ECONOMIES OF SCALE EFFECTS IN COMMODITY TRANSPORT MODELS: THE PARADIGM OF MODAL COMPETITION IN HINTERLANDS

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ABSTRACT

This paper develops an approach for mapping economies of scale and supply-demand interactions within the hierarchical transport modelling framework. The competition between different modes for hinterland transportation is studied: Truck transportation is assumed to be a homogeneous service regardless of who will carry out the task, whereas intermodal transport is seen as consisting of slightly different services.

The problem is formulated as a two stage choice problem using a nested-logit framework: The upper level deals with shippers’ choices between truck and intermodal transport. Relating to intermodal transport, shippers choose between different terminals. The market for terminal services is mapped as monopolistic competition. The terminals are located in space according to the Free-Economic-Energy approach.

Having reached an equilibrium in shippers’ choices and terminal location patterns, exogenous parameters are adjusted to influence shippers’ and terminal operators’ choices. Two effects occur. Firstly, economies of scale caused by a higher workload of the remaining terminals are likely to be realised. Secondly, the distance to the next terminal increases for the majority of consumers. It is analysed which effect will prevail and to what extent the remaining terminals will be able to absorb the former customers whenever an intermodal service is ceased.

Keywords: economies of scale, free economic energy, freight transport, dynamic demand supply interactions, dynamic transport modelling, meso logistics, colloidal structures, clusters, discrete choice theory

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INTRODUCTION

Economies of scale are important to be included in freight transport models since they represent the main drivers of freight companies for growing and to continuously offer new services. A port manager might want to increase its hinterland influence to attract more commodities, an inland terminal operator might want to increase the frequency of rails and barges to attract more customers to his infrastructure and a rail operator might want to increase the run of his rolling stock to give service to more commodities. For operators of logistics and transport systems it is crucial for their economic survival to attain a certain critical size. On the other hand, over investments should be avoided, too, because of the linked high fixed cost.

From these simple considerations we could describe the competition of transport logistics services as monopolistic competition. Two observations support this assumption: Firstly here are a few examples of transport network service providers making significant profits over a long period. Secondly, with today’s open markets, it is always possible that operators from the destination region or operating on alternative corridors begin to compete with existing service providers.

From the point of view of transport policy, the phenomenon of economies of scale in logistics system is also from a high relevance: On the one hand, public authorities are in charge of financing and constructing the macro logistics infrastructure (railway system, road system, components of the ports, intermodal terminals). But on the other hand, a political wish alone to establish a well-functioning port or intermodal transport system is not a guarantee that private operators will successfully provide the related transport services.

The sketched phenomena and problems occur, for instance, in the field of hinterland transportation. Two main transport modes compete with each other: “Green” intermodal transportation which fully depends on economies of scale (due to high capital costs required) and road haulage. Therefore, it is mandatory to establish the link between economies of scale and its typical characteristics. These characteristics are mainly: the existence of many equilibria and the incurred dynamic interactions.

Using the traditional tools of transport modelling, it is difficult to map the sketched competition situation in hinterland transport: Discrete choice modelling assumes given and fixed alternatives (their service quality might decrease with an increasing number of users). Research on logistics network design in a competitive environment normally assumes convex functions for the user cost (see, for instance, Nagurney (2009)).

A possible solution might be to extend a spatial computable general equilibrium (SCGE) model. However, these models are too exhaustive and depend totally on the volume of data. Another disadvantage for the purpose of this study is that SCGE fixes the location of customers and only their properties change without any kind of movement (e.g. customers reaching a terminal). In fact, until now, there has been a certain gap between the way of how SCGE are presenting economies of scale and transport modelling. Since the aim is to
include only the freight transportation sector, some basic principles of SCGE models – namely the mapping of economies of scale and the assumption of certain competition forms – can be considered in order to develop a tailored model for mapping modal competition.

Starting with the work of Krugman (1991) the so called new economic geography was established in order to explain and predict how economic activities and the structure of space influence each other. In the following years, several so-called spatial computable general equilibrium (SCGE) models such as RAEM3.0 and CGEurope have been set up in order to apply and extend the findings of Krugman. Furthermore, the main contribution of Krugman was to add the spatial patterns to the main findings of Dixit and Stiglitz (1977). Hence, it is important to consider the results of the latter authors and including the spatial dimension without going to deep in the inclusion of the whole economy.

The model explained in this paper deals with similar questions to the proposed in economic geography, but limits itself to the hinterland transportation sector of a seaport. It combines the concept of Free Economic Energy (based on Dixit and Stiglitz, 1977) with the well known nested logit approach developed by Ben-Akiva.

Interesting to know is the possibility to include economies of scale into the productions cost functions and make customers to bear a part of the “guilty” to join a given alternative in economic terms. In other words, customers are able to know their opportunities and risks when leaving an alternative and joining another one without an influence of the predetermined one.

In order to introduce the modal competition in hinterlands this paper has been structured as follows. After the present introduction, Section 2 is dedicated to the literature review supporting the principles of the spatial interactions between demand and supply under economies of scale. Section 3 introduced the model developed for this study including the mathematical deduction for the distribution of terminals, actors, regions and the universe, as well as the program developed for the same purposes. In Section 4 the results of the model are shown and briefly discussed. Finally, the conclusions on the suitability of the model for further applications as well as the main barriers for its construction and calibration are documented.

THEORETICAL FRAMEWORK

With services that only differ in their location the model reminds of the work of Hotelling (1929). In contrast to him, consumers do not only buy the services from the closest supplier due to the stochastic element of the discrete choice theory. Furthermore, truck transportation represents a mobile alternative that is not located at a certain place while intermodal alternatives do have a distinguished location depending on the density of demand.

