# BUS ARRIVAL TIME ESTIMATION USING TRAFFIC SIGNAL DATABASE & SHOCKWAVE THEORY

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## ABSTRACT

Intersection delays are one of the major sources of uncertainty in real-time bus arrival time estimation. Only a few studies have included signal delay in estimating bus arrival time. In this paper, a new approach is developed to incorporate intersection delays in bus arrival time estimation. The approach integrates information from: 1) an existing signal database, 2) estimated speed of shockwaves created by a red signal and 3) bus speed available from GPS tracking. From the online access to a signal timing plan, the exact signal phasing of any fixed signalized intersection is known in real time. Hence, the signal phase and elapsed time of that phase can be determined. In addition, the developed framework assumes that traffic conditions as well as bus speed are known or can be determined. Estimated bus travel time between a bus stop and a signalized intersection is divided into two parts: 1) cruising travel time and 2) travel time in queue. Cruising travel time is defined as the bus travel time required to join the backward propagating shockwave from the traffic light. Travel time in queue is obtained from the estimated delay in the queue. Two illustrative numerical examples show that this approach can incorporate the intersection delay well in real-time bus arrival time estimation.

*Keywords: bus arrival time, existing signal database, shockwaves*

# 1. INTRODUCTION

Delays at signalized intersections are important factor to consider in estimating real time bus arrival time at a given bus stop location. A number of models have been developed so far to provide real time bus arrival information [1,2,4,6,10,13-15,18] based on real-time bus GPS tracking systems. In these models, delays due to a signalized intersection are incorporated indirectly. None of existing models consider intersection delays explicitly and the effect of queue formation due to a red signal. During the red signal, a backward (primary) shockwave which travels opposite to the intersection approach is formed and defines the rear end of the queue. During the next green phase, another backward shockwave, known as the recovery shockwave forms as well. The speed of the recovery shockwave is greater than the primary shockwave and after some time it catches the primary shockwave and restores the normal traffic condition. Based on online monitoring of prevailing traffic condition (i.e. flow, density) speed of a shockwave can be determined by applying classical shockwave theory to a pre-timed signal [7, 9]. When the bus departs from the bus stop, its real time speed can be estimated by integrating information from: 1) real-time GPS tracking and 2) the historical bus speed data. After obtaining bus speed and shockwave speed, the link bus travel time can be estimated by considering various traffic conditions that the bus might face.

# 2. LITERATURE REVIEW

Detailed reviews of different approaches to bus travel time estimation are discussed by Chien et al [2]. Most available models can be classified into the following groups:

- Regression [13]
- Analytical [8,10,15,16]
- Kalman Filters [4,6,14,18]
- Artificial Neural Networks (ANN) [2,6]
- Combination of ANN & Kalman Filters [1,12]

Regression models (both linear and nonlinear) are well suited for parameter estimation problems due to their simplicity and ease of use [13]. Kalman filtering approaches that have elegant mathematical representations (e.g. linear state-space equations) have been applied in the estimation of bus travel times. The Kalman filter approaches have potentials to incorporate traffic fluctuations with their time-dependent parameters (e.g. Kalman gain)[2]. Artificial Neural Networks (ANN) models can map the non linear relation among the dependent and independent variables.. Son et. al. [14] proposed the segmentation of link between bus stops at the signalized intersections; still the authors did not considered the effect of real time queue estimation. The combinations of ANN and Kalman Filter approaches have shown promising results in improving the estimation of bus travel time [1, 12]. However, none of these approaches have considered the delays due to a signalized intersection explicitly [4, 6, 18 2, 6]. In this paper we propose a method to estimate the bus delay at a signalized intersection, in real time. This delay can be incorporated with the existing bus travel time estimation methods to improve accuracy.

# 3. PROBLEM STATEMENT

Intersection delays are one of the major sources of uncertainty in real-time bus arrival time estimation. In this paper, a new approach is developed to incorporate intersection delays in bus arrival time estimation. During the red signal a backward shockwave is formed. This shockwave travels backward and defines the rear end of queue. When the traffic signal turns green a recovery shockwave is formed and travels backward too. The speed of recovery shockwave is greater than the backward shockwave and after some time it catches the primary backward shockwave and restores the normal traffic condition. When the bus departs from the bus stop, its speed can be estimated based on the archived speed data from historical GPS tracking. Details of the traffic signal timing plan are known for pre-timed signals and speed of shockwaves can be estimated based on prevailing traffic condition (i.e. flow, density). This study focuses solely on a pre-timed signal. This paper proposes a framework to integrate information from: 1) an existing signal database, 2) estimated speed of shockwaves created by a red signal and 3) bus speed available from GPS tracking. The major contribution of this study is to incorporate the delays created by traffic signal operation along the bus routes in bus travel time estimation.

# 4. PROPOSED APPROACH

The various possible bus trajectories for a pre-timed signalized intersection are illustrated in the Figures 1 and 2. Figure 1 illustrates the case of moderate congestion (i.e. no residual queue from previous cycle) and Figure 2 illustrates the case of oversaturated conditions where the traffic signal was not able to dissipate the queue completely.

