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A tool for dynamically predicting incident durations, secondary incident occurrence, and incident delays

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Abstract:

Understanding the characteristics of incidents can help decision-makers select better operational strategies. Using roadway inventory and traffic incident data, provided by the Hampton Roads Traffic Operations Center (TOC), traffic incidents were analyzed. Using the data, a practical online prediction tool (DSD-Duration-Secondary incident-Delay) was developed based on statistical models for incident duration, secondary incident occurrence, and associated delays. The tool can dynamically predict the duration of an incident (given that one has occurred), the chances of secondary incidents, and associated delays in real-time. The tool relies on available inputs about the roadway conditions, and incident information, e.g., location, time of day, and weather conditions. The tool can aid incident management by generating information about primary and secondary incidents and help effectively assign incident management resources.

INTRODUCTION

Traffic incidents are estimated to cause between 30 to 50 percent of the congestion problems on urban roadways (Skabardonis et al. 1995, Ozbay 1999, Kwon et al. 2006). They are associated with safety, energy, and environmental problems, such as blocking lanes or a shoulder, causing reduction of roadway capacity and creating congestion. Incident-induced queues can increase the potential for additional incidents, referred to as secondary incidents. Such incidents increase the time needed to return the traffic to normal flow. Based on past literatures, secondary incidents can occur in 3 to 20 percent of the cases after an initial incident (Moore et al. 2004, Hirunyanitiwattana and Mattingly 2006). While some secondary incidents can be relatively minor (e.g., fender benders or

vehicles running out of fuel), others can be more severe in terms of their congestion and safety impacts.

The primary and secondary incidents impose substantial travel uncertainty and associated costs on highway users. However, there is no practical tool to help traffic management centers predict how long the incident will last given the incident has occurred, the chances of secondary incident and the traffic delay caused by the incident. The main objective of this paper is to develop a methodology to dynamically predict incident duration, probability of secondary incident occurrence and associated delay.

LITERATURE REVIEW AND SYNTHESIS

Incident durations: associations with spatial, temporal, and operational factors

In general, the incident duration is associated with incident characteristics, temporal characteristics, environmental effects, geographic information, and operational factors. Variables that are positively associated with longer incident durations are: longer response times, accidents (as opposed to other types of incidents), lane blockage, adverse weather, more heavy vehicles involved in an incident (Khattak et al. 1995; Ozbay and Kachroo, 1999; Kim and Gang-Len Chang, 2008), injury or fatality, occurrence during peak hour (Nam and Mannering 1998; Ozbay and Kachroo, 1999; Kim and Gang-Len Chang, 2008), incident located farther away from a traffic operations center (partly due to longer response times), and more vehicles responding from various agencies (Kim and Gang-Len Chang, 2008). The research by Khattak et al. (2009) explored factors associated with longer incident durations and secondary incident occurrence. Some of the key factors associated with longer incident durations were accidents (as opposed to other types of incidents), freeway facility damage, more vehicles involved in incident, severe injuries, when incident affects the left shoulder or ramp, and longer lane closure times.

Secondary incident definition and associated factors

Raub (1997) defined secondary crashes using fixed temporal and spatial parameters. He assumed that secondary crashes are those occurring within 15 minutes of the clearance of an initial accident and within a distance of less than 1 mile. More than 15% of the crashes reported by police may be secondary in nature according to this study. Karlaftis et al. (1999) also adopted this fixed threshold to identify secondary crashes and found that more than 15% of all crashes in Indiana (Borman, N=741 over 5 years) might have resulted from an earlier incident. In more recent research, Chang et al. (2003) considered rubbernecking effects when identifying secondary incident and 6.8% of all incidents with lane blockage are identified as secondary incidents. Moore et al. (2004) extended the boundary to two hours and two miles, with conclusion that secondary accidents are considerably rarer events with lower frequencies (secondary crashes per primary crash ranged between 0.015 and 0.030). Hirunyanitiwattana et al. (2006) defined secondary crashes as any crash that results from the non-recurring congestion or emergency response associated with a primary crash, and he used 60minutes as temporal boundary, same direction no more than two miles upstream as spatial boundary. Zhan et al. (2008) used two miles upstream of the primary incident location (same direction) as spatial boundary and 15 minutes as temporal boundary. 7.9% were identified as primary incidents in their study.

