Second, only those transfer stops where at least one of the intersecting lines has a headway which is compatible with the main line on the analysis corridor are analyzed. Two lines are deemed compatible if their headways are identical or are related by an integer multiple. This headway compatibility criterion is important to ensure that a significant number of trips will benefit from the transfer coordination initiatives during the analysis period. Vehicles operating along lines – with compatible headways – will now meet at the transfer stops an integer times per hour. As a result, the possibility of making good connections greatly increases.

A typical (simplified) transit corridor is shown in Figure 1. This is a route with eight stops in each direction, five of which are timepoints and five of which are transfer stops. At each transfer stop, four lines intersect (the line definition in this paper is directional; a line is uni-directional element of the route). This transit corridor is used to illustrate the selection process. The timepoint restriction limits the number of transfer stops to be analyzed to three: stops 2, 5 and 7. To finalize the selection, the headways of the connecting lines at these stops are considered. Assume that line A (and B) of the corridor under analysis has a headway of 6 minutes, line C (and D) has a headway of 12 minutes, line G (and H) has a headway of 10 minutes and line K (and L) has a headway of 6 minutes. Since the headway of line C (and D) is double that of line A (and B), these lines are compatible and stop 2 will be included in the analysis. Similarly at stop 7, both lines K and L have headways which are identical to lines A and B, and thus this stop will also be included in the analysis. The incompatibility of headways on lines G and H with those on lines A and B disqualifies stop 5 from being included. Once the transfer stops have been selected, the operations planning model can then be applied to develop a new service timetable for the chosen corridor comprising lines A and B in this example. While generating a new timetable for these lines, it is assumed that the service schedules of all the routes connecting with the corridor at the selected transfer stops are constant. The schedules of lines C, D, K and L are thus kept unchanged in the process of setting a new timetable for lines A and B.

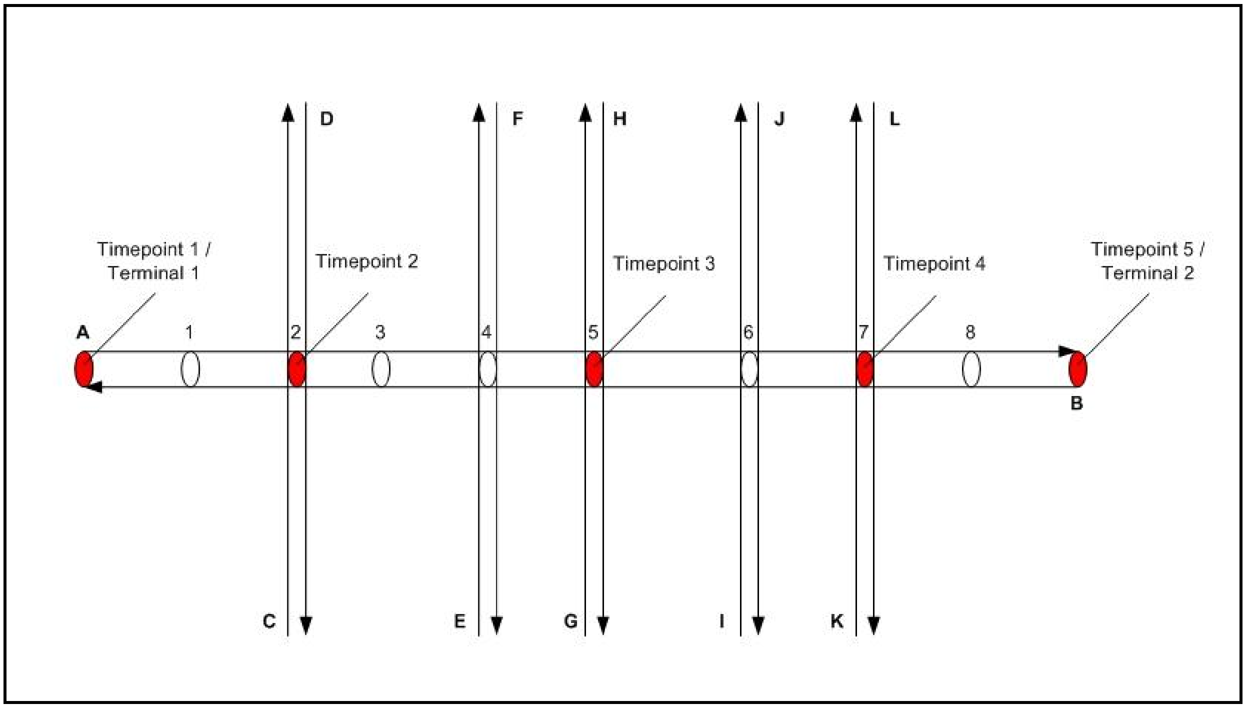

Figure 1 - Corridor Representation

Impacts of a New Service Timetable

Applying the operations planning model to a particular corridor is primarily meant to improve transfer coordination at selected transfer stops along that corridor by minimizing the inconvenience associated with the transfers. However, the new service timetable generated by this application should not only consider the savings in waiting times experienced by transferring passengers. The impacts of this timetable on non-transferring passengers should also be considered. Influenced passengers can actually be grouped into three categories at each transfer stop. These categories are illustrated below with reference to stop 7 in the figure above.

§ P1 Passengers

These are the passengers transferring between the vehicles on lines A and K. These transferring passengers are expected to benefit the most from coordination initiatives since they should experience shorter expected transfer waiting times. The passengers transferring from line K to A are at a greater advantage, however, since slack time can be added to the schedule of the vehicle on line A at stop 7 if that will minimize the number of missed connections.

§ P2 Passengers

These passengers arrive at the transfer stop onboard the vehicle on line A and proceed past that stop to alight at downstream destinations. If slack time is introduced in the schedule at that stop, these passengers are adversely affected since they are now delayed onboard for the duration of the slack.

§ P3 Passengers

These are the passengers who board the vehicle on line A at stop 7. These boarding passengers are assumed to know the schedule of their vehicle beforehand. Their arrival patterns at stop 7 are assumed to remain the same with or without coordination, and hence their waiting time is unchanged.

As discussed earlier, changing the vehicle departure time from the starting terminal and/or inserting slack time in its schedule at intermediate timepoints is not allowed to increase the cycle-time. Therefore, the operating cost to the transit agency does not change as a result of this model's application since the number of vehicles and the number of vehicle-hours remain the same.

Objective Function

A number of the planning models developed in the literature base the optimal schedule on the benefits accruing to transferring passengers only, with the sole measure being the reduction of transfer waiting time [Rapp and Gehner, 1976; Andréasson, 1977; Hall, 1985; Knoppers and Muller, 1995]. Changing the service timetable, however, can impact not only transferring passengers but also non-transferring ones especially if slack is introduced en route. In this case, transfer waiting time should not be the sole objective in timetable development. A timetable that minimizes the waiting time of transferring passengers while causing significant delays to non-transferring ones is unlikely to be optimal. Such a timetable may degrade the overall performance of the route and worsen the overall service quality offered to passengers. The design of a service timetable must be based on both the benefits and delays to all potentially affected passengers.