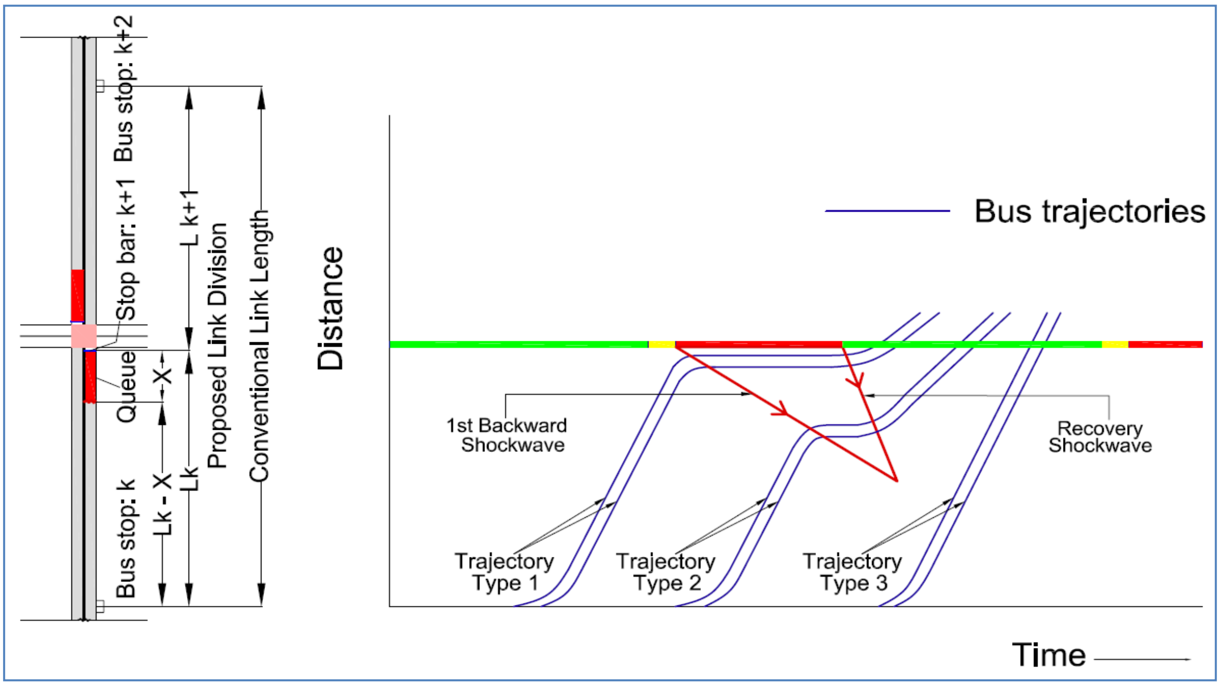


**Figure 1:** Time-space diagram without residual queue

As can be seen from Figure 1 the bus can follow different travel trajectories between bus stop *k* and stop line k+1. This bus trajectory depends on the departure time of the bus from the bus stop k as well as prevailing traffic conditions. Without any residual queue (the queue from the previous cycle is fully discharged in the last green phase), the bus can possibly arrive at the traffic signal k+1 during the red time period, so it will have to wait (Trajectory Type1 in Figure 1). Another possible trajectory represents the bus arrival at the signal just when the signal turns green. In that latter case, the bus may experience some delays due to the time needed to discharge the queue in front of the bus (Trajectory Type 2). Another possible trajectory represents the bus passing the traffic signal without any delay (Trajectory Type 3).

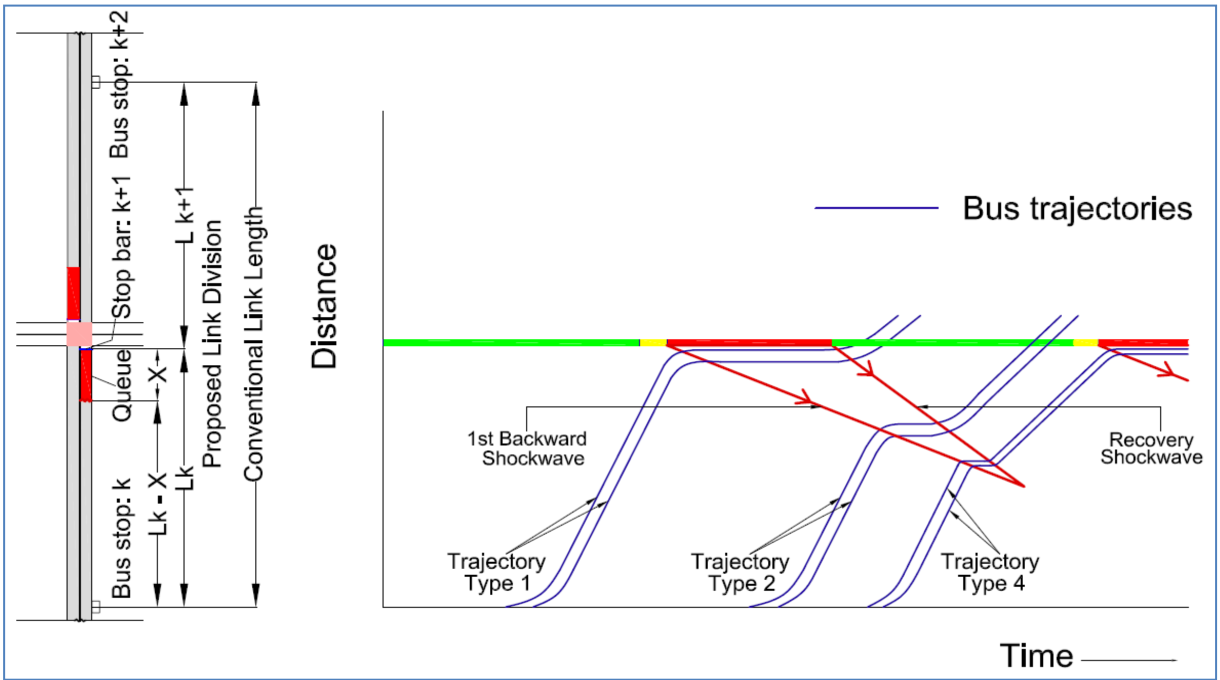


**Figure 2:** Time-space diagram with residual queue

Figure 2 illustrates the possible trajectories with the presence of a residual queue from a previous cycle. It shows another possible bus trajectory where the bus joins the intersection queue during the green phase but faces the back propagating shockwave and hence is not able to cross the intersection within that green phase (Trajectory Type 4).

The above possible trajectories show that the bus arrival times at downstream bus stop *k+2* are not similar and vary significantly depending on the prevailing traffic conditions and signal condition at the traffic signal *k+1.* By tracking busses with GPS, the real-time bus departure time from a bus stop k can be obtained. Prevailing traffic conditions at the intersection at a given time interval can be estimated from historical data. In addition, the exact signal phasing for pre-timed signals are also available and can be used to obtain the exact signal condition corresponding to a given bus departure time from a bus stop. These different sources of information can be integrated to estimate the bus trajectories thus bus travel time. Cruising bus speeds as well as speed of shockwaves are estimated for the link k after evaluating the state of downstream signalized intersection. The estimation process can be divided into main 4 steps:

*Step 1: Estimation of bus speed*
Since the length of the road link is known, mean bus cruising speed can be estimated using the real-time as well as historical GPS speed track database.

*Step 2: Evaluation of traffic signal*
The exact signal phasing of pre-timed traffic signal at the downstream intersection can obtained on real-time. So the signal state corresponding to the bus departure time can be determined.

*Step 3: Estimation of shockwave speed*
Prevailing traffic condition (i.e. flow, density, speed) is obtained in real-time from advanced detector data. Thus, the speed of shockwaves (i.e. backward or forward) can be estimated from classical shockwave theory.

*Step 4: Estimation of bus link travel time*
After getting the estimated bus and shockwave speeds, link travel time of bus can be estimated as explained in Section 6.