All of above methodologies of classifying secondary accidents are using the static thresholds. Sun (2005, 2007) proposed an improved dynamic threshold methodology to extract secondary accidents from an incident fusion database. The dynamic spatial threshold is derived from a master incident progression curve. The analysis shows that the static and dynamic methods can differ by over 30% in terms of identifying secondary incidents. Also, Zhan et al. (2009) developed a method to identify secondary incidents based on estimating the maximum queue length and the associated queue recovery time for incidents with lane blockages. 4.98% were identified as primary incidents, which are substantially lower than the 7.94% incidents identified by the static method in their previous study (Zhan, 2008).

Studies have found that various factors are associated with the occurrence of a secondary incident. The peak hour and weekdays are associated with more secondary incidents, and the clearance time is also associated with secondary incidents occurrence (Raub, 1997). In the study by Karlaftis et al. (1999), clearance time, season, vehicle type (car, semi) and lateral location are the most significant factors for higher secondary incident likelihood. Odds of a secondary crash increase by 2.8% for each minute the primary incident is not cleared. Chang (2003) stated that the likelihood of having secondary incidents increases consistently with the primary incident duration and congestion level based on statistical data. Hirunyanitiwattana (2006) found that secondary crashes occur more often during rush hour and rear end collision is the predominant secondary collision type, which accounts for about two thirds of all secondary crashes. He also found that the typical secondary crash on the State of California Highway System is a rear-end, property damage only crash on a greater than a four lane urban freeway that occurs during one of the peak periods and is caused by excessive speed. Zhan et al. (2008) identified five major factors influencing secondary incidents, which include the number of involved vehicles (in the primary incident), the number of lanes, the duration of primary incident, the time of day, and the primary vehicle rolling over. In a later paper, Zhan et al. (2009) found four factors were associated with likelihood of secondary crashes: primary incident type, primary incident lane blockage duration, time of day, and whether the incident occurred on northbound I-95. Khattak et al. (2009) demonstrated that primary incident duration and secondary incident occurrence are statistically interdependent and the key factors associated with higher occurrence of secondary incidents included longer primary incident duration, primary incident is a crash and occurs during peak hours, with more vehicles involved, and on higher AADT roadways.

Incident-induced delays

Generally accepted methods for delay calculations are: deterministic queuing (Moskowiz and Newman, 1963; Morales, 1987) and shock-wave analysis (Lighthill and Whitham, 1955; Richards, 1956, Wirashinghe, 1978,). Many studies calculated delays using either of these two methods. To check the interrelationship and consistency of these two models, Chow (1974), Rakha and Zhang (2005) conducted their investigations and both studies found the results from these methods to be identical if a unique flow-density relationship is applied in the shock-wave analysis. However, both methods are limited by static demands, which is unrealistic under peak hour or flow fluctuation situations. Khattak et al. (2004) used the FREEVAL model, which faithfully replicates the freeway facility methodology in Chapter 22 of the 2000 Highway Capacity Manual (HCM 2000)

to estimate incident induced delay for prioritizing and expanding freeway safety patrols service. But the above-mentioned methods do not consider dynamic route diversion that would be expected in practice, Al-Deek et al. (1995) proposed a loop-detector based method to estimate single incident or multiple incident induced delays on freeways by capturing traffic demand variation due to diversion of traffic. With the development of more sophisticated car-following and lane change models, microscopic simulation is a tool that can be easily used to estimate the incident-induced delays.

Few studies have examined the delay relating to the primary and secondary incident pairs. Only one recent study (Sun and Chilukuri 2005) uses accident data from Missouri, and focuses on analyzing the safety impacts. For incidents of various sizes, clearance times can have a relatively wide range (from a few minutes to several hours), depending on the incident type, number of vehicles involved, time of day, and response by various agencies. Zhang et al. (2009, 2010) used simulation methods showing that total delays substantially increase as the time gaps between primary and secondary incidents increase; and for those secondary incidents that end after their associated primary incidents, increasing the distance between the locations of primary and secondary incidents lessens the delays.