The concept of Free Economic Energy was developed by Carrillo and Liedtke (2008). It combines the two driving forces in a market under monopolistic competition. On one hand,
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there are consumers who demand for many different varieties and on the other there are suppliers who want to realise economies of scale. The demand of the consumers is modelled in terms of entropy maximisation, according to the work of Wilson (1970). Together with the opposing force of producers tending to minimise their costs, they are combined in one formula:

$$ FEE = C - \frac{1}{2\alpha} S $$  \hspace{1cm} (1)

With $S$ denoting the entropy, $\alpha$ a heterogeneity parameter and $C$ the total costs for producing the good. Carrillo Murillo (2010) adds a spatial aspect to his thoughts, so that the distance between two adjacent intermodal terminals results in a minimisation of the FEE.

Afterwards the choice between shipping the container directly per truck or via one of the terminals located by the free economic energy is done through a nested logit model. The latter is a member of the group of discrete choice models. There, a consumer chooses exactly one item out of a set of several mutually exclusive items. Its choice depends on an observable and deterministic part $V$ and a stochastic part $\varepsilon$ resulting from incomplete information of the modeller. Together, they make up the utility function:

$$ U = V + \varepsilon $$  \hspace{1cm} (2)

according to which a consumer draws its decision. Because of the existence of a stochastic part, only the probability of choice can be obtained from a discrete choice model. This probability depends on the distribution of the values of $\varepsilon$. In the case of a multinomial logit model, the error terms are independently and identically Gumbel distributed. The probabilities of choice then, can be calculated according to Maier and Weiss (1990):

$$ p_i = \frac{\exp(\alpha \cdot V_i)}{\sum_{i=1}^{n} \exp(\alpha \cdot V_i)} $$  \hspace{1cm} (3)

With $\alpha$ denoting the scale parameter of the Gumbel distribution of the error term. If the error terms of some members of the choice set are correlated, these alternatives can be grouped into nests. Within each of these nests a multinomial logit submodel is applied. These submodels are linked by an expression called logsum or expected maximum utility (EMU). The latter is calculated by (e.g. Ben Akiva, 1985):

$$ EMU = \frac{1}{\alpha} \ln \sum_{j=1}^{n} \exp(\alpha V_j) $$  \hspace{1cm} (4)

On an upper level, another multinomial logit model is placed, using another parameter, say $\beta$, and the logsums of the respective nests act as a proxy for the utility functions of their members. So the probability of choosing item $i$ from nest $j$ is (e.g. Anderson et. al., 1992):

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The two parameters $\alpha$ and $\beta$ are a measure for the intragroup (in case of $\alpha$) and intergroup (in case of $\beta$) heterogeneity. The smaller they are, the less different the products or product groups in various nests will be considered to be by the consumer (Anderson and de Palma, 1991). The logsum is not only a link between two levels of decision but also a term expressing the desire for variety. If the number of alternatives contained in a nest increases, the logsum and thus, the probability of choice also increases. A single consumer can represent the behaviour of a whole group of decision makers by multiplying the respective probabilities of choice by the number $N$ of people the group consists of.

The model outlined below will apply a nested logit choice process with two levels of decision and two nests. This kind of model and the partial analysis coming along with are quite common in spatial economics. In a first step, a representative consumer has to choose the mode for shipping containers to the seaport and if the decision is in favour of intermodal transport, a terminal has to be chosen in a second step.

The nested structure has some links to the model of monopolistic competition as described by Dixit and Stiglitz (1977) and extended to a spatial dimension by Krugman (1991). In both researches, there is an economic sector considered as being perfectly competitive and the sector more in focus of research meets the requirement for being under monopolistic competition. In this case, the two sectors also exist, but the one to be assumed monopolistic competitive consists of firms supplying completely identical services. Here, the Armington’s assumption (1969) is made like in some models of the New Economic Geography (NEG). It is assumed, that goods, that are actually identical are considered to be different by consumers just because of their different places of origin. Trips starting at various terminals are seen as different goods because of the distance and the costs involved in order to reach the latter.

Another similarity to the NEG is the existence of different actors whose behaviour is influenced by their location. Different locations of commercial partners account for transport costs. In the models based on the findings of Krugman often the Iceberg approach of Samuelson (1954) is used for modelling these costs. Here, the question of transport costs not only arises in terms of how to model them, but also in terms of what to consider as transport costs when the good itself is made up of transportation. Combining this with the Armington’s assumption one could argue, that there are only transport costs for intermodal transport, as they are charged in addition to the basic service of bringing a container from the region of origin to the seaport, i.e. the costs for the feeder truck.

An important feature of the NEG is the application of constant elasticity of substitution (CES) utility functions. They have some similarities with the nested logit model, as shown for example by Anderson and de Palma (1987). However, there are some differences. As opposed to the spatial discrete choice model of Heikkinen (2003) the model applied in this
case does not know a budget constraint. This could be circumvented by defining only such choice sets, consumers can afford, but it prevents the model from having a closed structure like spatial computable general equilibrium models.

The main difference of this model to those of the New Economic Geography is that there are loose ends. The question where the consumers receive the money from to pay for the transport is not answered, as well as it remains unclear, what inputs the shippers use to produce their services. This narrows the possibilities of the actors to influence each other. A high density of terminals can not bring consumers to relocate towards a place from where the port is better accessible, for example.