## 5. DETERMINATION OF SHOCKWAVE SPEED

Traffic condition is mainly defined by the following three fundamental traffic parameters: space speed, density and flow. When these parameters change suddenly such as in the case of a red light or sudden increase in density, an edge or boundary is established that makes a distinction between two flow conditions. This boundary is called a shockwave in the context of traffic flow theory [3]. Wirasinghe [19] showed how individual and overall traffic delays at various types of bottlenecks could be estimated using shockwave analysis. The behavior of traffic shockwaves was demonstrated by Lighthill and Whitham [9]. Liu et al [11] estimated queue lengths by tracing the trajectory of shockwaves based on the continuum traffic flow theory. Traffic shockwave can be described by using flow-density (q-k) diagram. The backward shockwave, $V_2$ and the recovery shockwave, $V_3$ are shown in the Figure 3 and are explained next.

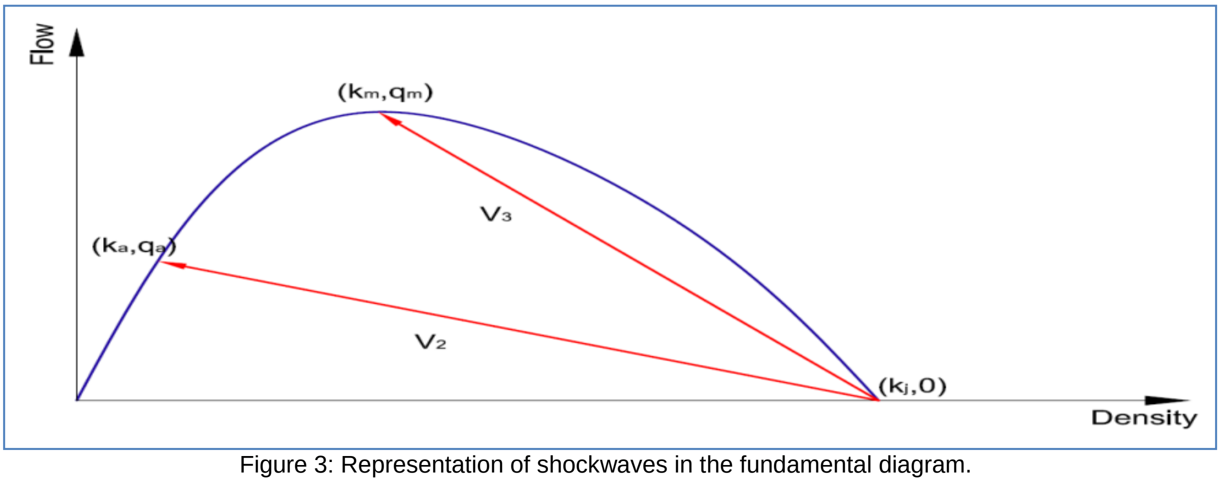


Figure 3: Representation of shockwaves in the fundamental diagram.

Let us denote, $q_1$, $k_1$, $u_1$ are the flow, density and velocity of the upstream region and $q_2$, $k_2$, $u_2$ are the flow, density and velocity of the downstream region. From Lighthill-Whitham-Richards (LWR) traffic flow model, the shockwave velocity can be written as below:

$$V = \frac{\Delta q}{\Delta k} = \frac{q_2 - q_1}{k_2 - k_1}$$

Garber, N.J. and Hoel, L.A. (7) suggested a more simplified formula to estimate shockwave speed for the traffic approaching a red light. Backward shockwave speed is estimated as: $V_2 = V_f \eta_1$ and recovery shockwave is estimated as: $V_3 = V_f$, where $V_f$ is free flow speed and $\eta_1$ is the ratio of upstream density and jam density. Thus, when flows and densities are known, shockwave velocities can be determined.

# 6. MODEL DESCRIPTION

Travel time between a bus stop and a signalized intersection is divided into two parts:

- Cruising travel time of bus, $T_{c,k}$ which is defined as the travel time of a bus to reach the queue, i.e. time required to travel the path ($L_k$ – X) as indicated in Figure 4,
- Travel time in queue, $T_{d,k}$ which is defined as the estimated travel time in the queue i.e. time required to travel the distance X as indicated in Figure 4

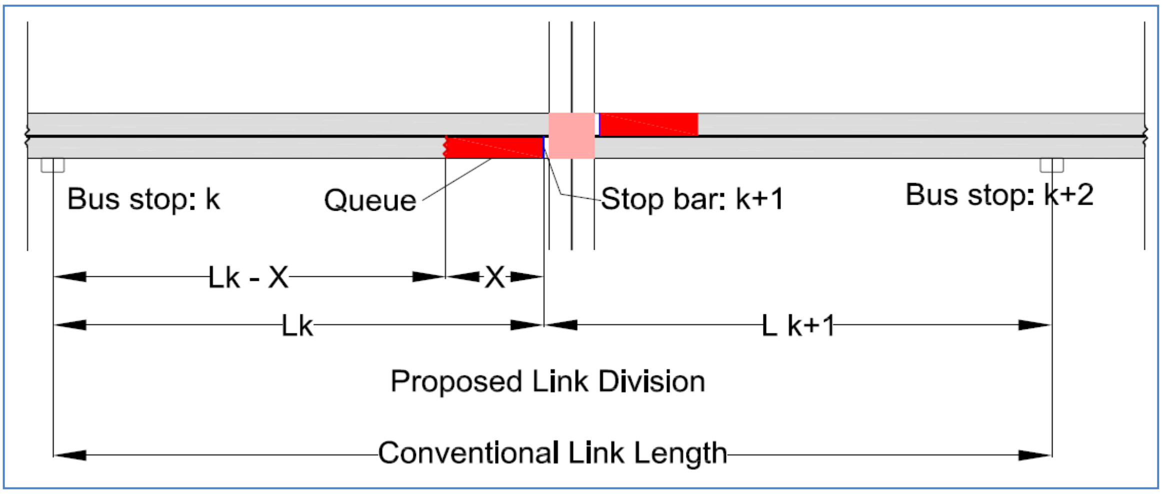


Figure 4: Layout of hypothetical road link.