In this paper, the measure used as the basis for selecting an optimal timetable will be the total expected waiting time which is defined to include both in-vehicle delay and out-of-vehicle wait. In-vehicle delay is suffered by passengers onboard a vehicle which is waiting at a stop while out-of-vehicle wait is experienced by passengers transferring between vehicles. Since waiting time is the major source of variability in any trip, it makes sense to base the schedule on the net change in the passengers' waiting time.

Model Assumptions

This section presents the assumptions made in the operations planning model. These assumptions allow the application of this model to transit networks with dynamic traffic conditions, multiple transfer stops and multiple routes connecting at these stops.

§ Minimum Connection Times

Minimum connection times are defined as the walk time needed by transferring passengers between their arriving vehicle and their destination one. Minimum connection times can vary by stop depending on the layout and physical characteristics and are included to make sure that all the transferring passengers make their connections successfully. The model assumes that the minimum connection times are deterministic.

§ Headways

The planning model deals with a predetermined network of transit routes with the headways on each route assumed constant for the duration of the time period being analyzed. Once the schedule for the first trip on a line in the corridor is set, this predefined sequence of headways is used to generate the schedule of all following trips on that line.

§ Dwell Times

The dwell time of a vehicle at a stop is the time needed by that vehicle to complete its passenger processing. This time depends on many factors including the vehicle's design, the fare payment method, and the expected numbers of boardings and alightings. The numbers of boardings and alightings will vary with the vehicle's preceding headway. However, since headways are assumed constant in this planning model, it is reasonable to assume that dwell times are also constant but differ between stops.

§ Half-Cycle Times

The expected travel times between any two stops are assumed constant in the model, as is the recovery time needed at each terminal. Consequently, the scheduled half-cycle times in each direction remain unchanged. This is an important assumption which implies that the offset time at terminal 2 is the same as that at terminal 1. It also implies that any slack time included in the schedule in one direction will not affect the expected arrival/departure times at the transfer stops in the reverse direction.

§ Vehicle Arrival Time Distributions

Many of the planning models in the literature assume deterministic vehicle arrivals [Rapp and Gehner, 1976; Andréasson, 1977; Klemt and Stemme, 1988], and those studies which use stochastic vehicle arrival time distributions are developed only for a single transfer stop [Hall, 1985; Lee and Schonfeld, 1991; Wirasinghe and Liu, 1995; Knoppers and Muller, 1995]. The planning model developed in this research assumes probabilistic vehicle arrival times at the selected transfer stops for both the analysis route and its connecting routes. It accommodates any arrival time distribution input by the user and estimates the corresponding total expected waiting time.

§ Passenger Demand

Passenger demand usually varies with the level of service provided by the transit agency. As the service quality on a line improves, the demand for that line is likely to increase. However, the change in the level of service of a line should not affect passenger demand for that line if the planning horizon is relatively short [Rapp and Gehner, 1976; Andréasson, 1977]. In this model, it is assumed that both the passenger demand and transfer flows associated with any line are fixed and are not affected by any change in that line's service timetable.

§ Passenger Time Values

Passengers perceive in-vehicle delay differently from out-of-vehicle time, with the latter generally being more onerous. The operations planning model differentiates between these time perceptions. Incorporating distinct values of out-of-vehicle time and in-vehicle delay leads to more realistic evaluations of the costs and benefits of any timetable.

Waiting Time Calculation

As mentioned earlier, the measure that is used for selecting a new schedule for the corridor under study is the total expected waiting time along that corridor, TWT. This total expected waiting time changes with the service timetable on the corridor. For a particular schedule, TWT is the sum of the expected waiting times at the selected transfer stops on both lines of the analysis route (Equation 1). Table 1 summarizes the notation used for the waiting time calculation.

$$TWT = \sum_{i=1}^{2} \sum_{x=1}^{\max\_n} WT_{x,i} \tag{1}$$

The expected waiting time at any transfer stop in the direction of i, $WT_{x,i}$, is the sum of the waiting times experienced by the different impacted passengers at that stop (Equation 2 where c1 and c2 represent the lines connecting with the main corridor at the transfer stop). As mentioned earlier, a new schedule affects the passengers transferring between the connecting routes and the analysis corridor as well as the through passengers along the corridor. Due to the probabilistic nature of vehicle arrivals at any transfer stop, the calculation of WT is determined from the joint probability distributions of these arrivals. However, since vehicle arrivals are assumed to vary independently on each route, the joint probabilities of arrivals may be obtained by simply multiplying the probabilities obtained separately from each vehicle arrival time distribution.

$$WT_{x,i} = \sum_{\min\_a_x(i)}^{\max\_a_x(i)} \left\{ \begin{array}{l} \sum_{\min\_a_x(c1)}^{\max\_a_x(c1)} \left[wt_x(i,c1) + wt_x(c1,i) \right] * P_{a(c1)} + \\ \sum_{\min\_a_x(c2)}^{\max\_a_x(c2)} \left[wt_x(i,c2) + wt_x(c2,i) \right] * P_{a(c2)} + wt_x(thr,i) \end{array} \right\} * P_{a(i)} \quad (2)$$

The waiting time experienced by passengers transferring from line i to line j at stop x, $wt_x(i,j)$, depends on the arrival time of the vehicle on line i and the departure time of the vehicle on line j from stop x. Two cases are considered. First, the vehicle on line j departs before the passengers transferring from line i complete their connection. These transferring passengers now have to wait for the next arriving vehicle on line j (Equation 3). Second, transferring passengers from line i are able to connect with the current vehicle on line j. This can occur when the transferring passengers from line i arrive either before (Equation 4) or after (Equation 5) the arrival time of the vehicle on line j but before its departure time from stop x.

$$wt_x(i,j) = p_x(i,j) * \{\alpha * [a_x(j+1) - a_x(i) - tt_x(i,j)] + [d_x(j+1) - a_x(j+1)]\} \quad (3)$$
$$wt_x(i,j) = p_x(i,j) * \{\alpha * [a_x(j) - a_x(i) - tt_x(i,j)] + [d_x(j) - a_x(j)]\} \quad (4)$$
$$wt_x(i,j) = p_x(i,j) * \{d_x(j) - a_x(i) - tt_x(i,j)\} \quad (5)$$

Table 1 - Notation for Waiting Time Calculation

$a_x(i)$	=	arrival time of current vehicle on line i at stop x
$a_x(i+1)$	=	arrival time of following vehicle on line i at stop x
$d_x(i)$	=	departure time of current vehicle on line i at stop x
$d_x(i+1)$	=	departure time of following vehicle on line i at stop x
$holding$	=	required holding time (min)
$\min\_a_x(i)$	=	earliest arrival time of current vehicle on line i at stop x
$\max\_a_x(i)$	=	latest arrival time of current vehicle on line i at stop x
$\max\_n$	=	number of stops analyzed including the current transfer stop
$P_{a(i)}$	=	probability of a vehicle arriving at time "a" on line i at stop x
p_{thr}	=	number of through passengers onboard the vehicle on the main line at the transfer stop (pax)
$p_x(i)$	=	number of passengers waiting to board a vehicle on line i at stop x (pax)
$p_x(i/hold)$	=	number of passengers who board a vehicle on line i at stop x during the holding time (pax)
$p_x(i,j)$	=	number of passengers transferring from line i to line j at stop x (pax)
$pr\_h_x(i)$	=	preceding headway of the vehicle on line i at stop x (min)
$s_x(i/hold)$	=	time savings of passengers who board a vehicle on line i at stop x during the holding time (min/pax)
$TN_{x,i}$	=	transfer stop x on line i