DATA USED

The incident database was provided by Hampton Roads Traffic Operations Center (HRTOC), located in Virginia Beach, VA. The data were vehicular based, covering the period from January 2004 to June 2007. For analysis, they were converted to incident-based records, i.e. incidents involving multiple vehicles were aggregated into only one record with a unique incident ID, and a new variable, the number of incident involved vehicles, was created. The HRTOC data are based primarily on Safety Service Patrol (SSP) records. SSP offer free assistance to motorists experiencing problems on freeways. They cover more than 100 miles, from Newport News to Virginia Beach, 24 hours a day, and 7 days a week. There are two main limitations of the HRTOC incident database: first, the exact positions of these archived incidents are not provided. The only available location information is the code of road segment (average 1 mile in length in HR) where the incident occurred. This creates difficulties in the secondary incident identification process since these segments are typically 1 mile in length. Therefore, the location of an incident was assigned randomly within the road segment in delay prediction. Secondly, the incident duration reported in the database is when the TOC staff opened/closed an “incident window”. This is not the “true” incident duration since SSPs typically need response time before arriving on an incident scene. We recognize that the incident duration data may have measurement error in the direction of short incidents since incidents may not be detected for some time after they occur and it may take 2 to 3 minutes on average (of course longer in some cases) to detect an incident, based on communication with incident managers at the HRTOC. Only incidents that occurred on freeways are considered; five major interstates freeway are considered: I-64, I-264, I-464, I-564 and I-664.

METHODOLOGY

Incident Duration prediction

We estimate Ordinary Least Squares (OLS) regression model for incident duration, using detection sources, vehicles count, etc. as predictors.

$$\text{Incident Duration} = \beta_0 + \beta_1 (\text{TOD}) + \beta_2 (\text{Weather}) + \beta_3 (\text{Location}) + \beta_4 (\text{AADT}) + \beta_5 (\text{Detection}) + \beta_6 (\text{Vehicles}) + \beta_7 (\text{Type}) + \beta_8 (\text{Laneclose}) + \beta_9 (\text{EMS}) + \beta_{10} (\text{Rt_shoulder}) + \beta_{11} (\text{Ramp}) + \beta_{12} (\text{Lf_shoulder}) + \varepsilon$$

Where:

Duration = Incident duration (minutes)

TOD = Time of day (1= Peak, 0 = Off-peak)

Weather = Bad weather or not (1= Bad, 0 otherwise)

Location= incident location (categorical variable)

AADT = Average annual daily traffic (per 1000 vehicles)

Detection = Incident detection source (categorical variable)

Vehicles = Number of vehicles involved

Type = Incident type (categorical variable)

Lanceclose = Whether lane closed or not (1=Yes. 0=No)

EMS = Emergency management service responded or not (1=Yes. 0=No)

Rt_shoulder = Right shoulder affected (1= Yes, 0 = No)

Ramp = Ramp affected (1= Yes, 0 = No)

Lf_shoulder = Left shoulder affected (1= Yes, 0 = No)

ε = the error term.

The prediction of incident duration can facilitate incident management, but most incident duration models have limited operational value since they require knowledge about all incident variables at the time of prediction. However, in most real-time situations, incident information such as the variables in the equation mentioned above cannot be obtained at the same time, instead, information is acquired sequentially. This makes it is difficult to reflect the time dimension in a model (Khattak, 1995), thus it is difficult to make dynamic predictions. Furthermore, the independent variables requested for the model are not always available. For instance, for a certain incident with few available details, operations managers may only know the weather conditions, incident location, time of day and detection source. An incident duration prediction model can be used, based on known incident information available at this time. But when more information about the incident becomes available, e.g., the incident type (accident or disabled, etc), and multiple vehicles involved, the incident duration needs to be updated. In some situations, the incident type, number of vehicles involved, etc., may be known at the outset. Therefore, an ideal dynamic prediction model must be capable of accepting various combinations of predictors and allow users to update the predictions when more variables become available.