Estimation of bus link travel time for link, k+1 can be directly determined from the estimated speed of the bus on this link where the link travel time is the link length divided by the bus speed. The detailed procedure for determination of $T_{c,k}$ and $T_{d,k}$ for the link, k are discussed below. In what follows, bus speed is denoted as, $V_{1,i}$ , backward shockwave speed due to red signal is denoted as $V_{2,i}$ and recovery shockwave speed due to green signal is denoted as $V_{3,i}$. For a moderate congestion condition i.e. no residual queue at a signalized intersection, these speeds are assumed to be known. In this analysis, green signal time includes the green and yellow phases and the aforementioned three speeds are assumed to be constant for simplicity. When a bus departs from a bus stop, the signal of the downstream intersection may be either red or green. If the signal is red, there will be a moving backward shockwave towards the bus and if the bus stop is close enough, the bus can meet this shockwave within this red phase but if bus stop is far away from the intersection, the bus may take more than one cycle to cross the intersection. The schematic diagram of estimation of link travel time is shown in the following figure:

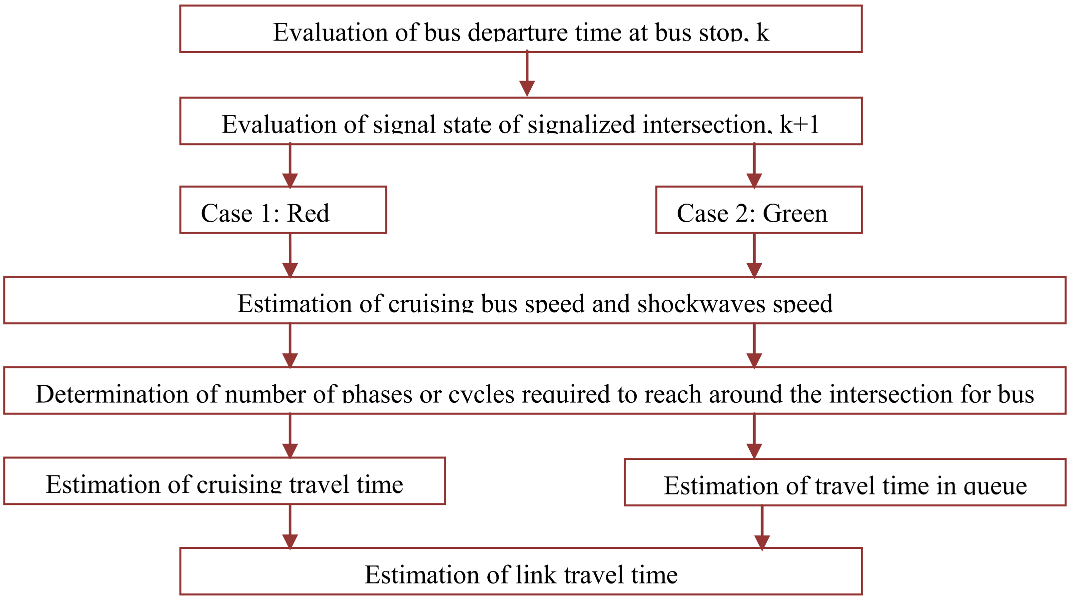


Figure 5: Schematic diagram of bus link travel time estimation

As can be seen in Figure 5, in this approach, the 2 possible cases of signal state are examined:

- Case 1: downstream signal is red when the bus leaves the upstream bus stop and
- Case 2: downstream signal is green when the bus leaves the upstream bus stop

As mentioned previously, the required travel time for the bus to cross the intersection is computed based on: estimated bus speed, 2) estimated shockwave speed and 3) prevailing signal conditions.

## Case 1: Traffic Signal is Red at Downstream Intersection k+1 when the Bus Departs from Bus Stop k.

Duration of Red phase = $R_i$ , Total Green time Duration = $G_i$, Elapsed time (i.e. time from start of red) = $E_i$, Remaining time for the signal to turn green= ($R_i$-$E_i$), Distance between bus stop and stop line = $L_k$ (Figure 4). For both cases, the number of phases or cycles required for the bus to reach the downstream intersection is first determined then the link travel time of bus is estimated. This will be achieved by applying the following steps:

*Step 1:*

This step checks whether the bus will be able to arrive at the intersection (k+1) within the current red phase (phase i). This will mainly depends on: 1) the link length, 2) bus speed, 3) shockwave speed and 4) duration of red phase. As indicated in Figure 6 below, the queue length created by backward shockwave $V_{2,i}$ in elapsed time, $E_i$ is $V_{2i}$ *$E_i$. Due to the propagating backward shockwave, this queue will be growing till the signal turn greens. Thus, the additional queue length till the signal turns green is $V_{2i}$ *($R_i$-$E_i$). The bus is travelling at a speed of $V_{1,i}$ when leaving the upstream bus stop. The distance travelled by bus in time interval ($R_i$-$E_i$) is $V_{1i}$*($R_i$-$E_i$). If the summation of these distances is equal or greater than the link distance, $L_k$ then the bus will join the queue within this phase; otherwise this bus will not be able to cross the intersection within this red phase. This criterion can be written mathematically as below:

$$\text{Checking whether, } V_{2i}*E_i + V_{2i}*(R_i-E_i) + V_{1i}*(R_i-E_i) \geq L_k \quad \text{----- (1)}$$

If the summation of the left side of Eqn.1 is greater than the right side, the bus will join the queue within the current red phase; otherwise we have to check again for the next green phase as illustrated in step 2. Let's denote $t_{i,}$ as the time needed for the bus to join the intersection queue as indicated in Figure 6:

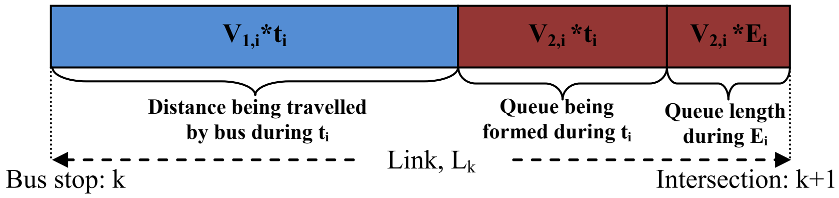


Figure 6: Schematic diagram of road link, Lk

If the bus meets the back propagating shockwave during the first red phase, implying that the summation of the three distances indicated in Figure 6 is equal to link distance, $L_k$. this can be written mathematically as follows:

$$V_{2,i}*E_i + V_{2,i}*t_i + V_{1,i}*t_i = L_k \quad \text{----- (2)}$$

$$\Rightarrow t_i = \frac{L_k - V_{2,i}*E_i}{V_{1.i} + V_{2,i}}$$

Since during time period $t_i$, the bus will not face any intersection queue, **Cruising travel time, $T_{c,k}$ is equal to $t_i$.** The total queue length in front of bus (i.e. red shaded area in Figure 6) is estimated as $V_{2,i}$ *$E_i$ + $V_{2,i}$ *$t_i$. If average spacing of vehicles is $S_v$, then number of vehicles, $N_{k,i}$ in front of bus can be estimated as below:

$$N_{k,i} = \frac{V_{2,i}*E_i + V_{2,i}*t_i}{S_v}$$

After joining the queue at the downstream intersection, the bus will remain stopped till the start of next green phase. The bus waiting time within red signal time is computed as:[$R_i$ – ($E_i$+$t_i$)]. When the traffic signal turns green, the queue in front of bus will start to dissipate at saturation flow rate. The time required to dissipate the queue in front of bus is [$N_{k,i}$/ $q_s$ (t)], where $q_s$ (t) is saturation flow rate. Bus **travel time in the queue is computed as:**

$$T_{d,k} = [R_i - (E_i + t_i)] + \frac{N_{k,i}}{q_s(t)}$$

*Step 2:*

If $V_{2i} *E_i + V_{2i} *(R_i-E_i) + V_{1i}*(R_i-E_i) < L_k$, the bus will not be able to arrive at the intersection (k+1) within the first red phase (phase i) as indicated in Figure 7, below:

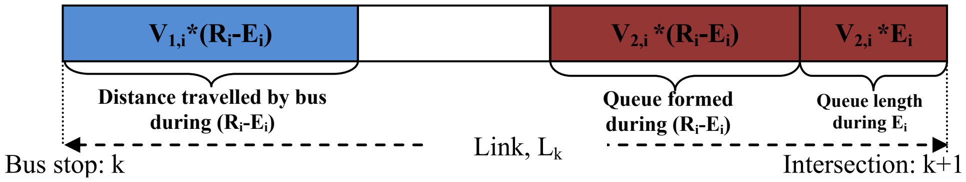


Figure 7: Schematic diagram of road link, $L_k$

Thus, in this case, the bus will not join the back of the queue during the current red phase. However, depending on the link length, bus speed, shockwave speed and duration of the next green phase, the bus might still be able to cross the (k+1) within this green phase (phase i).

If time required for the queue formed in the last red phase to be dissipated is $d_i$ , $V_{2,i(G)}$ is the backward shcokwave speed during this green phase and $V_{3,i}$ is the recovery shockwave speed due to green signal, $d_i$ can be computed as:

$$V_{3i}* d_i = V_{2i} *R_i + V_{2i(G)} * d_i$$

$$=> d_i = (V_{2i}*R_i )/( V_{3i} - V_{2i (G)})$$

During the time $d_i$, there will be some additional queue forming until the recovery backward shockwave catches the previous primary backward shockwave. Thus, the bus might stil be able to catch the back of the queue during $d_i$,this might happen if:

$$V_{1,i} *(R_i -Ei) + V_{1,i(G)} * d_i + V_{2,i} *R_i + V_{2,i(G)}*d_i -V_{3,i} *d_i \geq L_k \text{--------(3).}$$

Where, $V_{1,i(G)}$ is bus speed during this green phase. If Eqn.3 is satisfied, then the link diagram can be presented as illustrated in Figure 8 below:

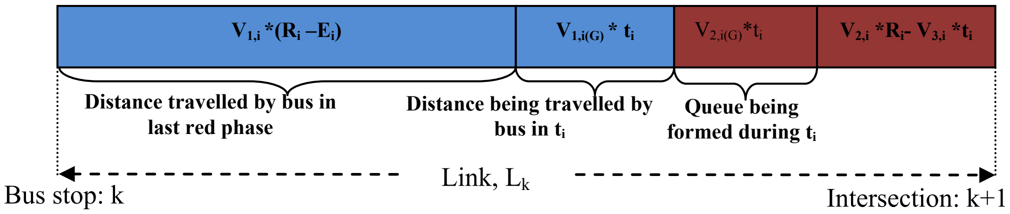


Figure 8: Schematic diagram of road link, $L_k$

At the start of the green phase (i.e. during $d_i$), there will be additional queue forming until the recovery backward shockwave catches the previous primary backward shockwave. If the bus comes at intersection in this duration (i.e. during $d_i$), it will face the queue and the time required,$t_i$ for the bus to meet the queue can be written as below:

$$V_{1,i} *(R_i -E_i) + V_{1,i(G)} * t_i + V_{2,i} *R_i + V_{2,i(G)}*t_i -V_{3,i} *t_i = L_k \text{ --------- (4)}$$

$$=> t_i = \frac{L_k - V_{1,i} *(R_i - E_i) - V_{2,i} * R_i}{V_{1.i(G)} + V_{2,i(G)} - V_{3,i}}$$

So, Cruising travel time: Tc,k = (Ri –Ei) + ti and Travel time in queue can be estimated as $T_{d,k} = \frac{N_{k,i}}{q_s(t)}$ , $N_{k,i} = \frac{V_{2,i} * R_i + V_{2,i(G)} * t_i - V_{3,i} * t_i}{S_v}$

If equation 3 is not satisfied, we have to go to step 3 as illustrated in what follows.

*Step 3:*

In this case, $V_{1,i} *(R_i - Ei) + V_{1,i(G)} * d_i + V_{2,i} *R_i + V_{2,i(G)}*d_i - V_{3,i} *d_i < L_k$. Thus, the bus will not join the back of the queue during this green phase. However, depending on the link length, bus speed, shockwave speed and duration of the green phase, the bus might still be able to cross the intersection during the green phase. This will happen if Eqn. 5 below is satisfied:

$$V_{1,i} *(R_i - Ei)+ V_{1,i(G)} * G_i > L_k \text{ --------(5)}$$

If $V_{1,i} *(R_i - Ei)+ V_{1,i(G)} * G_i > L_k$, then bus will be able to cross the intersection within this green phase without facing any intersection delay, the schematic link diagram will look like as Figure 9:

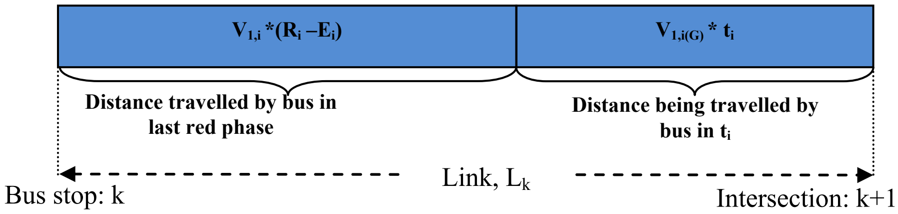


Figure 9: Schematic diagram of road link, Lk

As shown in Figure 9, in such situation (i.e. $V_{1,i} *(R_i - Ei)+ V_{1,i(G)} * G_i > L_k$) , bus will not face intersection delay. So, $V_{1i} *(R_i - E_i) + V_{1,i(G)} * t_i = L_k$, => $t_i = \frac{L_k - V_{1,i} *(R_i - E_i)}{V_{1.i(G)}}$

**Bus Cruising Travel time:** $Tc = (R_i - Ei) + ti$. In this case, there is no waiting time for bus in this intersection. So **travel time in queue:** $T_{d,k}$ **= 0 s**

If equation 5 is not satisfied then, the model will proceed to the next step 4.