$tt_x(i, j)$	=	minimum connection time from line i to line j at stop x (min)
TWT	=	total waiting time in the system
wt_{thr}	=	waiting time of through passengers onboard the vehicle on the main line at the transfer stop (min/pax)
$wt_x(i)$	=	waiting time of passengers boarding a vehicle on line i at stop x (min/pax)
$wt_x(i, j)$	=	waiting time of passengers transferring from line i to line j at stop x (min/pax)
α	=	ratio of out-of-vehicle wait to in-vehicle delay
$\lambda_x(i, j)$	=	passenger transfer rate from line i to line j at stop x (pax/min)
λ_{thr}	=	passenger through rate at the transfer stop (pax/min)
$\lambda_x(i)$	=	passenger arrival rate to board a vehicle on line i at stop x (pax/min)

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Finally, the waiting time experienced by through passengers on line i at stop x, $wt_x(thr,i)$, depends on the arrival and departure times of the vehicle on line i at the transfer stop (Equation 6). Recall that vehicles are never permitted to depart before their scheduled departure time. However, if a vehicle is delayed beyond this scheduled departure time, it will depart immediately upon completion of passenger processing.

$$wt_x(thr,i) = p_x(thr,i) * \{d_x(i) - a_x(i)\} \qquad (6)$$

Model Structure

The main aim of the operations planning model is to identify a new service timetable for a particular corridor that improves the transfer performance at selected transfer stops. Figure 2 shows the general structure of this model. In summary, the model first calculates the total expected waiting time associated with every feasible offset/slack time combination, and selects the combination that minimizes the total expected waiting time along the corridor.

The model first initializes offset time to the minimum allowed value from the offset range input by the user. It also initializes slack times at the selected transfer stops in direction 1 along the analysis corridor to zero. Using this offset/slack time combination, the expected arrival and departure times at each selected transfer stop in direction 1 are determined based on the expected link travel times which are assumed constant in the model. The expected waiting times at each of these stops are then calculated based on Equation 2, which accounts for the probabilistic nature of vehicle arrivals.

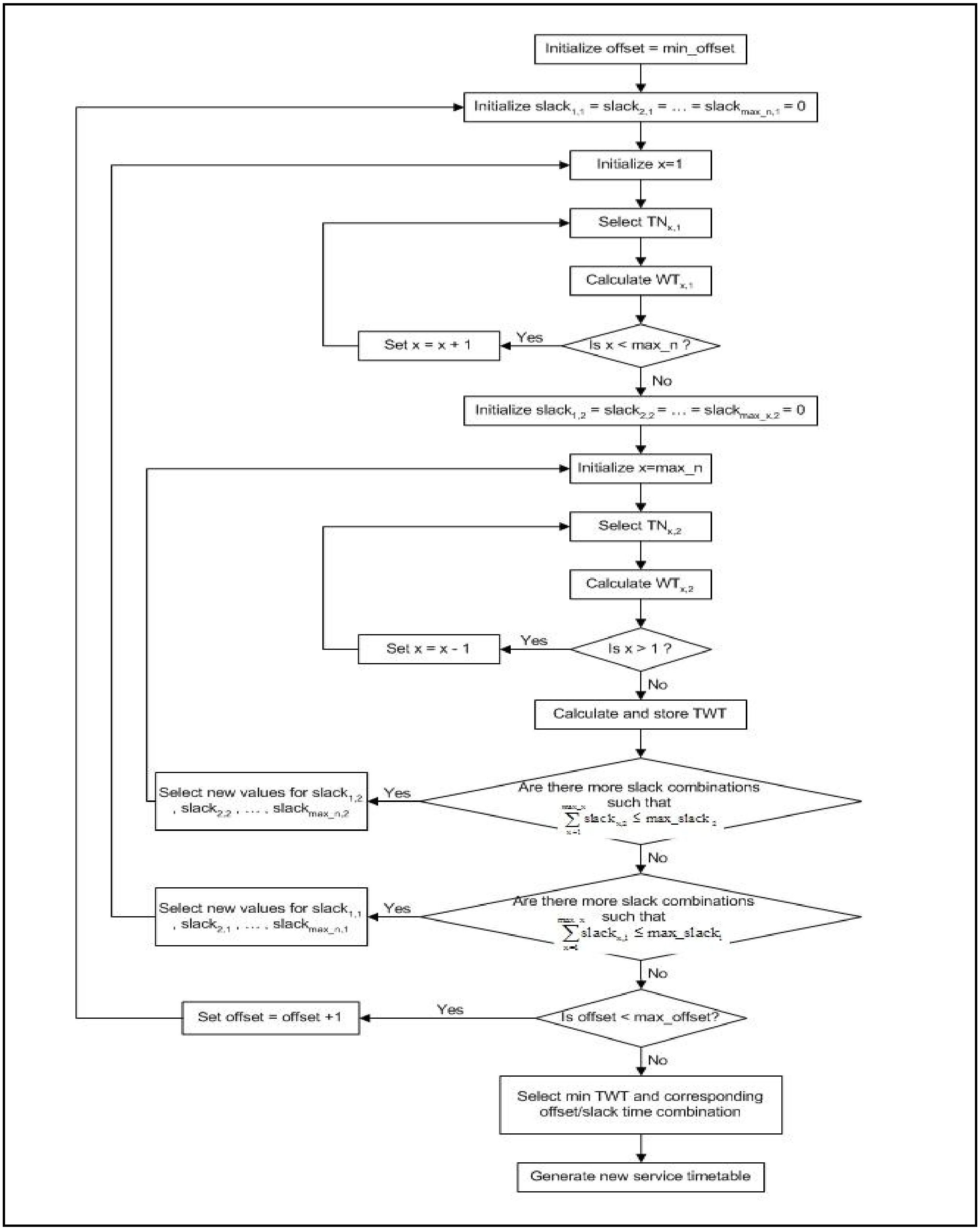

Figure 2 - Operations Planning Model Structure

The slack times at the selected transfer stops in direction 2 along the analysis corridor are now initialized to zero. Using these slack time values and the offset at terminal 2 (which is equal to that at terminal 1), the expected arrival and departure times at the selected transfer stops in direction 2 are determined. The expected waiting times at each of these stops are then calculated based on Equation 2. The total expected waiting time along the corridor is finally calculated according to Equation 1 for the given offset/slack time combination.

Since changing the schedule at the transfer stops in one direction (in terms of adding slack time) does not affect the operations plan in the opposite direction, the slack times at the transfer stops in direction 2 are now altered such that their sum does not exceed the maximum amount of slack available for that direction. The expected waiting times at the transfer stops in direction 2 as well as the total expected waiting time along the corridor are evaluated again for that offset/slack time combination. This process is repeated until all the slack time combinations in direction 2 have been analyzed for the same offset and slack time combinations in direction 1. The model then chooses another combination of slack times at the transfer stops in direction 1 such that the sum of these slack values does not exceed the maximum amount of slack available for that direction. The whole process of evaluating the different slack time combinations in direction 2 is then repeated with the TWT calculated and stored in each case.

