OPTIMAL RAMP METERING OPERATIONS WITH PROBIT-BASED IDEAL STOCHASTIC DYNAMIC USER OPTIMAL CONSTRAINTS

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ABSTRACT

An understanding of drivers' route choice behavior on the expressway-arterial network is important in the ramp metering efficiency studies. Field studies have shown that drivers divert to alternative routes after the installation of ramp metering on the network. An efficient ramp metering rate scheme should be determined by considering the drivers' route choice behavior. This paper proposes an optimal ramp metering scheme that adopts the probit-based ideal stochastic dynamic user optimal (p-SDUO) model to describe the drivers' route choice behavior. The probit-based model is chosen in light of its capability to overcome the limitations in logit-based model. The p-SDUO is formulated as a fixed point problem which is solved by using the method of successive averages (MSA). A modified cell transmission model (MCTM) is used to simulate the traffic propagation on the network. MCTM is appropriate for this study since it can capture the horizontal queue and shockwaves; fulfill first-in-first-out principle and model dynamic traffic interactions across multiple links. A non-linear optimization model, with p-SDUO as constraints, is developed to optimize the ramp metering operation. The objective of the model is to find the optimal ramp metering rate scheme that minimizes the total travel time of the expressway-arterial network. The proposed model is tested with an illustrative case study of I210W expressway-arterial network in Pasadena, California. A comparison study of no-metering and with-metering case is carried out. The results show that the proposed model could find a better solution compared to the initial one. There is significant number of drivers diverted from the expressway to the arterial streets when the ramp metering is installed. In addition, drivers would consider the queuing condition at the on-ramps and choose to enter the expressway through the less congested on-ramps.

Keywords: Ramp Metering, Stochastic Dynamic User Optimal, Cell Transmission Model, Probit Model, Genetic Algorithms

INTRODUCTION

Ramp metering regulates vehicles at the on-ramps from entering into the expressways by a proper metering rate pattern. It is a practical traffic control strategy to mitigate traffic congestion on the expressway-ramp system. A survey study carried out by Cambridge Systematics (2000) have demonstrated the benefits of ramp metering, such as increasing expressway's throughput, reducing total system travel time and enhancing traffic safety. Papageorgiou and Kotsialos (2002) gave a comprehensive study on how and why ramp metering improves traffic flow. They showed that by using ramp metering, the expressway mainline throughput can be increased for about 10-15% which in turn decreased the travel time. Theoretically, ramp metering is effective only if the traffic volume on the mainline expressway at the section immediately upstream of the ramp is less than the capacity. Under this condition, the application of ramp metering could ensure that the traffic volume delivered downstream of the ramp does not create a bottleneck due to the excessive demand from the upstream. While it is impossible to control the vehicles traveling on the expressway, the best choice is to regulate the entry vehicles from ramp. By ramp metering, it could break the "platooning" of entering vehicles for a more efficient merging. It could also reduce the demand of the expressway-ramp system by encouraging the diversion of vehicles to the surface streets (Wu 2001).

There are three types of ramp metering operations, namely fixed-time, reactive or responsive and coordinated ramp metering. Fixed-time ramp metering strategies defined the ramp metering rate pattern using the historical flow and demand pattern of the expressway-ramp system, without real-time measurement. Wattleworth and Berry (1965) were the first researchers to propose this type of ramp metering algorithm. The drawbacks of the strategy can be easily pointed out. Demands on the network are not constant but change with different time period. The occurrence of unexpected events may perturb the traffic condition on the road, which lead to travel time variance with the historical data. Reactive or traffic responsive ramp metering can address these limitations well. The metering rate pattern will be adjusted according to the real-time traffic condition on the expressway-ramp network system. For example, ALINEA proposed by Papageorgiou *et al.* (1991) is a typical example of this type of metering strategy. If a few of vicinity ramp meters are considered and coordinated, the strategy is termed as area wide coordination strategy. For example, ZONE (Lau 1997), METALINE (Papageorgiou *et al.* 1997), and FLOW (Jacobson *et al.* 1989) are typical examples of this type of strategy.