Such loose ends are not uncommon in spatial discrete choice models (e.g. Malchow, Kanafani (2004)). A reason for this are problems resulting from the iceberg approach, such as the ones described by McCann (2005). This way of capture transport costs leads to either an imbalance in the mass balance or the flows of money, at least in the very basic models like the one of Krugman (1991). This shortcoming can be circumvented by establishing commodity or transport agents performing transport services (e.g. Bröcker (1998)). However, these agents cause additional complexity in calculating the results as well as in information gathering.

Another advantage of a partial analysis like the one chosen in this case is, that there is no need to model other agents than the ones that are involved directly. Dealing with the question of inputs, consumers and forwarders need for ordering or performing the service, could entail biases in the results due to incomplete or doubtful information.

METHODOLOGY

The examined scenario is set up as follows: A region is located at a distance $d_p$ from the next seaport. The region can approximately be modelled as a line of length $L$. Along this line, a certain demand for the shipment (i.e. containers) to the seaport exists and is represented by the demand density per unit length $\rho$. The shippers have two alternatives to get their containers to the port. The first one is that they are picked up by truck and then forwarded directly to the port, whereas the second alternative is that they are brought to an intermodal terminal where they are transshipped on a train and then forwarded to the port. Each shipper has to choose, which alternative to take.

It seems to be obvious, that a discrete choice model is applicable in this case. Shippers choose between different local forwarders and these ones have different preferences concerning the terminals and the operators. The services offered might differ in terms of frequency, connectivity and cost. Because of its convenience and as it was chosen in similar situations (de Palma, 1994; Gelhausen, 2006; etc.) a logit model is used. A consumer (shipper) has the deterministic (dis)utility function:

$$V_{\text{mode}} = C_{\text{mode}}$$

(6)
For the two modes available, equation (1) becomes:

\[ V_{\text{truck}} = c_T \cdot d_p \]

\[ V_{\text{intermodal}_j} = c_R + c_f \cdot d_{ij} + c_{ch} \quad i = 1, \ldots, n \quad j = 1, \ldots, m \]  

(7)

(8)

Where:
- \( c_T \): costs for truck transport per kilometer
- \( c_R \): costs for rail transport per container and trip
- \( d_p \): distance between the region and the seaport
- \( d_{ij} \): distance between shipper \( i \) and terminal \( j \)
- \( c_f \): costs for the transportation by feeder truck
- \( c_{ch} \): costs to change the transport mode, i.e. costs for reloading the container from the feeder truck to the train

From equation (8) it can be seen, that there are \( n \) different shippers and \( m \) terminals. As there is also a probabilistic part in the utility function, it might well be possible for a terminal to be chosen, although it is not the closest and thus cheapest one. These specifications are quite general and raise some questions:
1. How many terminals are there? If their number \( m \) has to be specified, what will be the optimal quantity?
2. If question number one is answered, the locations \( l_j \) of the terminals have to be determined. How is this done, and what is the objective in choosing the locations?
3. Nothing has been said about the location of the customers so far. However, the choice crucially depends on this location as it influences the distance to the terminals and hence the cost component \( c_f \cdot d_{ij} \)
4. The cost and thus disutility functions have not been fully specified up to now. Especially the components \( c_T \) and \( c_R \) have to be more detailed.

Question number four is perhaps the easiest one to answer and it entails some consequences influencing the answers to the other questions. As far as truck transportation is concerned, the cost function remains unchanged, \( c_T \) and \( d_p \) can be adjusted later if necessary. The former covers all related costs divided by the distance incurred. In this model, as well as in reality, almost perfect competition is assumed as there are many rather small suppliers so that a unique market price can be justified. Intermodal transport, on the other hand, has a more complicated cost function.

\[ V_{\text{intermodal}_j} = \frac{S + F}{N_j} + c_r \cdot d_p + c_f \cdot d_{ij} + c_{ch} \]

(9)

Where \( S \) denotes the fixed set up costs per terminal and \( F \) the fixed costs for the first train starting from there. For the time being, it is assumed, that only one train per day is departing from each terminal. \( N_j \) is the number of containers handled per day at terminal \( j \). The
expression $N_j$ will cause problems, when setting up the model for an initial scenario, as it leads to feedbacks between the different levels of decision. Independently of the exact shape of the model, the utilisation per terminal is determined as follows:

$$N_j = \sum_{i=1}^{N} p_{ij} N_i$$

with: $\sum_{i} N_i = \sum_{j} N_j = N$ (10)

being the total quantity of containers per day in the system. There is a difference between the system and the region that will be explained later. $p_{ij}$ denotes the probability for consumer $i$ of choosing terminal $j$ and depends on $V_{truck}$ and $V_{intermodal}$, so $p_{ij} = p_{ij}(...,N_j)$ and thus, there is $N_j$ on both sides of the equation.

Values are assigned to $p_{ij}$ according to a nested logit model as displayed in figure 1.

![Decision tree of the nested logit model](image)

Equation (10) is too general in order to reach a solution and needs further specification. For the reasons stated above, some simplifications are made. In order to observe the responses of the system because of changing parameters on the customer as well as on the supplier side, an initial state of the system has to be set up. Therefore, the following assumptions are made:

The consumers are uniformly distributed over the region and do not differ in their choice behaviour. Therefore, the density of demand is equal at any location.