This is carried out for each allowable offset value so that, at the end, the total expected waiting times of all the feasible offset/slack time combinations have been evaluated. The minimum total expected waiting time along the analysis corridor is then selected along with its offset/slack time combination. Knowing the starting time from the terminal, the link travel times, the slack times at the transfer stops in each direction and the recovery times at the two terminals, the new schedule for the first trip on that route is fully determined. The schedule for subsequent trips can then be easily obtained since headways are assumed constant for the duration of the study.

The operations planning model recommends a new service timetable that minimizes the total expected waiting time for the first trip on the analysis corridor. The choice of the offset/slack time combination is, therefore, optimal for this first trip. If the headways on all the connecting routes are equal to that on the analysis corridor, this choice of offset/slack time combination is also optimal for all the subsequent trips in the time period under study. However, if some or all of these connecting route headways are not equal to the headway on the analysis corridor, the timetable recommended by the operations planning model might not result in the minimum total expected waiting time for all the trips. This is because a schedule which produces good transfer connections between certain trips may produce poor connections between other trips if the headways on the connecting routes are not equal.

For example, suppose we have a main line A and a connecting line B with headways of 10 and 20 minutes respectively. Suppose that the operations planning model recommends a new schedule on line A such that the first trip on that line (A1) arrives at the transfer stop at time t and completes its connection with the first trip on line B (B1), assuming a minimum connection time of zero minutes between the lines. Let A2 and B2 be the next trips on lines A and B after trips A1 and B1. The arrival times of A2 and B2 at the transfer stop would then be t+10 and t+20 minutes respectively assuming deterministic conditions. Although the schedule recommended by the operations planning model results in no transfer waiting time for the connection from A1 to B1, this schedule produces a wait of 10 minutes for the connection from A2 to B2. In fact, this wait will be experienced by passengers transferring from every second trip on line A. The waiting time experienced by passengers transferring from line B to line A – under the recommended schedule – will always be zero because of the higher frequency on the latter line.

In summary, the total expected waiting time calculated by the operations planning model only accounts for those trips which are similar to the first trips on the connecting routes. If the headways on all these routes are the same, then the timetable recommended by the model would produce the greatest benefits. Otherwise, the expected waiting times experienced by passengers on the remaining trips should be calculated separately for all offset/slack time combinations and the total expected waiting times along the corridor should be adjusted. A new timetable should then be recommended based upon the adjusted total expected waiting times and not upon those obtained directly from the planning model.

THE OPERATIONS CONTROL MODEL

Over the last decade, information technologies have advanced greatly so that more transit agencies are starting to make use of Automatic Vehicle Location (AVL), Automatic Passenger Counters (APC), and Automatic Fare Collection (AFC) systems to support their operations control decisions (among many other applications). One such operations control decision, which has attracted much attention in recent years, is vehicle holding at transfer stops to allow passenger transfers between connecting vehicles. For any vehicle, which is ready to be dispatched from a transfer stop, the question is whether to dispatch it immediately or to hold it for an arriving vehicle with connecting passengers. The model described in this section addresses this question. The main aim of this real-time dispatching model is to make recommendations as to whether a vehicle arriving at a transfer stop should be held for feeder vehicle arrivals. This evaluation of the holding decision is based upon the net passenger-minutes saved for both transferring and non-transferring passengers. Wong [2000] developed such a model that was applied to the transfers between the Red and Green Lines at the Park Street Station of the Massachusetts Bay Transportation Authority (MBTA). The model used in this research is an extension of Wong's since it can be applied to all connection types. Many of the underlying principles and assumptions comprising this model are similar to those developed by Wong.

Basic Elements

The principles and elements underpinning this real-time dispatching model are explained first, including: feasibility of a hold, impacts of a hold, and basis for a hold.

Feasibility of a Hold

Not all vehicles arriving at a transfer stop are deemed eligible for holding. The feasibility of a hold is based on three factors: the schedule of the vehicle being considered for a hold, the estimated arrival time of the following vehicle on the same line, and the estimated arrival time of the closest incoming vehicle on a connecting line.

The first factor considers the schedule of a vehicle y as it arrives at the transfer stop x. The maximum allowed holding time for this vehicle is calculated as the time remaining in the schedule of that vehicle at that point on the route excluding the minimum recovery time that is reserved at the terminal. The second factor ensures that the held vehicle y will depart the transfer stop x before the expected arrival of the following vehicle on the same line. The maximum holding time is calculated based on the following headway. The last factor is used to determine the holding time that is required by the transferring passengers to make their connection. This includes the estimated time for the incoming vehicle to arrive at the transfer stop and the minimum connection time for the passengers to transfer.

Holding is considered a feasible option only if the required holding time needed by the transferring passengers to complete their connection is less than or equal to the available maximum holding time based on both the schedule and following headway. Equation 7 depicts this feasibility requirement where H_z refers to the holding time calculated based on factor z.

$$H_3 \leq \min(H_1, H_2) \tag{7}$$

Impacts of a Hold

Holding a vehicle at a transfer stop is primarily intended to reduce transfer waiting time at that stop. Specifically passengers transferring to the held vehicle will benefit by experiencing no waiting time for the transfer. However, transferring passengers are not the only ones affected by this decision. The impacts of holding on other passenger types at the transfer stop and at downstream stops should also be considered. There are generally six different types of passengers affected:

§ P1 Passengers

These are the passengers connecting to the held vehicle from an incoming feeder vehicle. P1 passengers benefit the most from holding since they can now complete their connection to the destination line. Instead of waiting for the next incoming vehicle on the destination line, these passengers experience no wait time if a hold is implemented.

§ P2 Passengers

These through passengers originate at stops upstream of the transfer stop and alight at downstream destinations. P2 passengers are adversely affected by a hold since they are delayed in-vehicle for the duration of the hold.

§ P3 Passengers

These are the passengers accumulating at the transfer stop and at downstream stops over the preceding headway on the destination line. P3 passengers board the vehicle during the dwell time regardless of whether or not a hold is implemented. As such, these passengers are also negatively affected by a holding decision since they are delayed onboard the vehicle for the duration of the hold.

§ P4 Passengers

P4 passengers arrive at the transfer stop and at downstream stops during the holding period and are able to board the held vehicle as a result of the hold. These passengers save time amounting to the following headway on the destination line.

§ P5 Passengers

P5 passengers are a fraction of the P2 passengers who are destined to transfer at downstream stops to connecting vehicles. Holding influences these passengers' transfer waiting time at these downstream stops, either positively or negatively. As a result of holding their vehicle at the current stop, P5 passengers might miss their connections at downstream stops for instance.

§ P6 Passengers

These are the passengers connecting to the vehicle on the main line at downstream stops. P6 passengers are also affected, either positively or negatively, by a holding decision depending on whether they make or miss their connections.

The additional cost to the transit agency as a result of a holding decision is assumed zero since a hold is not permitted to extend the half-cycle time of a vehicle. Holding a vehicle at a particular transfer stop is considered a viable option only if there is enough time remaining in the schedule of that vehicle excluding the minimum recovery time at the terminal.