The research studies on the optimization of the effectiveness of ramp metering operation can be divided into two categories, i.e. the practical and theoretical studies. In practical studies, simulation models are used as the modeling tool to describe the traffic flow. The objective of most of the studies aims to evaluate the effectiveness of the ramp metering algorithm used. To name a few, Hellinga and Van-Aerde (1995) evaluated the effectiveness of ramp metering implementation using INTEGRATION; Lahiti *et al.* (2002) attempted to quantify the impact of simple ramp metering on average speed using CORSIM at the merge influence area under different traffic and geometric conditions; Hasan *et al.* (2002) used MITSIM to evaluate and compare the effectiveness of ALINEA and FLOW algorithms; Chu *et al.* (2004) employed

PARAMICS to evaluate three metering algorithms, namely ALINEA, BOTTLENECK and ZONE. In theoretical study, the objective is to find the optimum ramp metering rate scheme that can improve the efficiency of ramp metering operation. Optimal control theory is widely used to find the optimal ramp metering rate scheme. Papageorgiou *et al.* (1990) adopted the linear quadratic (LQ) optimization technique, a well known method of Automatic Control Theory to derive control strategies for traffic responsive and coordinated ramp metering. Zhang *et al.* (2001) extended the study by introducing the nonlinear feature in the control strategy. In addition, Zhang *et al.* (1996) provided an in-depth study on the benefits of ramp metering by formulating the ramp metering system using optimal control theory. They found that ramp metering does not improve the total travel time for a uniformly congested expressway unless there is diversion to alternative roads. Kotsialos and Papageorgiou (2004) adopted a constrained discrete time nonlinear optimal control problem in the formulation of coordinated ramp metering system. Yang and Yagar (1994), on the other hand, used bi-level programming model to solve the ramp metering problem. One of the significant limitations of the theoretical studies is employing simple traffic models to describe the dynamic of traffic flow. For example, Papageorgiou *et al.* (1990) and Zhang *et al.* (2001) adopted LWR model, Chang and Li (2002) adopted Payne model, and Kotsialos and Papageorgiou (2004) adopted the second order model, Metanet. These are point queue models which are limited in describing the real queue situation especially at the on-ramps. In addition, most of them do not consider the effect of on-ramp control on drivers' route selection (Bellemans *et al.* 2003; Hegyi *et al.* 2002; Kotsialos *et al.* 2002). Nevertheless, in the real situation, a significant number of drivers have rerouted due to the installation of ramp meters (Wu 2001). The accuracy of the ramp meter study might be affected if the drivers' route choice is not incorporated in the ramp metering optimization study.

This study intends to find the optimum ramp metering rate scheme that can optimize the efficiency of ramp metering operation. It is formulated as a bi-level programming model, in which the upper level is the nonlinear programming model that aims to find the best ramp metering rate scheme; while the lower level is a route choice model. An expressway-ramp-arterial network system is considered in the problem formulation. The arterial streets serve as the alternative routes for the drivers who choose to divert from the on-ramps. Besides, this study could address some of the limitations of the previous studies. For example, the modified cell transmission model (MCTM) (Meng and Khoo, 2009) is employed to describe the dynamic of traffic flow. It could capture the horizontal queue formation and dissipation. This is important in the ramp metering context considering the need to model the on-ramps queue. Besides, MCTM could capture shockwaves; maintain first-in-first-out (FIFO) principle and model dynamic traffic interactions across multiple links. A probit-based dynamic stochastic user optimal (p-DSUO) is employed to describe the route choice behavior of the drivers. p-DSUO assumes the drivers are not identical in making choice decision and do not have full knowledge of the network. It models the variation of drivers' perception with normal distribution. The actual route travel time used in the p-DSUO is derived from the MCTM. The p-DSUO is then formulated as a fixed point model. By doing so, the bi-level programming model for the ramp metering operations can be simplified into a single level optimization with pDSUO flow as constraints.