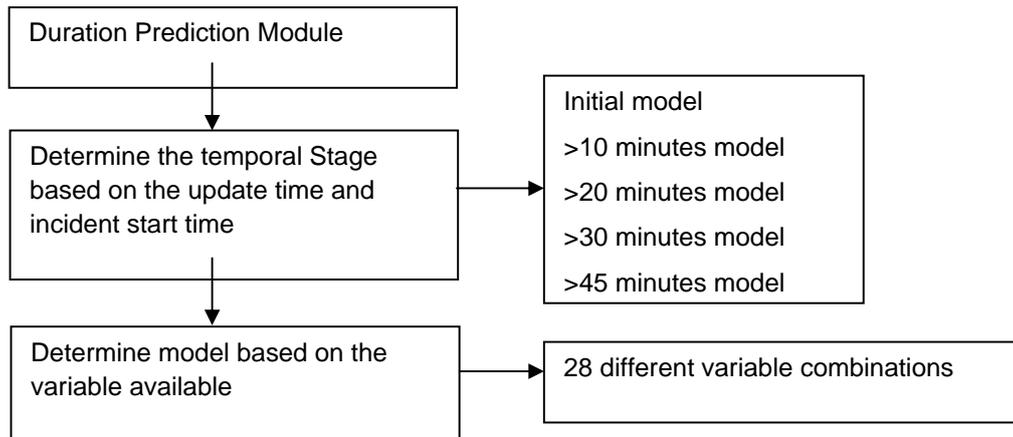


Figure 1: Methodology of Real-time Incident Duration Prediction

Dynamic incident duration prediction methodology is shown in Figure 1. To develop such operational models (for use in TOCs) which can be adaptive to different temporal and variable combinations, we used statistical techniques to estimate a set of models using different variable combinations. In this tool, there are five temporal stages (counting past the start of the incident, e.g., 10 or more minutes since the incident began). These include the initial incident occurrence stage, longer than 10 minutes, longer than 20 minutes, longer than 30 minutes, and longer than 45 minutes. For each stage, there are 28 models to deal with different variable combinations. The variable combinations are shown in Table 1. The tool uses an initial model from the beginning based on the available variables at that time. Within each temporal stage, if more information arrives, then the user can insert it and update the duration prediction. Even without additional information, the user can update the prediction as time goes by since the tool will automatically switch from the initial (or previous) model to the next stage model. For example, the model (> 10 minutes) is based on the historical incident data which excludes those incidents whose durations are less than 10 minutes.

Dynamic incident duration models predict incident duration more accurately since different time stages will support successively more information as an incident progresses. The methodology accounts for the dynamic nature of the information acquisition process at a TOC. Both Ordinary Least Squares and truncated regression models were tested. Truncated regression models were found to under-predict durations. Therefore, OLS models were embedded in the module. In this study, the dynamic models are based on data from the Hampton Roads area for 2006.

Secondary incident probability prediction

Secondary incidents were identified based their temporal and spatial proximity to primary incidents. If an incident occurs within the duration of the primary incident and within the queue created by it, then it is categorized as secondary incident (Khattak et al 2010). Secondary incidents can occur in the same or opposite directions, and a primary incident can be associated with multiple secondary incidents. A new variable was created in the incident dataset, identifying if the incident was a secondary or not.

Given all or partial independent variables (incident characteristics such as type, lane blockage, truck involved, road geometry and traffic information), the possibility of secondary incident is estimated from the binary Logit model (Khattak et al. 2010). The dynamic model provides the probability of a secondary incident occurring when a limited set of variables are available. Similarly, 28 models with the same data structure as in the Table 1 are used for secondary incident occurrence prediction.

Table 1 Variable combination for incident duration model

	Cons.	time	Weather	Location				AADT		Detection source				# of veh involve	Incident type			Lane close	EMS	Lane affected			
	_cons	tod	Bad weather	I264	I464	I564	I664	aadt	aadt_dy	cctv	vsp_radio	phone	other	vehicles	Dis.	Acci.	Aband.	laneclose	ems	Right shoulder	ramp	Left shoulder	
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Mandatory input
 Optional input
 Note: Cells with white color means that the corresponding variable is not included

Dynamic Incident delay prediction (Queuing model)

The main inputs to the dynamic incident delay prediction model are: 1) incident severity which is directly related to incident reduced capacity, 2) incident duration, which affects the length of time it takes to clear the incident, 3) arrival rate (traffic demand) and road geometry information such as the number of lanes. The predictions include: clearance time, total delay, and maximum queue length. Currently, a simple D/D/1 (deterministic queuing) model has been implemented in the tool.