Basis for a Hold

Real-time transfer coordination can improve the overall performance of a transit network by improving service reliability and reducing the disutility associated with transfers. Some of the benefits of holding include the minimization of missed connections and the shortening of transfer waiting time. However, these measures cannot be used as the sole criteria for holding a vehicle since they reflect the effect of such a decision on transferring passengers only. Such control decisions should consider the overall network effects and should ensure that the level-of-service experienced by non-transferring passengers is not significantly reduced.

The measure that will be used as the basis behind any holding decision will be the net passenger-minutes saved from a hold. This total net benefit measures the difference in waiting times for passengers benefiting from the hold and those being delayed by it. Since waiting time is the major source of variability in any trip, it makes sense to base any holding recommendation on the net change in the passengers' waiting time and to use such a measure for evaluating the effectiveness of a hold.

Some transit agencies, however, might not feel comfortable basing their dispatching decision on just a positive total net benefit measure. If the net passenger-minutes saved is negligible, holding a vehicle is a plausible but not necessarily wise option since it will not result in significant benefits. That is why a minimum holding criterion is utilized as the basis for holding decisions. Real-time holding is implemented at a transfer stop only when the net passenger-minutes saved exceeds this minimum holding threshold.

Model Assumptions

Some of the principles and assumptions behind the operations planning model also hold true for the operations control model. This section discusses new issues that are introduced to this real-time dispatching model as well as areas that are treated differently than in the operations planning model.

§ Holding Characteristics

Holding time is defined as the time vehicle x waits at a transfer stop for an incoming feeder vehicle y. The start of this holding time depends on the estimated arrival time of vehicle x and the nature of the transfer stop. If vehicle x arrives at a transfer stop, which is a timepoint, and completes its passenger processing before its scheduled departure time from that timepoint, then holding begins at the scheduled departure time. Otherwise, holding begins after the conclusion of the initial dwell time of vehicle x. The duration of the hold depends on the arrival time of vehicle y. It is assumed that if vehicle x is held, it will not leave the transfer stop until all the passengers from the feeding vehicle y have transferred. It is also assumed that all these transferring passengers are able to board the current held vehicle x without capacity concerns.

§ Vehicle Characteristics

A real-time information system is capable of predicting the arrival times of vehicles at the stops based on AVL data. In the absence of AVL data, the agency can only resort to communication between its field supervisors, vehicle operators and control center to get approximate locations of its vehicles and estimate their arrival times at stops. In most cases it will not be practical to employ holding in the absence of an AVL system. Such a system is key to predicting the arrival times of the arriving and following vehicles at the transfer stop and at downstream stops and thus for estimating the waiting times experienced by the different passenger categories, both with and without holding.

§ Passenger Characteristics

Real-time information systems can also generate estimates of the number of passengers onboard a vehicle approaching a transfer stop based on historical manual or APC system data and headways from an AVL system. Historical rates can be used to estimate the number of passengers transferring from that vehicle at that stop as well as the number of non-transferring passengers who will alight at downstream destinations. Passenger through, arrival, and transfer rates at all stops on all lines are assumed to be deterministic in the operations control model. The implication of this assumption can be reduced by shortening the analysis periods to minimize the variability of passenger rates within each period.

Waiting Time Calculation

The measure that is used for evaluating any holding decision is the total net benefits resulting from such a decision. To arrive at this measure, the waiting time of all impacted passengers at, and downstream of, the transfer stop must be estimated for both the holding and no-holding scenarios. Before the waiting time calculation though, the number of impacted passengers in each category must be estimated. Table 1 summarizes the notation used for estimating the number of impacted passengers and their waiting time period.

The number of through passengers onboard the vehicle on the main line at the transfer stop is estimated as shown in Equation 8 by multiplying the through passenger rate and the preceding headway of that vehicle at that stop.

$$p_{thr} = \lambda_{thr} * pr\_h_x(i) \tag{8}$$

The number of passengers waiting to board a vehicle on the main line at or downstream of the transfer stop is estimated in a similar fashion according to Equation 9.

$$p_x(i) = \lambda_x(i) * pr\_h_x(i) \tag{9}$$

The waiting time experienced by each of these two passenger types is estimated by subtracting the departure time of the vehicle on the main line from its arrival time at a particular stop.

$$wt_{thr} = wt_x(i) = d_x(i) - a_x(i) \tag{10}$$

The number of passengers transferring from line i to line j at a particular stop is estimated by multiplying the passenger transfer rate from line i to line j and the preceding headway on line i at the stop.

$$p_x(i,j) = \lambda_x(i,j) * pr\_h_x(i) \tag{11}$$

Similarly to the operations planning model, the waiting time experienced by each of these transferring passengers is a function of the arrival time of the vehicle on line i and the departure time of the vehicle on line j at stop x. Equation 12 shows the waiting time calculation if the transferring passengers miss their connection. The waiting time experienced by passengers who are ready to transfer before the arrival of their destination vehicle is shown in Equation 13. Finally, Equation 14 shows the waiting time calculation when passengers transferring from line i arrive after the arrival of their destination vehicle but before its departure from stop x.

$$wt_x(i,j) = \alpha * \left[a_x(j+1) - a_x(i) - tt_x(i,j)\right] + \left[d_x(j+1) - a_x(j+1)\right] \tag{12}$$

$$wt_x(i,j) = \alpha * \left[a_x(j) - a_x(i) - tt_x(i,j)\right] + \left[d_x(j) - a_x(j)\right] \tag{13}$$

$$wt_x(i,j) = d_x(j) - a_x(i) - tt_x(i,j) \tag{14}$$

If the vehicle on the main line is held at the transfer stop for an incoming connecting vehicle, then there is one other passenger category affected by such a decision. These passengers arrive during the hold and save time since they can now board the current vehicle on the main line without waiting for the following one. The number of these passengers and their waiting time savings are estimated in Equations 15 and 16 respectively.

$$p_x(i/hold) = \lambda_x(i) * holding \tag{15}$$

$$s_x(i/hold) = \alpha * \left[a_x(i+1) - d_x(i)\right] + \left[d_x(i+1) - a_x(i+1)\right] \tag{16}$$

The total waiting time of all impacted passengers is estimated by summing the different waiting times at, and downstream of, the transfer stop where a hold is considered as shown in Equation 17, where c1 and c2 correspond to the lines connecting with the main line at the transfer stops.

$$\begin{aligned} TWT = {} & wt_{thr} * p_{thr} + \sum_{x=1}^{max\_n} [wt_x(i) * p_x(i) + wt_x(i,c1) * p_x(i,c1) \\ & + wt_x(i,c2) * p_x(i,c2) + wt_x(c1,i) * p_x(c1,i) + wt_x(c2,i) * p_x(c2,i)] \end{aligned} \tag{17}$$

This is carried out for both the holding and no-holding scenarios. (If a “ready” vehicle is held at a transfer stop, its departure time is delayed by the required holding time). Total net benefits or the net passenger-minutes saved are then calculated by subtracting the total waiting time for the no-holding scenario from the total waiting time for the holding scenario (which also includes the time savings of passengers now able to board the current vehicle). If this difference exceeds the minimum holding threshold set by the transit agency, the “ready” vehicle is held at the transfer stop for the arrival of the incoming connecting vehicle.