Having specified this density, the initial utilisation $N_0$ as well as the distance between terminals $\delta$ are calculated according to the formulas derived by Carrillo Muri (2010).

$$\delta = \frac{1}{\rho^3 \left( \frac{1}{2} \sqrt{ \frac{c_f}{S+F} } \right)^3 + \left( \frac{\beta}{S+F} \right)^3}$$ (11)
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\[ N_0 = \frac{1}{(\frac{1}{2} \sqrt{\frac{c_f}{S+F}})^3 + \left(\frac{\beta}{S+F}\right)^3} \]  

(12)

With these results, the locations \( l_j \) can be obtained. In order to obtain a model, where various scenarios can be compared, the zero point of the scale is set in the middle of the region, where also a terminal is located. Starting from this point, terminal \( j \) is located at \( l_j = [j \cdot \delta] \), i.e. the next pair of terminals is at \( \pm \delta \), the following at \( \pm 2\delta \) and so on. As a consequence, there will always be a terminal at the zero point, regardless of its economic reasonableness.

As the costs and thus the probability of choice depend, among others, on the distance between consumers and terminals, the locations of the former have to be specified. It is obvious, that the same scale is used as for the terminals. In order to position the customers, the region is subdivided into \( w \) different zones, each of them covering the area \( a = \frac{L}{w} \). The customer is located in the centre of a corresponding zone. The location of each zone depends on whether the number of zones is odd or even. In the first case, the situation in figure 2 occurs. The situation in the second case is shown in figure 3.

Figure 2: Distribution of an odd number of customers

\[ l_1 = -\frac{L}{2w} \]

\[ l_2 = -\frac{3L}{2w} \]

\[ l_3 = -\frac{L}{w} \]

Figure 3: Distribution of an even number of customers

\[ l_1 = -\frac{L}{2w} \]

\[ l_2 = -\frac{L}{w} \]

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The work of Carrillo Murillo assumes infinite space and thus, there are two problems that have to be dealt with. On the one hand, infinite space entails an infinite number of terminals and each of them is likely to be chosen by the customers following a distribution that depends on the distance for joining each terminal. This is due to the fact that the choice probability has the shape of a fraction with exponential functions in the numerator as well as in the denominator. As they always have positive values, every item is chosen as soon as it is relevant on the local choice (all alternatives are assumed to be latent). Therefore, unrealistic choices are prevented (i.e. that containers are brought to terminals although they are too far away) by limiting the influence in function of the distance to incur for joining a terminal \( p_{\text{min}} \). On the other hand, there are terminals in adjacent regions that are also likely to be chosen. Moreover, there are customers in other regions choosing terminals in the region under examination. These overlaps, as well as the truncated areas of choice can be seen in figure 4.

In order to set the boundaries of the region with respect to customer and supplier relations, a finite set of consumers as well as of terminals has to be found. For this purpose, an ideal customer is regarded. For simplicity, we assume, that he is located in the halfway between two terminals as shown in figure 3. The location of the consumer is set to zero. As a consequence of this, the next pair of terminals is \( \frac{\delta}{2} \) units of length away. The next but one pair is located at \( \pm \frac{3\delta}{2} \) and some arbitrary pair at \( \frac{(s-1)}{2} \cdot \delta \), where \( s \) denotes the number of all terminals in the catchment area of the consumer. This area is now defined by means of a minimum probability of choice \( p_{\text{min}} \). Starting with the pair located most closely to the consumer, more remote pairs are added sequentially until \( p_{\text{min}} \) is undercut for the most distant terminal. Formally written, the condition:

\[
 p_{\text{min}} \leq \exp\left(-\alpha_2 \cdot V_{\text{Intermodal}_{i}}\right) \leq \sum_{k=1}^{s} \exp\left(-\alpha_2 \cdot V_{\text{Intermodal}_{k}}\right) \tag{13}
\]
The right hand side of formula (13) can be written more detailed, so that:

\[
p_{\min} \leq \frac{\exp(-\alpha_2 \cdot V_{TF}) \cdot \exp\left(-\alpha_2 \cdot \left(\frac{t-1}{2} \cdot \delta\right) \cdot c_f\right)}{\exp(-\alpha_2 \cdot V_{TF}) \cdot \sum_{k=1}^{t} \exp\left(-\alpha_2 \cdot \left(\frac{k-1}{2} \cdot \delta\right) \cdot c_f\right)}
\]

(14)

\[
p_{\min} \leq \frac{\exp\left(-\alpha_2 \cdot \left(\frac{t-1}{2} \cdot \delta\right) \cdot c_f\right)}{2 \cdot \sum_{k=1}^{t} \exp\left(-\alpha_2 \cdot \left(\frac{k-1}{2} \cdot \delta\right) \cdot c_f\right)}
\]

(15)

With \( V_{TF} \) denoting the components of equation (9) that are independent from \( \delta \) and cancel each other out. Starting from \( s = 1 \), the number of pairs is now increased as long as the condition in formula (15) holds. As a result, the catchment area \( range \) covers \( \pm \frac{s-1}{2} \cdot \delta \).

This definition yields unrealistic results when the distance between terminals \( delta \) grows. The reason is the dependence of \( p_{\min} \) on the choice set. Even if the former is very high, the choice set includes very remote terminals if \( \delta \) increases. A trade-off was found, so that equations (13) through (15) are applied, if \( \frac{range}{2} \) stays below 20% of the distance to the port \( dP \). If this is not the case, range is set at \( 0.4 \cdot dP \).