*Step 4:*

If $V_{1,i} *(R_i - Ei)+ V_{1,i(G)} * G_i < L_k$, bus will not be able to cross the intersection within this green phase (i.e. phase i). Similar to step 1, Check for the next red phase whether $V_{1,i} *(R_i - Ei)+ V_{1,i(G)} * G_i + V_{1,i+1}*R_{i+1} + V_{2,i+1}*R_{i+1} \geq L_k$. If yes, then use $V_{1,i} *(R_i - Ei)+ V_{1,i(G)} * G_i + V_{1,i+1}* t_{i+1} + V_{2,i+1}* t_{i+1} = L_k$

=> $t_{i+1} = \frac{L_k - V_{1,i} *(R_i - E_i) - V_{1,i(G)} * G_i}{V_{1,i+1} + V_{2,i+1}}$ **Cruising Travel time:** $T_{c,k} = (R_i - E_i) + G_i + t_{i+1}$

**Travel time in queue:** $T_{d,k} = (R_{i+1} - t_{i+1}) + \frac{N_{k,i+1}}{q_s(t)}$ Where, $N_{k,i+1} = \frac{V_{2,i+1} * t_{i+1}}{S_v}$

*Step 5:*

If $V_{1,i} *(R_i - Ei)+ V_{1,i(G)} * G_i + V_{1,i+1}*R_{i+1} + V_{2,i+1}*R_{i+1} < L_k$, then bus will not be able to cross the intersection within this red phase (phase i+1). As discussed in step 2, if total Green time =

$G_{i+1}$ and time required to be dissipated the queue formed in the last red phase is $d_{i+1}$ then $V_{3,i+1}* d_{i+1} = V_{2,i+1} *R_{i+1} + V_{2,i+1(G)} * d_{i+1}$, => $d_{i+1} = (V_{2,i+1}*R_{i+1})/(V_{3,i+1} - V_{2,i+1(G)})$ and if $V_{1,i} *(R_i - Ei)+ V_{1,i(G)} * G_i + V_{1,i+1}*R_{i+1} + V_{1,i+1(G)}*d_{i+1} + V_{2,i+1}*R_{i+1} + V_{2,i+1(G)}*d_{i+1} - V_{3,i+1}*d_{i+1} > L_k$ then $V_{1,i} *(R_i - Ei)+ V_{1,i(G)} * G_i + V_{1,i+1}*R_{i+1} + V_{1,i+1(G)}*t_{i+1} + V_{2,i+1}*R_{i+1} + V_{2,i+1(G)}*t_{i+1} - V_{3,i+1}*t_{i+1} = L_k$ , so

$$t_{i+1} = \frac{L_k - V_{1,i} *(R_i - E_i) - V_{1,i(G)} * G_i - V_{1,i+1} * R_{i+1} - V_{2,i+1} * R_{i+1}}{V_{1.i+1(G)} + V_{2,i+1(G)} - V_{3,i+1}}$$

**Cruising travel time:** $T_{c,k} = (R_i - E_i) + G_i + R_{i+1} + t_{i+1}$ and **Travel time in queue:** $T_{d,k} = \frac{N_{k,i+1}}{q_s(t)}$

Where, $N_{k,i+1} = \frac{V_{2,i+1} * R_{i+1} + V_{2,i+1(G)} * t_{i+1} - V_{3,i+1} * t_{i+1}}{S_v}$

*Step 6:*

If $V_{1,i} *(R_i - Ei)+ V_{1,i(G)} * G_i + V_{1,i+1}*R_{i+1} + V_{1,i+1(G)}*d_{i+1} + V_{2,i+1}*R_{i+1} + V_{2,i+1(G)}*d_{i+1} - V_{3,i+1}*d_{i+1} < L_k$, bus will not face the queue in this green phase at least. Then it will be checked whether $V_{1,i} *(R_i - Ei) + V_{1,i(G)} * G_i + V_{1,i+1}*R_{i+1} + V_{2,i+1}*R_{i+1} + V_{1,i+1(G)}*G_{i+1} > L_k$. If yes, then $V_{1,i} *(R_i - Ei) + V_{1,i(G)} * G_i + V_{1,i+1}*R_{i+1} + V_{2,i+1}*R_{i+1} + V_{1,i+1(G)}*t_{i+1} = L_k$

=> $$t_{i+1} = \frac{L_k - V_{1,i} *(R_i - E_i) - V_{1,i(G)} * G_i - V_{1,i+1} * R_{i+1} - V_{2,i+1} * R_{i+1}}{V_{1.i+1(G)}}$$

**Cruising Travel time:** $T_{c,k} = (R_i - E_i) + G_i + R_{i+1} + t_{i+1}$ And there will be no waiting time for bus. So **travel time in queue:** $T_{d,k}$ **= 0 s**

*Step 7:*

If $V_{1,i} *(R_i - Ei)+ V_{1,i(G)} * G_i + V_{1,i+1}*R_{i+1} + V_{2,i+1}*R_{i+1} + V_{1,i+1(G)}*G_{i+1} < L_k$, bus will not be able to cross the intersection within this green phase. And the above procedure will be repeated for the next signal phases until it will get the solution.