PROBLEM STATEMENT

Consider an expressway-ramp-arterial network system, $G=(N,A)$ where N is the set of nodes; A is the set of directed arcs denoting the on-ramps, the off-ramps, the expressway stretches, and the arterial roads, connecting two consecutive points meeting an on-ramp or an off-ramp, namely, $A \subset N \times N$. The network system comprises expressway segments, A_E, a group of on-ramps with ramp meters, A_{on}^{+} or without ramp meters, A_{on}^{-}, the off-ramps, A_{off} and arterial road segments, A_{art} that stretches parallel to the expressway is linked with it through these on-off-ramps. More specifically, $A = A_{on}^{+} \cup A_{on}^{-} \cup A_{off} \cup A_E \cup A_{art}$. Assume R and S denote the sets of origins and destinations, respectively, where $R,S \subseteq N$. Let K_{rs} denote the set of all routes from an origin $r \in R$ to a destination $s \in S$, namely OD pair *rs*.

Let $[0,T]$ denote the time horizon for the dynamic ramp metering operations, which is long enough to allow drivers departing during the time horizon to complete their trips. The time horizon is discretized into T equal time intervals of δ, namely, $[0,\delta,2\delta,3\delta,\cdots,(T-1)\delta]$. Let $q_{rs}(t)$ be the fixed traffic demand between OD pair *rs* departing from the origin r during time interval $[t,t+\delta]$, $t \in \{0,1,2,\cdots,T-1\}$; $f_k^{rs}(t)$ be route flow on route $k \in K_{rs}$ between O-D *rs* departing from origin r during time interval $[t,t+\delta]$, $t \in \{0,1,2,\cdots,T-1\}$. Without loss of generality, it is assumed that $\delta = 1$ hereafter. According to the flow conservation law, it follows that:

$$\sum_{k \in K_{rs}} f_k^{rs}(t) = q_{rs}(t), r \in R, s \in S, t = 0,1,2,\cdots,T-1 \tag{1}$$

For the sake of presentation, two row vectors are introduced associated with route flows:

$$\mathbf{f}(t) = \left(f_k^{rs}(t) : k \in K_{rs}, r \in R, s \in S\right) \tag{2}$$

$$\mathbf{f} = \left(\mathbf{f}(t) : t = 0,1,2,\cdots,T-1\right) \tag{3}$$

Eqn. (2) indicates that the flow on all the used routes k for OD pair rs in the expressway-ramp-arterial network system at departure time t. On the other hand, eqn. (3) denotes the route flow on all the departure time during the time period considered.

Given a route flow solution, $\mathbf{f}$ satisfying the flow conservation eqn. (1), referred to as a feasible route flow solution, the actual route travel time on route, $k \in K_{rs}$ for OD pair rs for flows departing at time t should be a function of the departure time, t and the feasible route flow solution, denoted by $\eta_k^{rs}(\mathbf{f},t)$. The objective of the ramp metering operations optimization model is to minimize the total travel time as shown in the following equation by deciding the optimal ramp metering rate:

$$F(\mathbf{Z}) = \sum_{\forall rs} \sum_{\forall k} \sum_{\forall t} \eta_k^{rs}(\mathbf{f},t) f_k^{rs}(t,\mathbf{Z}) \tag{4}$$

where $\mathbf{Z} = \left(Z_a(t), a \in A_{on}^{+}, t = 0,1,\cdots,T-1\right)$ is a row vector that denote the dynamic ramp metering rate solution of all on-ramps time dependent metering rates.

PROBIT-BASED IDEAL DSUO

Fixed Point Formulation

Since there is variation in perception and exogenous factors, actual route travel times are perceived differently by individual driver. It is more realistic to assume that driver's perceived actual route travel time is a random variable being equal to the actual route travel time plus a random error term, which random error term has mean of zero, namely:

$$C_k^{rs}(t,\mathbf{f}) = \eta_k^{rs}(t,\mathbf{f}) + \xi_k^{rs}(t), k \in K_{rs}, r \in R, s \in S, t = 0,1,2,\cdots,T \quad (5)$$

where $E\left[\xi_k^{rs}(t)\right] = 0$.