The queue length at a given time and the remaining total delays on a specified freeway segment are illustrated in Figure 2. Traffic arrives at the incident location according to curve $A_c(t)$. The departure curve $D_c(t)$ shows the departure from the incident bottleneck. The departure flow rate is initially μ^* , the reduced capacity of the bottleneck and then after the incident is cleared at time T_c , is the restored capacity, u . The variables t_{n-1}, t_n represent the (n-1) th and nth time intervals from the incident start (the time interval usually is 15 minutes, representing the minimum period when a traffic arrival rate remains steady). The traffic arrival curve consists of a number of small time-dependent arrival rates at small time intervals.

The current queue length for a given time t_i can be expressed as:

$$\begin{aligned} q(t_i) &= q(t_{n-1}) + (t_i - t_{n-1})(\lambda_n - \mu^*) && \text{for } t_{n-1}, t_i < T_c \\ q(t_i) &= q(t_{n-1}) + (t_i - t_{n-1})(\lambda_n - \mu) && \text{for } t_{n-1}, t_i > T_c \end{aligned}$$

As long as all of the queue lengths for t_i, t_n, \dots, t_e are calculated, the remaining total delay for a given time t_i is the shaded area between t_i and T_e , (in the figure) which is the summation of small trapeziums between arrival and departure curve right after t_i . The areas of the first three trapeziums can be written as:

$$\begin{aligned} A_1 &= \frac{1}{2}(q(t_n) - q(t_i)) \times (t_n - t_i) \\ A_2 &= \frac{1}{2}(q(t_{n+1}) - q(t_n)) \times (t_{n+1} - t_n) \\ A_3 &= \frac{1}{2}(q(t_{n+2}) - q(t_{n+1})) \times (t_{n+2} - t_{n+1}) \\ &\dots \end{aligned}$$

Thus the remaining total delay at $t_i = \sum_{k=1} A_k$

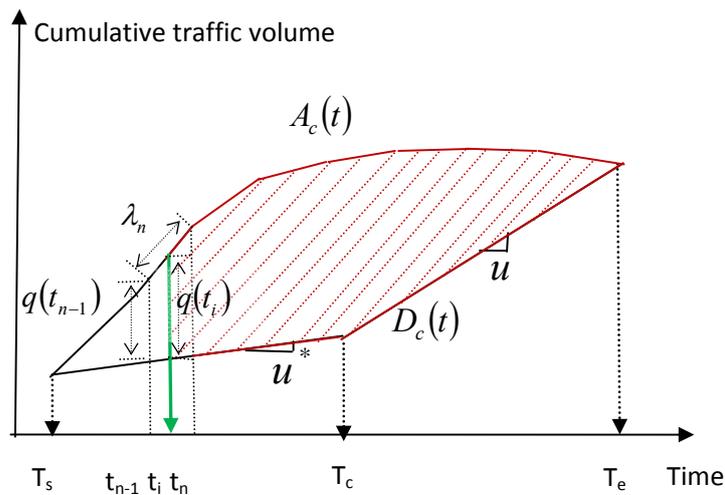


Figure 2: Illustration of the Queue Length and Remaining Total Delay at Time t_i .

ONLINE DSD PREDICTION TOOL

The online DSD (Duration-Secondary incident-Delay) prediction tool includes three major components: incident duration prediction module, secondary incident probability prediction module, and delay (and queue length) prediction module. Microsoft Visual Basic for Applications (VBA) in Excel was used to develop the tool. The framework of the tool is shown in Figure 3. Given that an incident has occurred, the key inputs of the tool include incident information such as start time, weather conditions, and incident location; additional inputs include type of incident (accident, disabled, abandoned, other), lanes closed, number of vehicles involved, detection sources, whether EMS (Emergency Medical Service) present, and the start and end time of lane closure. The key outputs include predicted incident duration, the chances of secondary incident and associated delays caused by the incident. The tool output window is shown in Figure 4.