Model Structure

This section describes the process for evaluating whether a vehicle, which is ready to be dispatched from a transfer stop, should be held for an incoming feeder vehicle arrival. To estimate the change in waiting times of the different passenger types, vehicle location data is needed to predict vehicle arrival times and passenger demand throughout the transit system. It is thus important that this holding or dispatching decision be updated whenever the control system has new vehicle location data.

Figure 1 again illustrates the general model. Assume that a vehicle x arriving at stop 2 on line A is ready to depart at its scheduled departure time. However, the next arriving vehicle y on connecting line C is expected to arrive in one minute and the next arriving vehicle z on connecting line D in three minutes. Vehicle x can either be dispatched immediately, held one minute for vehicle y, or held three minutes for both vehicles y and z, assuming a minimum connection time of zero minutes. The decision to hold vehicle x for vehicle y is evaluated first since the arrival time of that vehicle is the closest. If vehicle x is held for the arrival of vehicle y, the control model is applied again at the end of the holding period to decide whether vehicle x should then be dispatched immediately or held further for the arrival of vehicle z. This dispatching decision is based on an evaluation of the change in waiting times to impacted passengers both at transfer stop 2 and at the downstream stops 3 through 8. If holding vehicle x for the arrival of vehicle y cannot be justified, then vehicle x is dispatched immediately. The control model does not evaluate holding vehicle x for the arrival of vehicle z unless the former is already held for the arrival of vehicle y.

Figure 3 shows the general structure of the operations control model, which is split into two distinct phases. Phase I evaluates the feasibility of a holding action and estimates the holding time needed to allow transferring passengers to complete their connection. Phase II estimates the number of impacted passengers and their respective waiting time savings/delay leading to a recommended course of action based on maximizing the net passenger benefits.

Holding a “ready” vehicle is feasible only if Equation 7 is satisfied. The maximum time, H_1, vehicle x can wait at stop 2 without increasing its half-cycle time is first determined. This time is the leeway that is remaining in the schedule of vehicle x at that point on the route excluding the minimum recovery time required at terminal 2. The estimated arrival time of the following vehicle on the same line determines the following headway of vehicle x which in turn determines H_2, the maximum holding time at stop 2 that will result in any benefits. Finally, the minimum holding time H_3 that is needed by the passengers transferring from vehicle y to vehicle x at stop 2 to complete their connection is estimated based on the expected arrival time of vehicle y at stop 2 and the minimum connection time.

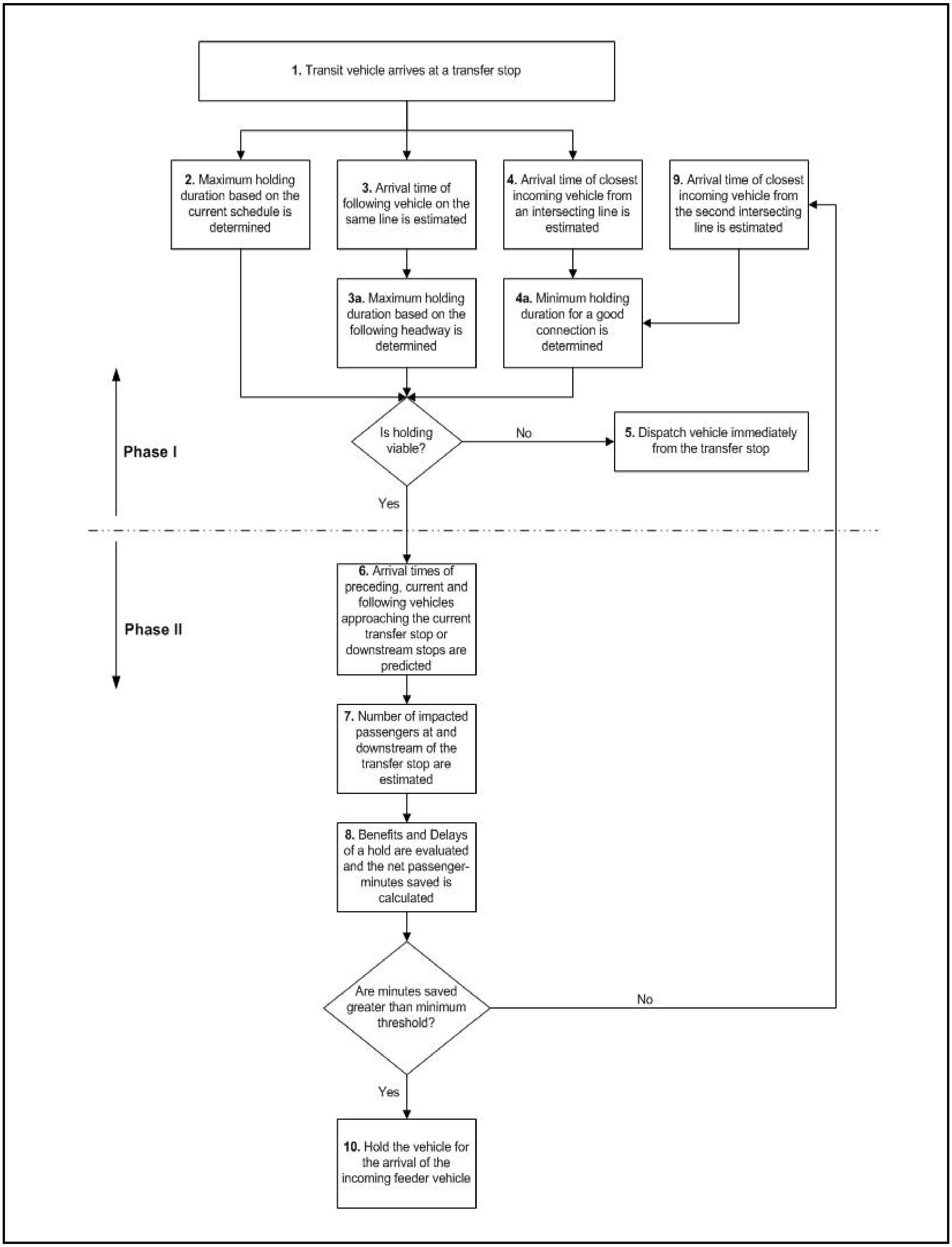

Figure 3 - Operations Control Model Structure

If Equation 7 is not satisfied, vehicle x is dispatched immediately from stop 2. Otherwise, holding is deemed a feasible option, the required holding time is H_3, and the model proceeds to Phase II.

Phase II estimates the number of impacted passengers and their respective waiting time savings/delay as a result of a hold according to the equations presented previously. Estimates of the arrival times of the preceding, current and following vehicles on all the lines approaching the current transfer stop and all downstream stops are first determined. These estimates are needed to determine the preceding and following headways on each line at each stop.

Preceding headways are used to estimate the number of impacted passengers in each category at each stop and the following headways are used to estimate the waiting time savings/delays for each passenger group. The overall net benefits to all impacted passengers along the transit corridor is finally estimated and compared to the minimum holding threshold set by the transit agency. If the net passenger-minutes saved equals or surpasses the minimum holding threshold, holding vehicle x for the arrival of vehicle y is appropriate.