It is further assumed that all the error terms associated with departure time t for OD pair rs follows a multivariate normal distribution, namely:

$$\left(\xi_k^{rs}(t) : k \in K_{rs}\right) \succ N\left(\mathbf{0}, \Sigma^{rs}(t)\right), t = 0,1,\cdots,T \quad (6)$$

where $\Sigma^{rs}(t)$ is the $\left(|K_{rs}| \times |K_{rs}|\right)$ covariance matrix, $|K_{rs}|$ represents the cardinality of set K_{rs}. The probability of a driver perceiving route $k \in K_{rs}$ as the shortest one among routes of OD pair rs can be expressed by:

$$p_k^{rs}\left(\mathbf{C}^{rs}(t,\mathbf{f})\right) = \text{Prob}\left[C_k^{rs}(t,\mathbf{f}) \le C_l^{rs}(t,\mathbf{f}), \forall l \in K_{rs} \text{ and } l \ne k\right] \quad (7)$$

Eqn. (7) denotes that route k being chosen by a driver who is traveling on OD pair rs if the driver perceived that the actual route travel time of route k is the minimum compared to the route travel time of all available alternative routes K_{rs}. As such, the route flow solution, $\mathbf{f}$ fulfils the probit-based ideal DSUO conditions if and only if:

$$f_k^{rs}(t) = p_k^{rs}\left(\mathbf{C}^{rs}(t,\mathbf{f})\right) q_{rs}(t), k \in K_{rs}, r \in R, s \in S, t = 0,1,2,\cdots,T-1 \quad (8)$$

Eqn. (8) shows the fixed point formulation of the ideal DSUO and is proved as follows:

Proposition: If all the actual route travel time functions, $\left\{\eta_k^{rs}(t,\mathbf{f}), k \in K_{rs}, r \in R, s \in S, t = 0,1,2,\cdots,T-1\right\}$, are continuous with respect to the route flow solution, $\mathbf{f}$, then the fixed-point formulation (8) has at least one solution.

Proof: According to Brouwer's Fixed Point Theorem, if $\mathbf{P}:\mathbf{f}\mapsto\mathbf{f}$, where $\mathbf{P}=\left(p_k^{rs}\left(\mathbf{C}^{rs}\left(t,\mathbf{f}\right)\right)q_{rs}\left(t\right):k\in K_{rs},r\in R,s\in S,t=0,1,2,\cdots,T-1\right)$ and $\mathbf{P}$ is continuous, there is at least one $f^*\in\mathbf{f}$, such that $f^*=\mathbf{P}f^*$. □

The actual route travel time function is derived from the MCTM. Although it is continuous, it is not monotonic w.r.t. flow. Lo and Szeto (2002) pointed out that under certain traffic flow condition such as congestion, the actual route travel time function w.r.t. flow is non-differentiable. Hence, the uniqueness of the solutions still remains an open question. Due to this limitation, the solution obtained might not unique.

An Approximation Solution Method

One of the main concerns when solving the DSUO is how to define the route choice set. For a small network, all the available routes between each OD pair can be enumerated. However, if the network size is large, such enumeration exercise may take a lot of time and resource. In such cases, one could reduce the route choice set to include only some of the prevailing routes that will considered by the drivers. Specifically, those routes that are long and less attractive are removed because they are less likely to be chosen by drivers. Since not all the routes are considered, the solutions obtained are approximated solutions.

Determination of sets of routes $\hat{K}_{rs}, r\in R,\ s\in S$

There are some existing studies on the determination of routes considered by drivers in the network loading process. Bovy and Catalano (2007) proposed a doubly stochastic choice set generation method to find the route choice set required by DSUO. By carrying out a repeated shortest route search, the method guarantees that attractive alternatives will be in the choice set with high probability while unattractive ones will have negligible probabilities. Besides, in an earlier research, Cascetta et al. (1997), Lo and Chen (2000) proposed that a reasonable subset of all the available routes for an OD pair that needs to be considered for a medium-sized network is 4-7 routes. They showed that by considering this number of routes, it is sufficient and representative.