![Figure 6: The most distant terminals a consumer will choose with a certain probability](image)

With the endogenous variable \( range \), the overlaps between different regions can be determined. There are two objectives:

1. The outermost consumers (in adjacent regions) that still choose terminals in the local region have to be found.
2. The outermost terminals patronized either by the last customer within the region or by the outermost one resulting from the objective above have to be found.
These two tasks have to be completed in order to specify the already mentioned system. The
difference between region and system is, that the system includes terminals that either
receive containers originating from the region or consumers who ship containers via
terminals, that are located in the region. Moreover, all terminals chosen by foreign
consumers who also send containers via terminals in the region belong to the system. This
has to be done to complete the choice sets of all consumers involved in business with local
terminals and to complete the catchment areas of all terminals in the region. Thus, the first
objective above can roughly be dealt with by adding half of the range to the location of the
last terminal in the region. So:

\[ l_{oc} = l_{LT} + \frac{\text{range}}{2} \quad (16) \]

With \( l_{oc} \) denoting the location of the outermost customer in the system and \( l_{LT} \) the location
of the last terminal in the region. This is obviously not exactly the location of the outermost
customer because the distribution of terminals is continuous, whereas consumers can only
reside in certain discrete points. The exact value of \( l_{oc} \) depends on the number of zones in
the region. If this number is odd, then:

\[
l_{oc} = \begin{cases} 
\left(\frac{l_{LT} + \frac{\text{range}}{2}}{a}\right) \cdot a & \text{if } l_{LT} \geq 0 \\
\left(\frac{l_{LT} + \frac{\text{range}}{2}}{a}\right) \cdot a - 0.5 \cdot a & \text{if } l_{LT} \leq 0
\end{cases} \quad (17)
\]

If the number of zones is even, then:

\[
l_{oc} = \begin{cases} 
\left(\frac{l_{LT} + \frac{\text{range}}{2}}{a}\right) \cdot a - 0.5 \cdot a & \text{if } \left(l_{LT} + \frac{\text{range}}{2}\right) - \left(\frac{l_{LT} + \frac{\text{range}}{2}}{a}\right) a \leq 0.5 \\
\left(\frac{l_{LT} + \frac{\text{range}}{2}}{a}\right) \cdot a - 0.5 \cdot a & \text{otherwise}
\end{cases} \quad (18)
\]

for \( l_{LT} \geq 0 \).
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\[
I_{OC} = \begin{cases}
\left( \frac{l_{LT} + \text{range}}{2} \right) \cdot a + 0.5 \cdot a & \text{if } \left( \frac{l_{LT} + \text{range}}{2} \right) \cdot a - \left( l_{LT} - \frac{\text{range}}{2} \right) \geq 0.5 \\
\left( l_{LT} + \frac{\text{range}}{2} \right) \cdot a + 0.5 \cdot a & \text{otherwise}
\end{cases}
\]  

(19)

for \( l_{LT} \leq 0 \)

In the course of the calculations, the number of terminals will decrease, so that the region only contains one at the zero point. The high value of \( \delta \) causing this fact is also responsible for the necessity of limiting range to \( 0.4dP \). If the area \( L \) covered by the region is rather large, it can happen, that consumers residing within the region will be excluded from the model. This is prevented by setting \( I_{OC} \) equal to the location of the last consumer within the region \( l_{LC} = \frac{L - a}{2} \) if equations (17) to (19) yield a result lower than \( \frac{L}{2} \).

The second objective entails the comparison of two possible locations. The first location is the last foreign terminal patronised by the outermost customer. Therefore, \( l_{LT} \) is set at \( r \cdot \delta \) and \( r \) is repeatedly increased by one as long as \( r \cdot \delta < I_{OC} + \frac{\text{range}}{2} \). Thus, \( I_{OR} = r \cdot \delta \). The second location regards the last customer residing within the region and determines the most remote terminal he patronises. Setting \( l_{LT} \) to \( s \cdot \delta \) the condition \( \frac{L - a}{2} + \frac{\text{range}}{2} > s \cdot \delta \) has to hold. Comparing both results, yields :

\[
I_{OR} = \begin{cases}
s \cdot \delta & \text{if } s > r \\
r \cdot \delta & \text{otherwise}
\end{cases}
\]  

(20)

Knowing the locations of all relevant consumers and terminals in the system, the numbers \( n \) and \( m \) have to be calculated. \( m \) denotes the number of terminals and \( n \) the number of consumers. It holds:

\[
m = 2 \cdot \frac{l_{OR}}{\delta} + 1
\]  

(21)

\[
n = 2 \cdot \frac{l_{OC}}{a} + 1
\]  

(22)

Later, it will be useful to know the number of terminals \( u \) and consumers \( w \) inside the region. Therefore, they are also stated here:
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\[ u = 2 \cdot \frac{L_{LT}}{\delta} + 1 \]  \hspace{1cm} (23)
\[ w = \frac{L}{a} \]  \hspace{1cm} (24)

After having done this, a \( n \times m \) distance matrix \( D \) can be calculated, whose entries show the distances between customers \( i = 1, \ldots, n \) and terminals \( j = 1, \ldots, m \). So \( d_{ij} = |l_i - l_j| \) denotes an entry of row \( i \) and column \( j \) in \( D \). In \( D \) there are pairs of consumers and terminals between which no traffic occurs. Therefore, a distinction has to be made between feasible and infeasible pairs because the latter do not enter the choice sets and thus, the probability formulas. A new \( n \times m \) matrix \( T \) is introduced with entries indicating the existence of traffic between customers and terminals. The following rules hold:

\[
t_{ij} = \begin{cases} 
1 & \text{if } d_{ij} < \frac{\text{range}}{2} \\
0 & \text{otherwise}
\end{cases}
\]  \hspace{1cm} (25)

With these results, \( V_{\text{intermodal}_j} \) can be specified completely.

\[
V_{\text{intermodal}_j} = \frac{S + F}{N_0} + c_r \cdot d_p + c_f \cdot d_{ij} + c_{ch_j}
\]  \hspace{1cm} (26)