In actual operation, vehicle locations in real time can be relayed to the control center and/or field supervisors. Estimates of arrival times and the number of through, transferring and arriving passengers can then be made for each vehicle at each stop. This data can be used to make an informed decision as to whether to hold or dispatch a "ready" vehicle at a transfer stop. In this research – due to the absence of such prediction capabilities - a simulation approach is adopted which draws from arrival time distributions input by the user for the preceding, current, and following vehicles on each line at each stop on the analysis corridor.

MAJOR FINDINGS

The operations planning and control models were tested on a hypothetical corridor to illustrate their applicability in coordinating transfers. This section briefly summarizes the major findings of this application. The sensitivity of the decision variables and the model results to exogenous factors including headways, bus arrival time variance, transfer volumes and values of passenger time are also explored.

Corridor Description

The corridor used for these applications is shown in Figure 4. The analysis on this corridor was restricted to a single direction to better explore the results of the transfer coordination models and to better illustrate their expected benefits. Line A runs between the two terminals, stops 1 and 7, making five intermediate stops along the way, only two of which are transfer stops. These two stops are also timepoints for which there are scheduled departure times. The analysis was carried out for off-peak period which previous studies showed to benefit the most from transfer coordination initiatives. The baseline values for the headways and other user-defined parameters were selected based on actual data from Chicago Transit Authority routes to represent realistic operating conditions. A symmetrical triangular distribution was used to represent bus arrival times, with buses assumed to arrive at a stop up to three minutes earlier or three minutes later than their scheduled departure times.

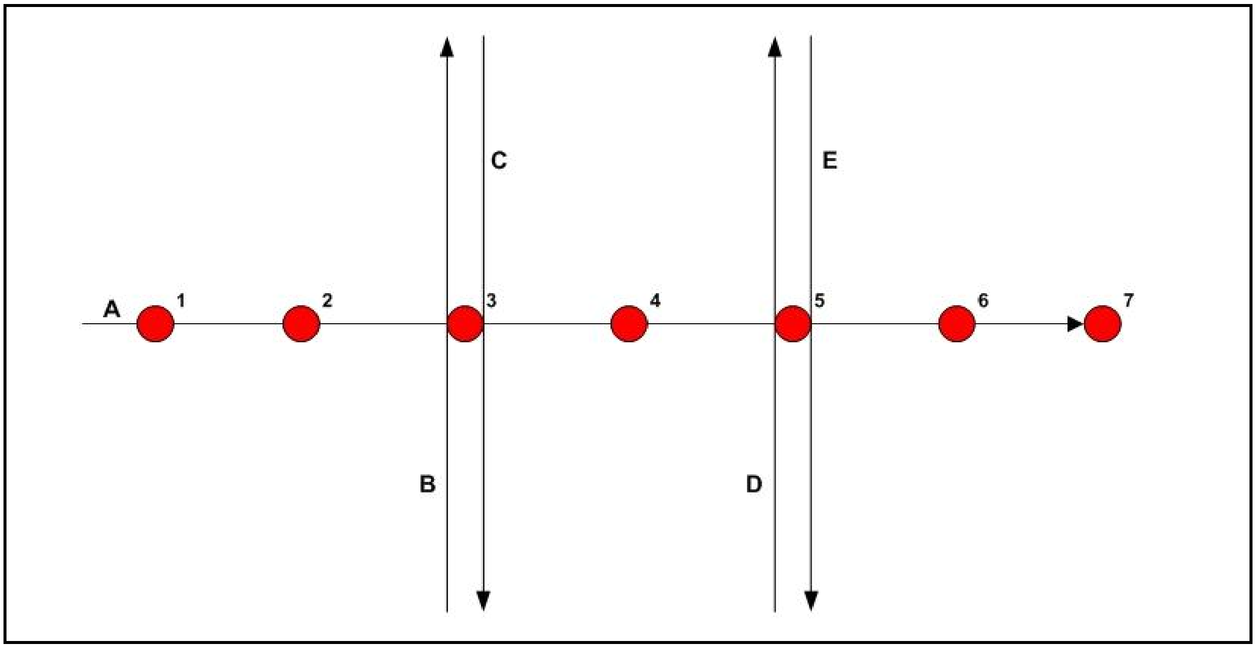

Figure 4 - Corridor Representation

Other baseline inputs are summarized in Table 2 below.

Table 2 - Baseline Application Parameters

	Line A	**Line B**	**Line C**	**Line D**	**Line E**
Headway (min)	10	20	20	10	10
Passenger Transfer Volume to Line A (pax/trip)		1	1	1	2
Passenger Transfer Volume from Line A (pax/trip)		1	1	2	1
	Stop 2	**Stop 3**	**Stop 4**	**Stop 5**	**Stop 6**
Through Passenger Volume on Line A (pax/trip)	30	25	21	18	10

Operations Planning Model Results

The results from the application of the operations planning model to the hypothetical route proved to be quite robust as the service timetable recommended by the model remained unchanged for the different sensitivity analyses. This timetable involved changing only the departure time from the terminal of all vehicles on the corridor with no addition of slack time to the schedule at the different timepoints. This is expected since slack is not usually introduced into the schedule when the service headway on the analysis route is short, the ratio of the number of transferring passengers to the number of through passengers at the transfer stops is low, and the standard deviations of vehicle arrival times at the transfer stops are high. Figure 5 shows the effect of changing the departure time from the terminal of Line A vehicles on the expected transfer waiting time per trip on that line for the two transferring passenger types: T1, passengers transferring to Line A, and T2, passengers transferring from Line A. As the offset time increases from 0 to 9 minutes, the expected transfer waiting time per trip changes. The minimum and maximum expected transfer waiting times per trip experienced by T2 passengers are higher than those experienced by T1 passengers due to the higher headways on lines C and D. Since the headway on line A is only ten minutes, vehicles arrive more frequently on this line and T1 passengers experience – on average – shorter expected transfer waiting times.