If the number of routes considered is fixed, $\hat{K}_{rs}$-shortest route algorithm could be applied to find the $\hat{K}_{rs}$-shortest route for each OD pair rs. Lim and Kim (2005) proposed an efficient algorithm that can minimize the degree of similarity when finding the $\hat{K}_{rs}$-shortest route. This is guaranteed by defining three parameters in the algorithm, namely overlapped ratio, maximum degree of overlapped and link penalty. However, the $\hat{K}_{rs}$-shortest route algorithm adopted in this study is modified from the original one. Instead of using the length of the overlapped route to calculate the overlapped ratio, the number of links that is overlapped is counted. In such a case, the overlapped ratio is more stable and well defined compared to the original version.

$\hat{K}_{rs}$*-Shortest Route Algorithm With Route Overlapped*

Let $\tau_{k_a}^{rs}$ is the given cost of link a, $a \in A$ on route k, $k \in \hat{K}_{rs}$ of OD pair rs, O_p is the maximum degree of overlap between links of routes allowable which is predefined. O_z is the link penalty, in which $O_z = \left[\frac{1}{O_p}\right]^{\alpha}$ where α is a positive parameter. Let $ol_{n/k}^{rs}$ is the number of overlapping links between route k and n, where $k,n \in \hat{K}_{rs}$ of OD pair rs. $n = k-1, k-2, \ldots 1$, and $Op_{n/k}^{rs} = \frac{ol_{n/k}^{rs}}{l_k^{rs}}$ is the overlapping ratio between route k and n with l_k^{rs} is the number of links that route k of OD pair rs has. When the overlapped ratio between current found route and the entire previous route that has been found is calculated, $O_{\max}^{rs/k}$ is taken as the maximum overlapped degree for the current k route of OD pair rs. In other words, $O_{\max}^{rs/k} = \max \left\{Op_{k-1/k}^{rs}, Op_{k-2/k}^{rs}, \ldots Op_{1/k}^{rs}\right\}$. Define also E_k^{rs} is the link set for the k-th route of OD pair rs which consisted of J number of links, i.e. $\mathrm{E}_k^{rs} = \left\{a_{k_1}^{rs}, a_{k_2}^{rs}, \ldots, a_{k_j}^{rs}, \ldots a_{k_J}^{rs}\right\}$, $a_{k_j}^{rs}$ indicates the link j for route k of OD pair rs, where $a_{k_j}^{rs} \in A$ and P^{rs} is the route set for OD pair.

The step-by-step procedure is illustrated as follows:

Step 0: (Initialization) Set a value for O_p, and set $rs = 1$, which indicate the first OD pair.

Step 1: (Shortest route algorithm) With any of the existing shortest route algorithm, find the first shortest route for OD pair rs, set $k = 1$. Add the links that form the route to E_k^{rs}, and add the route set $P^{rs} = \left\{p_k^{rs}\right\}$.

Step 2: (Link cost update) Examine the links in E_k^{rs}. Update the link cost by applying the penalty on the link. Essentially, $\tau_{k_l}^{rs} = \tau_{k_l}^{rs} \times O_z$, where $l = \{1, 2, \ldots, J\}$.

Step 3: (Route search and overlapping ratio calculation)

Step 3a: (Proceed to next route) Set $k = k+1$.

Step 3b: (Find shortest route) With link label setting algorithm, find k - shortest route. Add the links that form the route to E_k^{rs}, and add the route set $P^{rs} = \left\{p_k^{rs}\right\}$.

Step 3c: Calculate degree of overlap, $Op_{n/k}^{rs} = \frac{ol_{n/k}^{rs}}{l_k^{rs}}$, where $n = k-1, k-2, \ldots 1$.

Step 3d: Find the maximum overlapped degree from these routes, $O_{\max}^{rs/k}$.

Step 4: (Route convergence test) If $O_{\max}^{rs/k} > O_p$, proceed to Step 5. Otherwise, proceed to Step 2.

Step 5: (OD pair convergence test) If $rs = RS$, where all the OD pairs have found the routes, terminate the algorithm. Otherwise, set $rs = rs + 1$, proceed to Step 1.