## Case 2: Traffic Signal is Green at Downstream Intersection k+1 when the Bus Departs from Bus Stop k.

Total Green time = $G_i$, Elapsed time = $E_i$, Remaining time = $(G_i - E_i)$. As discussed for Case 1, Step 2, if the time required to dissipate the queue formed in the last red phase is $d_i$ then $V_{3,i}*d_i = V_{2,i} *R_i + V_{2,i(G)} *d_i$ => $d_i = (V_{2,i} *R_i)/(V_{3,i} - V_{2,i(G)})$

If the distance travelled by the bus during the remaining time of the green phase is greater than $L_k$, and if the Bus is not joining the back of the queue, the bus will not be delayed due to the queue and will only be travelling at the cruising speed. This situation will occur if $V_{1i} *(G_i - Ei) > L_k$ and $E_i > d_i$. Thus, the bus travel time on link $L_k$ can be determined as $t_i = L_k / V_{1i}$. Thus, **Cruising Travel time:** $T_{c,k} = t_i$ and waiting time in queue will be zero, so **travel time**

**in queue:** $T_{d,k}$ **= 0 s**. However, if the bus joins the back of the queue (i.e. if $E_i < d_i$) during this green time, then the bus will be travelling at 2 different speeds: cruising speed and speed in the queue. The time required to catch the primary backward shockwave by recovery shockwave is denoted as ($d_i$-$E_i$) and time required to meet the bus and the 1st backward shockwave is $t_i$. Thus, the bus will join the back of the queue if the following equation holds:
$V_{1,i}*(d_i - E_i) + V_{2,i}*R_i + V_{2,i(G)}*E_i + V_{2,i(G)}*(d_i - E_i) – V_{3,i}*(d_i - E_i) > L_k$ --------(6)

The cruising travel time of bus, $T_{c,k}$ = ti is estimated as follows:
$V_{1,i}*t_i + V_{2,i}*R_i + V_{2,i(G)}*E_i + V_{2,i(G)}*t_i - V_{3,i}*t_i = L_k$ => $t_i = \frac{L_k - V_{2,i}*R_i - V_{2,i(G)}*E_i}{V_{1,i} + V_{2,i(G)} - V_{3,i}}$

**And travel time in queue:** $T_{d,k} = \frac{N_{k,i}}{q_s(t)}$ Where, $N_{k,i} = \frac{V_{2,i}*R_i + V_{2,i(G)}*t_i - V_{3,i}*t_i}{S_v}$

If Eqn.6 does not hold i.e. $V_{1,i}*(d_i - E_i) + V_{2,i}*R_i + V_{2,i(G)}*E_i + V_{2,i(G)}*(d_i - E_i) < L_k$, the bus will not face the queue in this green phase and then $V_{1i}$ *ti = $L_k$ => $t_i = L_k / V_{1i}$. **Cruising Travel time:** $T_{c,k} = t_i = L_k / V_{1i}$ and waiting time in queue will be zero. So **travel time in queue:** $T_{d,k}$ **= 0 s.** If $V_{1i}*(G_i - Ei) < L_k$ then bus will not be able to cross the intersection within this green phase and the model will proceed to next red phase and the aforementioned steps of Case 1 will be applied.

For each green phase, the length of residual queue can be estimated by deducting the total discharged vehicles during a given green phase from the total vehicles arrived in the last red phase. For the case of presence of residual queue, the main difference will be in checking the effective link length. Instead of $L_k$, the check will be applied to ($L_k$ - $Q_{R,i-1}$), where $Q_{R,i-1}$ is the residual queue. The queue due to next red signal will form behind this residual queue. As for example, if signal phase is red when the bus leaves the upstream bus stop, condition: $V_{2i}*E_i + V_{2i}*(R_i-E_i) + V_{1i}*(R_i-E_i) > (L_k - Q_{R,i-1})$ is checked and the procedure will be same as the previously described that determines cruising travel time. In estimation of travel time in queue, another additional check has to be performed, whether the bus will be able to pass the intersection with the remaining available green time of the current phase. If time required to discharge of the queue in front of bus is more than the green time, then the bus will not be able to pass the intersection in this green phase as well as in the next red phase which can be estimated by simple modification in the aforementioned equations.

# 7. MODEL ILLUSTRATION BY NUMERICAL EXAMPLES

For simplicity, it is assumed that every signal cycle starts with its red phase in a two phase signalized intersection. If a bus departs from bus stop, k at $i^{th}$ cycle of downstream intersection, let us denote the following:
Cycle Length, $C_i$ = 30 sec, Red Duration, $R_i$ = 15 sec, Green Duration, $G_i$ = 15 sec
Here, two numerical examples are shown to represent two important scenarios. Example 7.1 corresponds to short distance between bus stop and downstream signal and Example 7.2 represents the case where the distance between bus stop and downstream signal is long

and more than one signal cycles may be required to cross the intersection, k+1 from bus stop, k for bus.

**Example 7.1**
Link length (distance between bus stop & stop line), $L_k$ = 70 meter
Departure time, $D_k$ = 9.00.000

***Step 1:*** *Evaluate the departure time of bus and traffic signal condition of next intersection at that time.*
Let's assume the signal of downstream intersection, k+1 is red when bus departs from bus stop, k.
Elapsed time, $E_i$ = 5 sec
Remaining time = ($R_i$ - $E_i$) = (15-5) sec = 10 sec

***Step 2:*** *Estimation of required information (i.e. using Kalman Filter, ANN or other available methods):*
Bus speed, $V_{1,i}$ = 18 km/hr (known from GPS tracking database) = 18*0.278 m/s = 5.004 m/s
1st backward shockwave speed due to red signal, $V_{2,i}$ =8 km/hr =8*0.278 m/s = 2.224 m/s

***Step 3:*** *Determination of number of phases or cycles required to reach around the intersection for bus*
(1) **Check for this red phase, whether** $V_{2i}*E_i + V_{2i}*(R_i-E_i) + V_{1i}*(R_i-E_i) > L_k$
2.224*5 + 2.224*10 + 5.004 *10 = 83.4 m > 70 m, so bus will meet the queue within this red phase. So if time required meeting the bus and 1st backward shockwave is $t_i$ then it can be written as: $V_{1,i}*ti + V_{2,i}*E_i + V_{2,i}*ti = L_k$

$$t_i = \frac{L_k - V_{2,i} * E_i}{V_{1.i} + V_{2,i}} = \frac{70 - 2.224*5}{5.004 + 2.224} = 8.146 \sec$$

***Step 4:*** *Determination of link travel time*
**Cruising Travel time:** $T_{c,k} = t_i$ = 8.146 sec
**Travel time in queue:**

$$T_{d,k} = (R_i - E_i - t_i) + \frac{N_{k,i}}{q_s(t)} = (15-5-8.146) + \frac{4.87}{0.44} = 12.922s \qquad N_{k,i} = \frac{V_{2,i}*E_i + V_{2,i}*t_i}{S_v} = \frac{2.224*5 + 2.224*8.146}{6} = 4.87$$

***Link travel time, $T_k$ =*** $T_{c,k}$ **+** $T_{d,k}$ =8.146+12.922 ***= 21.068 sec***

**Example 7.2:**
Link length (distance between bus stop & stop line), $L_k$ = 250 meter

***Step 1:*** *Evaluate the departure time of bus and traffic signal condition of next intersection at that time.*
Let's assume the signal of downstream intersection, k+1 is red when bus departs from bus stop, k and elapsed time, $E_i$ = 5 sec and remaining time = ($R_i$ - $E_i$) = (15-5) sec = 10 sec