Up to now, transportation by truck has been neglected. It is now inserted into the model as shown in figure 1. The choice probabilities \( p_{ij} \) and \( p_{i,\text{truck}} \) are calculated according to the following formulas:

\[
p_{i,\text{train}} = \frac{e^{-\alpha_i V_{\text{train}}}}{e^{-\alpha_i V_{\text{train}}} + e^{-\alpha_i V_{\text{EMU}}}}
\]  \hspace{1cm} (27)
\[
p_{i,\text{truck}} = \frac{e^{-\alpha_i V_{\text{truck}}}}{e^{-\alpha_i V_{\text{truck}}} + e^{-\alpha_i V_{\text{EMU}}}}
\]  \hspace{1cm} (28)

\[
EMU_i = -\frac{1}{\alpha_2} \ln \left( \sum_{j=1}^{m} t_{ij} \cdot \exp \left( -\alpha_2 V_{\text{intermodal}_j} \right) \right)
\]  \hspace{1cm} (29)

\[
p_{ij} = \frac{e^{-\alpha_i V_{\text{truck}}} t_{ij} \cdot e^{-\alpha_i V_{\text{intermodal}_j}}}{e^{-\alpha_i V_{\text{truck}}} + e^{-\alpha_i V_{\text{EMU}}} \sum_{k=1}^{m} t_{ij} \cdot e^{-\alpha_i V_{\text{intermodal}_k}}}
\]  \hspace{1cm} (30)

Having obtained these probabilities, especially those in equations (27) and (30), the containers waiting in each zone can be assigned to the transport modes and destinations. This yields:

\[
N_{i,\text{truck}} = p_{i,\text{truck}} \cdot \rho_i \cdot a
\]  \hspace{1cm} (31)
\[
N_{ij} = p_{i,j} \cdot \rho_i \cdot a
\]  \hspace{1cm} (32)

Looking at the terminals and at the trucking sector, the two final results are:
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\[ N_{\text{track}} = \sum_{i=1}^{n} N_{i,\text{track}} \]  
(33)

\[ N_j = \sum_{i=1}^{n} N_{ij} \]  
(34)

It is important to recognise, that \( N_j \) is very likely to be different from \( N_0 \) as it appears in equation (12). This happens if transportation by truck is chosen and is due to the fact, that in equation (12) only intermodal transport was considered. The consequence of this is, that the value of the fraction \( \frac{S + F}{\# \text{containers}} \), for example in equation (9) increases, what on its part makes \( p_{i,\text{train}} \) as specified by (28) decrease for any \( i \). As the formulas of Carrillo Murillo yield terminals with an equal demand in the beginning and nothing has been changed about that so far, all \( p_{ij} \) have the same value and thus \( p_{i,\text{track}} \) has to increase in order to fill the gap between \( p_{i,\text{track}} + \sum_{i=1}^{m} p_{i,j} \) and one for any \( i = 1, \ldots, n \). It is obvious, that an iterative process has to be established, in order to determine the initial distribution of containers. This could be done by an algorithm like the one outlined in a kind of pseudocode here:

Find initial solutions for \( m \), \( I_j \) and \( N_0 \)

Calculate the deterministic utility functions \( V_{i,\text{track}0} \) and \( V_{i,0} \) \( \forall i, j \)

Calculate the probabilities \( p_{ij0} \) and \( p_{i,\text{track}0} \) \( \forall i, j \)

Calculate the utilisations \( N_{j0} \) and \( N_{\text{track}0} \) \( \forall j \)

set \( t = 1 \)

Do

Update deterministic utilities with \( N_{j,t-1} \) and \( N_{\text{track},t-1} \) to get \( V_{i,\text{track}t} \) and \( V_{i,t} \) \( \forall i, j \)

Calculate \( p_{ij} \) and \( p_{i,\text{track}t} \) \( \forall i, j \)

Calculate the utilisations \( N_{j,t} \) and \( N_{\text{track}t} \) \( \forall j \)

\( t = t + 1 \)

Loop while \( p_{ij} = p_{ij,t-1} \) \( \forall i, j \)

The loop stops, if for every probability \( p \) at step \( t \) holds: \( p_i = p_{i,t-1} \). Up to now, it has been assumed, that only one train per day is departing from each terminal. This is reflected for example in equation (9). However, the possibility of running additional trains can be included. The demand a terminal \( j \) faces, is:

\[ N_j = \sum_{i=1}^{n} p_{ij} \cdot N_i \]  
(35)

if \( N_j > K_j \) (where \( K_j \) denotes the capacity of a train), the cost (and disutility) function will be:

\[ V_{\text{intermodal}j} = S + \left[ \frac{N_j}{K_j} \right] \cdot F + \ldots \]  
(36)

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These step costs are expected to limit the economies of scale. The problem here is, that \( N_j \) is the result of the old cost structure, where fixed costs consisted of \( S + \frac{N_j}{K_j} \cdot F \). This is a problem in formulas (26) and (12), where a similar iterative process takes place as outlined in the algorithm above.

Due to the shift of containers from intermodal to truck transport, the terminals are now located too close towards each other. Therefore, their distance has to be adjusted, because otherwise, the model would be in favour of intermodal transport. A new, relative, density of demand is calculated taking only those containers in account that are forwarded by train. With this new density:

\[
\tilde{\rho} = \left( 1 - \frac{\sum_{i=1}^{\infty} p_i,track}{w} \right) \cdot \rho
\]

(37)

A new distance is calculated according to equation (11). This new and higher value of \( \delta \) changes the values of \( \nu_{\text{intermodal}} \) leading to a changed choice behaviour of consumers. Therefore, the algorithm above has to be run through again, but not with \( \tilde{\rho} \) as new density of demand, because the number of containers to be shipped stays the same. Moreover, the choice situation faced by consumers now, is completely independent of the decisions drawn before the terminals were relocated. To give an example: A consumer who had resided very far away from the next terminal has now (after the redistribution) a terminal in the direct neighbourhood. He does not only decide what to do with the containers he shipped by train before, but rather shifts containers from truck to train. These procedure repeats, until a new equilibrium is found (c.f. figure 6).