In Step 1, any algorithm can be adopted to find the shortest route at each iteration, such as the link label setting algorithm and Dijkstra algorithm (Dijkstra 1959). In step 2, it could be seen that the link cost of the previous shortest route is increased by applying the penalty charges. This will deter these links from being chosen as members in the subsequent shortest route search. From Step 3c, the degree of overlap has the value of $[0,1]$, where 0 means no overlapped. If the route found is exactly the same as the previous one, the overlapped degree is 1. Hence, O_p is defined such a way to control the number of $\hat{K}_{rs}$-shortest route found. For instance, if only one route is needed, $O_p = 1$. Proper tuning of the parameter is thus very important.

Derivation of Covariance Matrix $\hat{\Sigma}^{rs}(t), r \in R, s \in S, t = 0,1,2,\cdots,T$

Sheffi (1985) has shown the derivation of the covariance matrix for the probit-based model used when performing the static version of the stochastic user equilibrium model. The concept however could be generalized into the dynamic case shown herein. Considering two routes, k and l, where $k,l \in \hat{K}_{rs}$ of OD pair rs available for drivers to choose from during departure time t, the covariance matrix to describe the overlapped of the routes is presented in the following equation:

$$Cov\left(\xi_k^{rs}(t),\xi_l^{rs}(t)\right) = \theta_{kl}^{rs}(t)\sum_{a\in A} t_a^0 \delta_{a,k}^{rs}\delta_{a,l}^{rs},\ k,l \in \hat{K}_{rs} \qquad (9)$$

where $\delta_{a,k}^{rs}$ is a binary parameter which takes value of 1 if route k of OD pair rs comprise link a, and it takes value of 0 if it is not; t_a^0 denotes the free flow travel time of link a. By doing so, eqn. (9) actually indicates that the covariance between the perceived travel times of two routes is the summation of the free flow travel time of the common links that shared by two routes. The $\theta_{kl}^{rs}(t)$ in eqn. (9) is interpreted as the drivers' variance of the perceived travel time over a road segment for both routes k and l of OD pair rs at departure time t. This parameter may change according to the different departure time t. It is clear that this study used the link free flow travel time to compute the covariance matrix for the overlapped route. Historical link travel time can be used if they are available.

Actual Route Travel Time: Determination and Properties

To obtain the actual route travel time on any used route of any OD pair, the development of a dynamic traffic flow model that can characterize traffic flow condition is utmost importance. This study adopted the modified cell transmission model (MCTM) to imitate the traffic flow condition. Details of MCTM can be found at Meng and Khoo (2009). Given the OD demand for the expressway-ramp-arterial network system, the MCTM is called to calculate the actual route travel time, $\boldsymbol{\eta}$, where $\boldsymbol{\eta} = \left(\eta_k^{rs}(t,\mathbf{f}) : k \in K_{rs}, r \in R, s \in S, t = 0,1,2,\cdots,T-1\right)$ by performing the DSUO assignment. By adopting the method introduced by Lo and Szeto (2002), the actual route travel time $\eta_k^{rs}(t,\mathbf{f})$ of flow $f_k^{rs}(t)$ can be computed through cumulative counts at the origin cell r and destination cell s. Figure 1 shows the cumulative count curves, $\chi_k^r(t)$ and $\chi_k^s(t)$ on route k between OD pair rs. ω denotes the arrival time of the packet of traffic. The actual route travel time experienced by traffic departing at time t, $\left(f_k^{rs}(t)\right)$, is the horizontal distance between the two cumulative curves as shown in the figure. Since time is discretized, there is no guarantee that the entire packet of flow will arrive at the destination in the same clock tick of time. Hence, the travel time averaging scheme is introduced so that the entire departing traffic has one uniquely determined average actual route travel time.

Eqn. (10) calculates the average actual route travel time:

$$\eta_k^{rs}(t,\mathbf{f}) = \frac{\int_{\chi_k^r(t-1)}^{\chi_k^r(t)} \left[\chi_k^{s^{-1}}(\nu) - \chi_k^{r^{-1}}(\nu)\right] dv}{\chi_k^r(t) - \chi_k^r(t-1)} \tag{10}$$

where

$$f_k^{rs}(t) = \chi_k^r(t) - \chi_k^r(t-1) \tag{11}$$

Eqn. (11) defines the departure flow at time t.