***Step 2:*** *Estimation of required information (i.e. using Kalman Filter, ANN or other available methods):*
Bus speed in red phase, $V_{1,i}$ = 18 km/hr = 18*0.278 m/s = 5.004 m/s
Bus speed in next green phase, $V_{1,i(G)}$ = 19 km/hr = 19*0.278 m/s = 5.282 m/s
1st backward shockwave speed due to red signal, $V_{2,i}$ =8 km/hr =8*0.278 m/s = 2.224 m/s
1st backward shockwave speed during the green phase, $V_{2,i(G)}$ =8 km/hr =8*0.278 m/s = 2.224 m/s
Recovery shockwave speed due to green signal, $V_{3,i}$ = 20 km/hr = 20*0.278 m/s = 5.56 m/s

***Step 3:*** *Determination of number of phases or cycles required to reach around the intersection for bus*
(1) **Check for this red phase, whether** $V_{2i} * E_i + V_{2i} * (Ri-Ei) + V_{1i} * (Ri-Ei) > L_k$
2.224*5 + 2.224*10 + 5.004 *10 = 83.4 m < 250 m, so bus will not be able to reach around the intersection within this red phase.

(2) **Check for the next green phase**
(i) Time required to be dissipated the queue formed in the last red phase = $d_i$
$V_{3i} * d_i = V_{2i} * R_i + V_{2,i(G)} * d_i$

$$d_i = \frac{V_{2,i} * R_i}{V_{3.i} - V_{2,i(G)}} = \frac{2.224*15}{5.56 - 2.224} = 10.000 \sec$$

(A) Check whether $V_{1,i} * (R_i - Ei) + V_{1,i(G)} * d_i + V_{2,i} * R_i + V_{2,i(G)} * d_i - V_{3i} * d_i > L_k$
=> 5.004*10 + 5.282*10 + 2.224*15 + 2.224*10 – 5.56*10= 102.86 m < 250m, so bus will not face the queue in this green phase at least.
(a) Check whether $V_{1,i} * (R_i - Ei) + V_{1,i(G)} * G_i > Lk$
=> 5.004*10 + 5.282*15 = 129.27 m < 250 m, bus will not be able to cross the intersection within this green phase.

(3) ***Check for the next red phase whether*** $V_{1,i} * (R_i - Ei) + V_{1,i(G)} * G_i + V_{1,i+1} * R_{i+1} + V_{2,i+1} * R_{i+1} > L_k$
**=>** 5.004*10 + 5.282*15 + 5.282*15 +2.502*15 = 241.86 m < 250 m, bus will not meet queue within this red phase

(4) ***Check for the next green phase***
(i) Time required to be dissipated the queue formed in the last red phase = $d_{i+1}$
$V_{3,i+1} * d_{i+1} = V_{2,i+1} * R_{i+1} + V_{2,i+1(G)} * d_{i+1}$

$$d_{i+1} = \frac{V_{2,i+1} * R_{i+1}}{V_{3,i+1} - V_{2,i+1(G)}} = \frac{2.502*15}{6.116 - 2.502} = 10.385 \sec$$

(A) Check whether $V_{1,i} * (R_i - Ei) + V_{1,i(G)} * G_i + V_{1,i+1} * R_{i+1} + V_{1,i+1(G)} * d_{i+1} + V_{2,i+1} * R_{i+1} + V_{2,i+1(G)} * d_{i+1} - V_{3,i+1} * d_{i+1} > L_k$
=> 5.004*10 + 5.282*15 + 5.282*15 +5.282*10.385 +2.502*15 + 2.502*10.385 – 6.116*10.385 = 263.35 m > 250m, so it can be written as below:
$V_{1,i} * (R_i - Ei) + V_{1,i(G)} * G_i + V_{1,i+1} * R_{i+1} + V_{1,i+1(G)} * t_{i+1} + V_{2,i+1} * R_{i+1} + V_{2,i+1(G)} * t_{i+1} - V_{3,i+1} * t_{i+1} = L_k$ or

$$t_{i+1} = \frac{L_k - V_{1,i} * (R_i - E_i) - V_{1,i(G)} * G_i - V_{1,i+1} * R_{i+1} - V_{2,i+1} * R_{i+1}}{V_{1.i+1(G)} + V_{2,i+1(G)} - V_{3,i+1}}$$
$$= \frac{250 - 5.004*10 - 5.282*15 - 5.282*15 - 2.502*15}{5.282 + 2.502 - 6.116} = 2.38\,\text{sec}$$

***Step 4:*** *Determination of link travel time*

**Cruising Travel time:** $T_{c,k}$ = ($R_i$ –Ei) +$G_i$ + $R_{i+1}$ + $t_{i+1}$ = 10 + 15 + 15 + 2.38 = 42.38 sec

**Travel time in queue:**

$$N_{k,i+1} = \frac{2.502*15 + 2.502*2.38 - 6.116*2.38}{6} = 4.82 \quad T_{d,k} = \frac{N_{k,i+1}}{q_s(t)} = \frac{4.82}{0.44} = 10.96\,\text{sec}$$

***Link travel time, T**k* **=** $T_{c,k}$ **+** $T_{d,k}$ = 42.38 +10.96 ***= 53.34 sec***

The aforementioned examples show that this approach can incorporate the intersection delay well in real-time bus arrival time estimation for short distance between bus stop and downstream signal as well as for the longer distance between bus stop and downstream signal.

# 8. CONCLUSIONS

Delays at signalized intersection are critical factors in the estimation of real-time bus arrival time. In this paper, a framework for incorporating this delay in the estimation of real-time bus travel time was proposed. This framework divides the estimation of link travel time into two components: 1) link cruising travel time estimation and 2) travel time needed to exit the queue at the intersection. Exact signal and traffic information from the signal database and surveillance system are assumed available online and integrated in this framework. The proposed framework is based on the analysis of all possible cases that the bus might encounter when travelling the distance between the bus stop and downstream signal. The approach is tested on two numerical examples. It is expected that bus delays due to presence of queues in signalized intersection can be addressed with reasonable accuracy in this model.

Future work in this research will expand on the case of actuated rather than just pre-timed signals. Future work will also incorporate Bus travel time on an arterial that considers the presence of signal progression which might reduce the effect of shockwave propagation. In addition, the framework could possibly be integrated with the online information on traffic condition provided by the MIST (Management Information Systems for Transportation) system available in more than half of the signalized intersections in Calgary.

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