Having achieved the equilibrium mentioned above, a change is made to one of the parameters on which equation (9) and (7) depend. This leads to a further run through the process of finding an equilibrium of the discrete choice model. After this second loop, a new relative density \( \tilde{\rho} \) is calculated according to (37). With this new density, the terminals are reallocated according to equation (11) and the process starts again (c.f. figure 6):
The loop finishes as soon as a certain threshold in the utilisation of the terminal located at the zero point is undercut. The model was implemented in Java, using a special class library enabling the program to write the results directly in an Excel file. For the calculation, an adjustment was made to formula (26). When specifying the region, some outside terminals also have to be considered for the sake of completeness. However, their catchment area is not completely within the boundaries of the system. It is impossible to do this, because then new and more remote consumers would enter the scene. They could only make a mode choice with a complete choice set entailing the addition of further terminals and so on. With an incomplete catchment area, $N_j$ is smaller than it would be in reality and thus $V_{intermodal,j}$ is too high. This bias in the probability of choice is circumvented by replacing $N_j$ of all terminals by $N_i$ (i.e. the utilisation of the terminal located at the zero point). This simplification is justified by supposing the region to be a part of a world with a recurring spatial structure.
The weakness of the model relates to the inaccuracies resulting from the definition of \( \text{range} \) and the missing redistribution of terminals once an equilibrium has been reached after introducing road transport. The first point of criticism is the part of the definition of \( \text{range} \) that results from inequalities (13) to (15). It is only exact, if the consumer is located in the middle of two terminals. Along with the fact, that consumers can only reside in discrete points, biases are very likely to happen. The location on the middle influences the number of terminals entering the choice set of a consumer. Most of them will have one terminal less in their distribution according to the consumer’s location. This effect is transmitted by the EMU to the upper level of decision resulting in a higher demand for road transport. Another impreciseness concerns the catchment areas of the respective terminals. The end of \( \frac{\text{range}}{2} \) (regardless of how it came about) does often not match with the location of a consumer. The last consumer residing in a catchment area can contribute too much to the utilisation of this terminal even if a part of his zone is beyond the catchment area. On the other hand, \( \frac{\text{range}}{2} \) can be extended into the zone of a consumer residing outside. The extent of this problem can be limited by choosing rather small sizes \( a \) for the zones.

The introduction of \( \text{range} \) in general is one of the problems arising from trying to extract a completed part of an infinite space and from the requirements of discrete choice theory. The solution above was chosen, because the criteria specified in formulas (13) till (15) do not depend on the number of containers handled at the respective terminal. Consequently, this issue entails two problems leading the model to be slightly inexact. The first one is, that the restriction of the choice set could be regarded as similar to forming truncated distributions \( p_{ij|j \in R} \) with \( R \) denoting the set of all terminals in the range of consumer \( i \). The number \( N_{ij} \) of containers from zone \( i \) to terminal \( j \) is now \( \frac{p_{ij}}{p_{j \in R}} \rho \cdot a \) in contrast to \( p_{ij} \cdot \rho \cdot a \) without assuming a finite range of terminals. After all, a limitation of the choice set has to be made for equation (30) to be applicable. This truncation of the distribution function leads to an overestimation of the terminals within the reach of the consumers. On the other hand, a reduced choice set decreases the value of equation (29) and thus, the probability of deciding in favour of intermodal transport on the upper choice level specified in figure 1.

**CASE STUDY AND RESULTS**

The present section aims to show the potentiality of the model developed in this study and gives suggestions for possible policy measures. The simulation is based on a hinterland transportation system. It contains two principal modes, namely, road haulage and intermodal transportation. The road haulage is shipping the container directly while the intermodal transport is considered to be only the road-haulage to reach a certain terminal, the transfer from the road to the rail and the rail haulage until reaching the supposed port. The former mode of transport is considered to take the form of perfect competition (being available at any time and place without capacity restrictions) while the latter is considered to take a monopolistic competition until a cost minimisation schema is reached (see FEE assumptions
in Carrillo Murillo, 2010). As described before, the present simulation targets a region market over a one-dimensional platform. Two main areas are considered for this case, the analysed region and the whole system.

In order to follow the iteration process, the chosen scenario relates to the terminals’ set up costs. The initial situation here assumes (parameter values are from Carrillo Murillo, 2010):

- Parameter $\alpha_1$ for the nest upper level = 0.005
- Parameter $\alpha_2$ for the nest lower level = 0.01
- $p_{\min} = 0.1$
- Length of the region = 200 km
- Industrial density = 15 TEU/km
- Road haulage cost = 1.33 Euro/km
- Rail transport cost = 0.8 Euro/km
- Distance to the port = 600 km
- Terminals’ set-up costs = 10 000 Euro/day
- Fixed costs per train = 5 000 Euro/day
- Rail capacity = 62 wagons/train
- Terminal handling costs = 15 Euro/TEU

The results show a distribution of 5 terminals in the analysed region while 11 were allocated to the system for the first 6 iterations or scenario changes, and then there is a drop of two terminals inside the region leading also to 7 terminals in the system. The process continues until only one terminal is standing at both the region and the system. The aggregated probability of choices along the scenario process is shown in fig. 9 by mode of transport.