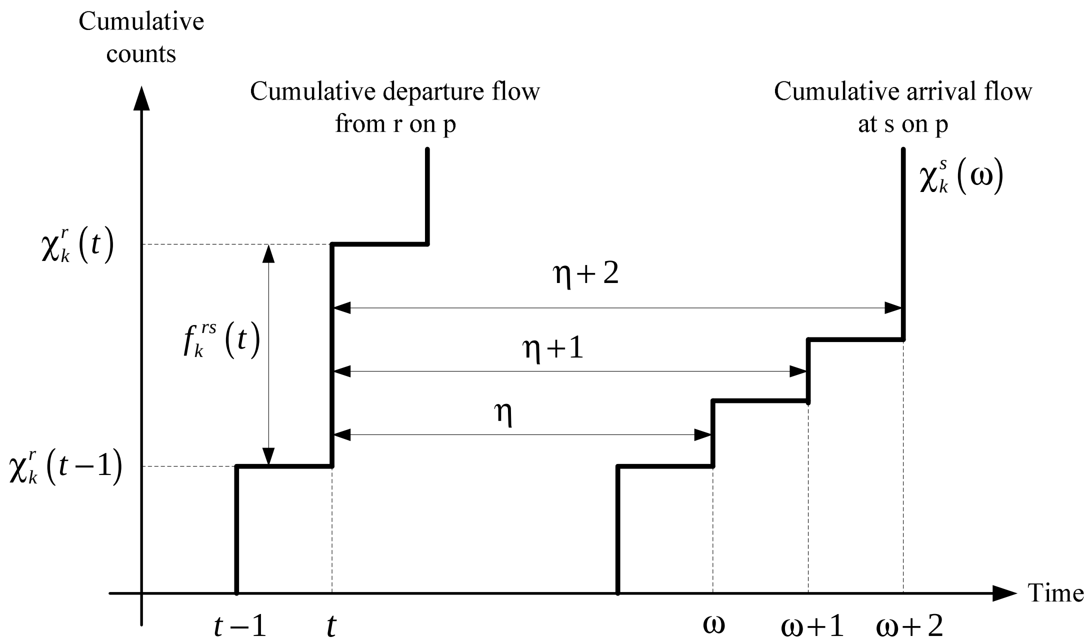

Figure 1 The traffic cumulative curve (Source: Lo and Szeto (2002))

Lo and Szeto (2002) showed that the actual route travel time function, $\boldsymbol{\eta}$ obtained using eqn. (10) is continuous w.r.t. to the flow, $\mathbf{f}$. However, it is also shown that the actual route travel time function may not be continuously differentiable with respect to $\mathbf{f}$. This may cause inexistence of solution for the probit-based DSUO assignment. In addition, it is indicated that the average actual route travel time is not monotonic with respect to the route flow by using MCTM. Thus, the solutions (if exist) may not be unique. An approximated solution or a tolerance-based solution could only be found due to this limitation. Szeto and Lo (2005) developed a tolerance-based deterministic dynamic user optimal (DDUO) principle, which allows the declaration of the user optimal flow even if the travel times of the used routes between the same origin-destination pair are not exactly the same. This could be regarded as a special case to the traditional DDUO.

This tolerance-based concept is applied here for the fixed point problem indicated in eqn. (8). The exact equality of the left hand side and the right hand side of the eqn. (8) could not be obtained in some cases. The solution is declared found if the difference between both sides is small enough, which can be pre-determined as a theoretical gap. It is shown in the following equation:

$$\phi = \max\left\{f_k^{rs}(t) - p_k^{rs}\left(\mathbf{C}^{rs}(t,\mathbf{f})\right)q_{rs}(t)\right\} \quad (12)$$

where ϕ is the theoretical gap.

Method of Successive Averages (MSA)

There are many methods available for solving the fixed point problem. To name a few: Ishikawa algorithm, Mann algorithm, Krasnoselskij iterations and so on (Berinde 2007). MSA is one of these methods that also can be adopted to solve the fixed point problem. It is adopted in this study to solve the fixed point formulation of the DSUO model with a predefined partial routes, $\hat{K}_{rs}$ and the relevant $|\hat{K}_{rs}|\times|\hat{K}_{rs}|$ covariance matrix of $\hat{\Sigma}^{rs}(t)$. In addition, Monte Carlo simulation method is adopted to perform the probit-based ideal DSUO loading, in which the probability of drivers choosing each route $k \in \hat{K}_{rs}$ is computed.