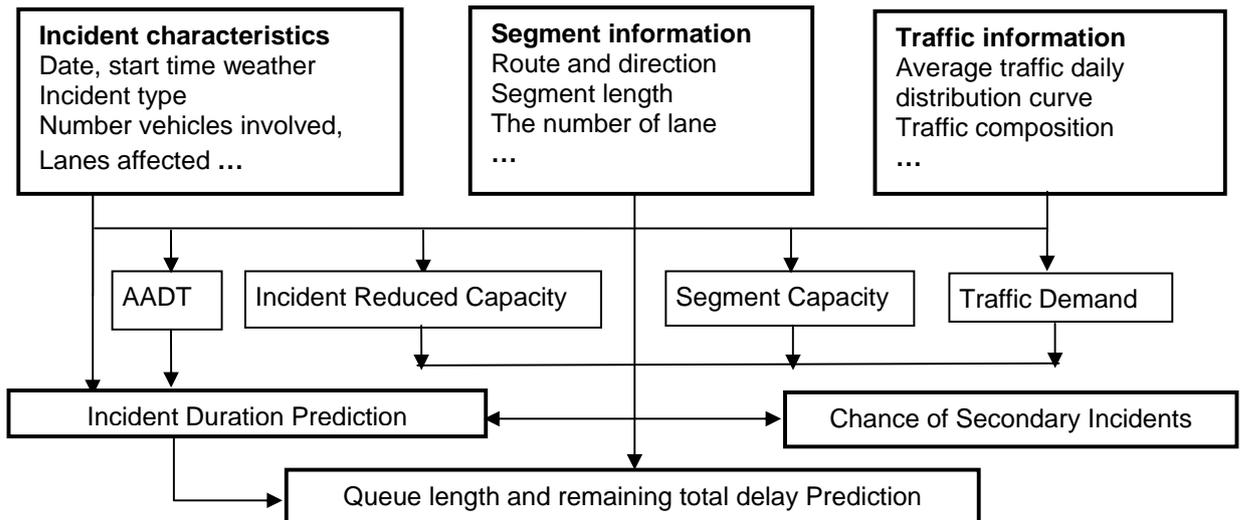


Figure 3 Illustration of prediction modules

The tool shows a duration prediction when the user presses the “update” button, which will start the duration prediction (shown in Figure 4). The remaining duration based on the current time, will be displayed in the predicted incident duration window. The tool will keep counting down the remaining duration until the next “update” is requested. This is typically when new information about the incident arrives.

The screenshot displays the 'Incident Online Prediction Tool (Hampton Roads) Beta' interface. It is divided into several sections:

- Input:**
 - Incident1 | + [Add Incident]
 - Date & Time:** Start Date: 10/30/2009, Start Time: 07:11:23 PM, Time Now: 07:13:20 PM
 - Define Current Weather at Incident Site:** Weather: Clear
 - Provide Incident Info:** ID: 2009-1030-7, Type: Accident, Num Vehs Involved: 1, Lane Blockage Situation: Middle Lane
 - Provide Segment Info:** Route: 64, Direction: East Bound, Seg. Code: E64-09, Length (mi): 2.17, # of Lane: 2, Segment Description: Militarv - 64/664 Marker
 - Verify Traffic Info:** AADT (veh): 37975, Demand (veh/hour): 2057, Capacity (veh/hour): 4162, Proportion of Capacity Available under Incident Condition (<= 1): 0.35
- Outputs:**
 - Predicted Incident Duration: 37.05 (Minutes)
 - Chance of Secondary Incident: 2.0 (%)
 - Total Delay Remaining: 147 (Veh-hour)
 - Predicted Queue Length: 1.54 (mile)
 - Affected Choke Point Name: [Empty]
 - UpdateTime: 07:13:14 PM
- Provide Response Info:**
 - Detection Source: SSP
 - Lane Closure Start: [Empty] REC
 - Lane Closure End: [Empty] REC
 - EMS Present:
 - No of Agencies Responding: [Empty]

Navigation buttons at the bottom include: <<Back, Next>>, Exit, and Update.