![Figure 9: Probability choices for the road haulage – Scenario on Terminals’ setup costs](image-url)
Clearly one can observe the mirrored choice probability given by the upper nest of the discrete choice model between both alternatives. However, the variability along the iterations is the result of the FEE interactions. In total 347 iterations of 1 000 Euro increase on the setup cost were generated for the case study showing a smooth interactions of customers among the road and intermodal alternatives. At first sight the figure shows a distribution for the road-haulage option of 16 per cent, while a drastic change is observed at the last two iterations from almost 65 per cent until all intermodal terminals drop off giving the maximum probability of distribution to the road alternative. The first and the last jumps in the probability distribution were the result of the discrete choice model at the upper level nest.

The results displayed in figures 9 and 10 also show possible implications for economic policy: If one assumes that the real set up costs amount to a level at which the choice probability for intermodal transport plummets to zero, all other points on the corresponding curve can only be reached if there are subsidies. Three different areas can be distinguished. In the rightmost part of the curve, a small initial funding leads to the establishment of the first terminal. From figure 10 it can be seen that in the beginning there is actually only one terminal. So one could suppose that economies of scale are stronger than the preference for variety. Thus, in this area less subsidies can have high impact as they have only to be paid for a low number of terminals that are planned or exist already. A different situation exists in the area on the left-hand side of the intersection of the two curves in figure 9. A wide range of subsidies per terminal has less impact on the decision of consumers. The number of terminals increases at only one point in this area (c.f. figure 10) entailing a higher increase in subsidies. Here only the subsidies per terminal are displayed on the x-axis. In other words: Tripling the total amount of subsidies makes less than ten percent of consumers change their mode choice. This could be an indicator that economies of scale also prevail in this area. The latter are outperformed by the preference for variety as one moves towards the left end of the graphs in figures 9 and 10. Here the question arises if a further improvement of area coverage is affordable and socially desirable.

With regards on the distribution of terminals figure. 10 shows the dynamic interaction among all alternatives.
However, on the right part of the graph in figure 10 it is possible to see full discrete choice theory behaviour among two alternatives. This is due to the lack of distribution among similar alternatives of the FEE. However, at the last two iterations we can see the FEE influence because in discrete choice theory it is impossible to see a reversible behaviour on customers choosing an alternative and retracting them to previous one. On this part of the graph we see also the similarity of the model to the classical mirrored probability choice shown in figure 10.

In the area on the left side of the graph, one can see two spikes resulting from the cutting out of a finite region of an originally infinite area. They occur, when a terminal approaches the edge of the region. Then, its catchment area extends into the neighbourhood of the region and thus the terminal gets attractive for customers from outside. If the distance between the terminals continues to increase, it is finally pushed out of the region. The next terminal within the region does not attract foreign customers so much because it is too far away from the border. The area left of these spikes is governed by this boundary effect, whereas on the right hand side of these spikes some interesting observations can be made.

After having left the region, the terminal gradually gets out of the reach of more and more consumers and the long distance makes the pre- carriage uneconomic. Looking at the curves for terminals 2 and 3 after the spike at about 32000 Euro, one can see that more demand shifts to the last remaining terminal than to road haulage. This effect continues until the two terminals have left the area under investigation. Not till then, the occupation of terminal 1 peaks and truck transportation catches up. This reinforces the observations made in figure 9, namely the sufficiency of just one terminal and the superiority of economies of scale over the preference for variety.

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CONCLUSIONS

The simulated modal competition on hinterlands has been developed along the study. It includes two types of model: a nested logit and the Free Economic Energy. It can be qualified as a combined pseudo dynamic model. For this purpose the model was programmed in Java and the resulted simulation shows the advantage of including many alternatives differentiated by the mode of transport.

The model examines the behaviour of the agents depending on their spatial location. Space was modelled by assigning locations to the terminals as well as to the consumers and by the density of demand. This density can be seen as a proxy variable for a variety of properties of the environment. It is obvious, that a limitation on these two parameters reduces the explanatory power of the model as well as the limitation to just one dimension does.

An extension to a two dimensional space seems to be possible, especially because the formulas for calculating the Free Economic Energy can be adjusted for this purpose. In this case the question would be, which metric to use and if or how to consider other spatial circumstances like the rail and road network. Once having set up such a realistic region, terminals can not be moved to every point in space anymore as they require a side track. Up to now, the model reaches equilibria between demand and supply in the discrete choice as well as in the Free Economic Energy parts. If this will still be the case after having imposed a traffic network and the resulting locations, in which terminals are allowed to be located, has to be examined first.

The simulation case shows the influence for the combined FEE and discrete choice models, with a special kind of customers’ behaviour that renders them unique in the frame of freight transport models. The classical probability choice distribution at the upper level nest shows a smoother interaction between the aggregated intermodal transport and the road haulage. However, when disaggregating the demand, it is possible to observe a more realistic distribution of the customers among all alternatives and modes whenever an intermodal alternative is closed. This is the main strength of the combined model: It is able to map the drastic changes of transport systems composed of interacting supply and demand actors. This model can be applied for policy scenarios avoiding “misuses” of the modeller on the final results. Those policy scenarios can open the discussions on the choice between intermodal subsidies and internalisations of external costs as well as on the permits to built and launch new services on inland terminals. Finally, it can be stated that modal competition in freight transportation can be mapped in hinterlands by complementing discrete choice theory models with both, economies of scale and dynamic interactions coming from the FEE.
REFERENCES