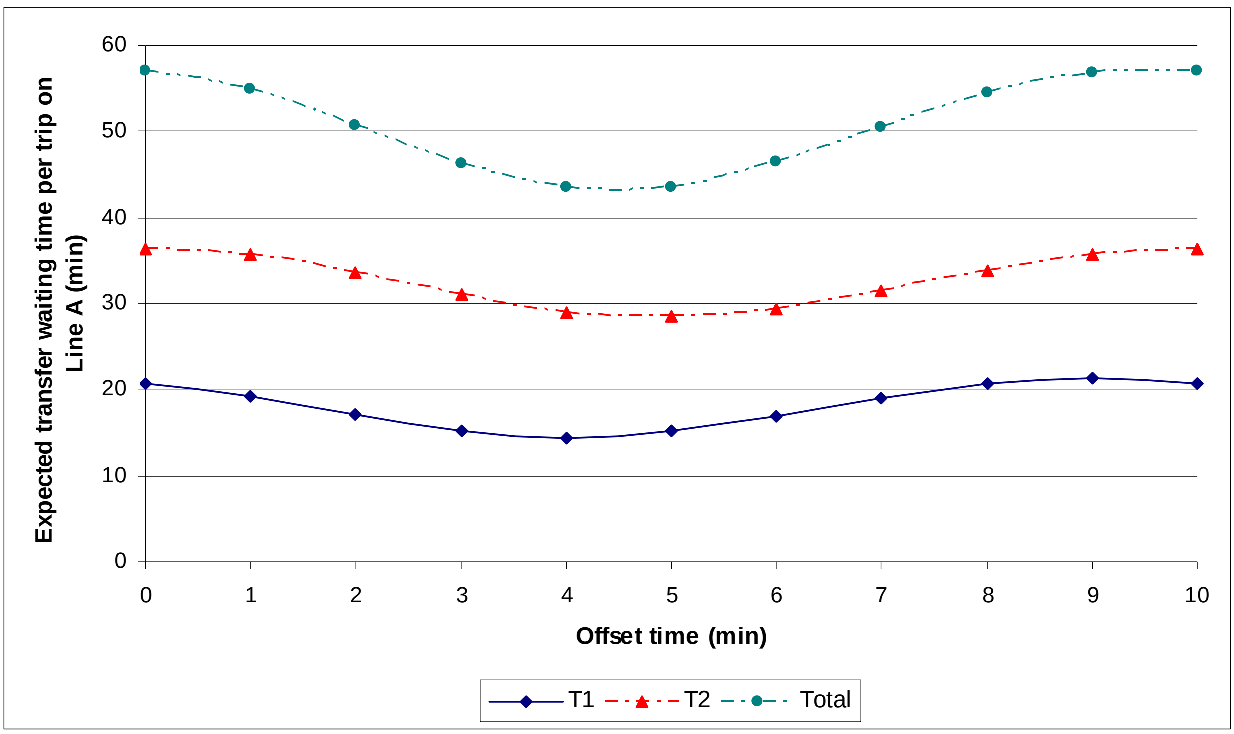

Figure 5 – Expected Transfer Waiting Time per Trip on Line A vs. Offset Time

Sensitivity analyses showed that there are very few benefits from coordinating transfers when the headway on the analysis corridor is short because short headways usually imply short expected transfer waiting times. However, as the headway on the analysis corridor increases, the benefits from transfer coordination increase substantially. At long headways, transfer coordination becomes an effective strategy since it can reduce the expected waiting times of both T1 and T2 passengers. Both passenger types suffer long expected waiting times, on average, if the arrival times of their connecting vehicles do not allow good connections. A similar trend is observed when the variance of vehicle arrival times is altered. The benefits accruing from better scheduling increase as the variability in vehicle arrival times decreases. In fact, the total expected transfer waiting time per trip on line A was reduced by approximately 48 percent when vehicle arrival times were assumed deterministic in the system. No change in the total expected transfer waiting time was observed, however, when the transfer demand to the analysis corridor was increased or the out-of-vehicle to in-vehicle time ratio was altered in the range 1 – 2. The same timetable was still recommended by the model.

A basic premise behind the operations planning model is that it only changes the schedule on the selected corridor while assuming that the schedules on its connecting routes stay the same. As such, it was expected that the recommended timetable would limit the actual benefits that could be obtained from better scheduling on the analysis corridor. Allowing the schedules of the connecting routes to also change verified the hypothesis since it resulted in greater expected benefits to all impacted passengers for the same vehicle arrival time distributions. The benefits from adopting a network approach to schedule synchronization became even more pronounced as the variability in vehicle arrival times decreased.

Operations Control Model Results

The operations control model was applied to the last transfer (stop 5) on the analysis corridor, with the impacts of any holding/dispatching decision studied both at that stop as well as at the one remaining downstream stop. The control model suggested that a vehicle arriving at stop 5 on line A had a 28 percent probability of being held for an incoming vehicle on a connecting line, assuming a zero minimum holding threshold. As the minimum holding threshold increases, the probability of holding a “ready” vehicle decreases. This is expected and agrees with Wong [2000] who showed that increasing the minimum holding threshold results in fewer but more substantial holds. The combination of low transfer demand to line A at stop 5 and frequent service on that line result in a small missed connection cost that cannot justify holding a vehicle if such an action must produce significant benefits.

It was also shown that, for a particular minimum holding threshold, the conditional probability of holding a “ready” vehicle decreases as the required holding time increases. This is expected because the longer the holding time, the greater the delays experienced by P2 and P3 passengers relative to the savings accrued by P1 and P4 passengers and hence the lower the net passenger-minutes saved. This conditional probability also decreases if the preceding headway of the vehicle is long and its following headway is short. This is expected since the vehicle to be held will have more through passengers and more boarders who will be negatively impacted by a hold. On the other hand, the missed connection cost of transferring passengers –

who are the main beneficiaries of a hold – will be small given the short following headway. As such the greatest benefits occur when the required holding time and the preceding headway of the vehicle on the destination line are short and its following headway is long.

At low transfer demand to the analysis corridor at the transfer stop under study, the probability of holding a "ready" vehicle increased only modestly with increase in the scheduled headway and/or with increase in the out-of-vehicle to in-vehicle time ratio. However, as the transfer demand increased, the probability increased significantly reaching approximately 100 percent when the number of transferring passengers was 30 percent more than the number of through passengers at that stop. The application of the control model to the transfer stop under study showed that transfer demand is a major driving factor behind any holding recommendation because it affects the missed connection cost which, at high transfer volumes, becomes the major part of the total cost.

As discussed earlier, the operations control model was tested and applied separately from the planning model in this research. However, the impact of adopting a sequential approach to transfer coordination which involved optimizing offset and slack times beforehand was also investigated. Results showed that the "ready" vehicle was dispatched immediately 91.8 percent of the time since its expected departure time from the transfer stop – under the new service timetable – now allowed transferring passengers to complete their connection. For the hypothetical corridor, it was shown that as the variance of the vehicle arrival times decreases, a "ready" vehicle will never be held at the transfer stop under study since the recommended operations plan will have already optimized the transfer performance along the corridor.

CONCLUSION

This paper presented operations planning and control models developed to improve connectivity on a transit corridor with multiple transfer stops. The operations planning model involved the simultaneous application of two planning strategies: changing the terminal departure time and inserting slack time. The operations control model determined the conditions under which holding a vehicle at a transfer stop for an incoming connecting vehicle is an appropriate strategy. The results of testing both models on a hypothetical corridor were also explored, along with the sensitivity of the decision variables and the model results to exogenous factors including headways, bus arrival time variance, transfer volumes and values of passenger time.

In this research, schedule coordination was applied at the route level. A new service timetable was recommended for the analysis corridor while the schedules on all the connecting routes were assumed constant. Results of the sensitivity analyses showed that greater benefits are expected if transfers are coordinated at the network level. Adopting a network approach is expected to increase the complexity of the planning model significantly because the schedules on all the routes now need to be optimized simultaneously. However, it is likely that a successful implementation of a network-wide schedule coordination system would produce significant passenger benefits and would have important applications to transit operations on the whole. It would be interesting to apply such a network approach to the operations control model as well.

Finally, the approach presented for coordinating transfers involved the separate and independent application of the operations planning and operations control models. A different approach that might yield greater overall benefits could be the simultaneous application of both models whereby control guidelines affect and feed into the recommended operations plan.

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