Figure 4 Predicted Results Output Display

Further statistics summary of the prediction results include four different graphs generated by the program (shown in Figure 5), which are respectively the average incident duration for each route, the predicted remaining total delay by different routes, the average queue lengths by different routes and the average probability of secondary incident in each routes. These graphs provide quantitative information to traffic managers about incident impacts based on existing conditions, and support their operational decisions regarding assigning response vehicles and other resources efficiently.

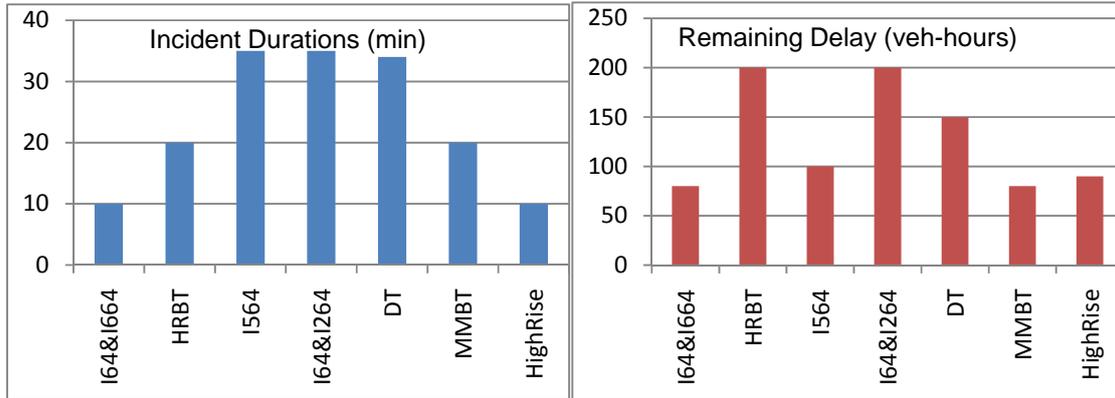
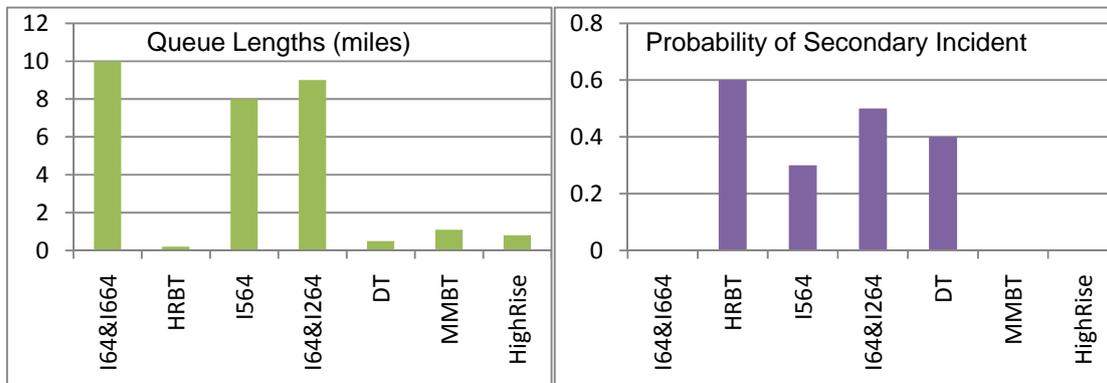


Figure 5(a) Predicted Incident Durations (b) Predicted Remaining Total Delay



(c) Predicted Queue Lengths at Choke Points (d) Predicted Secondary Incidents Probability

Note: These graphs are for demonstration only and do not reflect the actual conditions at these choke point locations.

SUMMARY OF FINDINGS

Based on freeway incident and roadway inventory data in Hampton Roads Area, the research team estimated incident duration and secondary incident occurrence models. The results were translated into an online tool that can be used in any traffic operations center to instantaneously predict important performance measures that include incident durations, chances of secondary incident occurrence, and delays. Specifically, the online DSD prediction tool can dynamically predict incident duration, the probability of secondary incident and the remaining delay based on the known information of the observed incident, which can aid in monitoring the frequency and durations of secondary incidents on freeways at the planning level. In an operational real-time setting, they can help develop effective incident management strategies to mitigate the impacts of secondary incidents. Although the tool is currently calibrated using the Hampton Roads incident data, the methodology is transferable to other regions.

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