The step-by-step procedure of MSA is shown as follows:

Step 1: (Initialization)

Step 1a: Define $\hat{K}_{rs}$-shortest route choice set for each OD pair rs based on the link free-flow travel time using the method present earlier.

Step 1b: Initialize the departure flow, $f_k^{rs(n)}(t)$ for each route k in the partial route choice set, $\hat{K}_{rs}$ for each OD pair rs at each departure time t randomly. The flow assigned must fulfill the following equation:

$$q_{rs}(t) = \sum_{k=1}^{\hat{K}_{rs}} f_k^{rs(n)}(t) \quad (13)$$

Set $n = 1$.

Step 2: (Calculate actual route travel time) Load the departure flow $f_k^{rs(n)}(t)$ to the MCTM. Run the MCTM to propagate traffic across the cells until all the vehicles arrived at the destinations. Compute the actual route travel time according to eqn. (10) for each traffic flow depart at time t for route k of OD pair rs.

Step 3: (Compute probability of route) Using Monte Carlo sampling method, sample the drivers' perceived actual route travel time according to eqn. (5), compute the probability of each route of OD pair rs being used by drivers, namely, $p_k^{rs(n)}\left(\mathbf{C}^{rs}(t,\mathbf{f})\right)$.

Step 4: (Compute auxiliary flow, $\tilde{f}_k^{rs(n)}(t)$) Calculate the auxiliary flow based on the probability using the following equation:

$$\tilde{f}_k^{rs(n)}(t) = p_k^{rs(n)}\left(\mathbf{C}^{rs}(t,\mathbf{f})\right)q_{rs}(t) \quad \forall k, rs, t \quad (14)$$

Step 5: (Calculate the flow for next iteration) The flow for next iteration is computed by the following equation:

$$f_k^{rs(n+1)}(t) = f_k^{rs(n)}(t) + \frac{1}{n}\left[\tilde{f}_k^{rs(n)}(t) - f_k^{rs(n)}(t)\right] \quad \forall k, rs, t \tag{15}$$

Step 6: (Convergence checking) If convergence criteria are satisfied, stop. Otherwise, set $n = n + 1$ and proceed to Step 2.

The stopping criteria of the algorithm in Step 6 are based on the calculation of the average absolute error shown in eqn. (16). There are two stopping rules that can be adopted based on this, one is to stop the algorithm when the ∇ in eqn. (16) is smaller than ϕ in eqn. (12), or to stop the algorithm when the ∇ in eqn. (16) is stable over N iterations. In this study, the latter rule is adopted.

$$\nabla = \frac{\sum_{\forall k}\sum_{\forall rs}\sum_{\forall t}\left|f_k^{rs(n)}(t) - p_k^{rs(n)}\left(C^{rs}(t,\mathbf{f})\right)q_{rs}(t)\right|}{\sum_{\forall rs}\hat{K}_{rs}T} \tag{16}$$

A solution exists if the actual route travel time used in eqn. (10) is continuous w.r.t. flow. The solution however is not unique because the actual route travel time is not monotone. MSA could solve the problem and produce a tolerance-based solution. Hence, it is stable.

THE OPTIMIZATION MODEL

The nature of the ramp metering operation allows it to be formulated as a bilevel programming model, in which the upper level problem aims to find an optimal metering rate that minimizes the total travel time while the lower level problem addresses the drivers' route choice decision behavior. Due to the fact that DSUO is chosen to formulate the lower level problem and it is formulated as a fixed point problem, the bilevel programming model of the ramp metering operation problem could be converted into a single level optimization model by adopting the DSUO flow as a constraint. Given any feasible metering rate solution, $\mathbf{Z}$, the DSUO flow obtained from the network system is actually subjected to the metering rate used. In other words, the DSUO flow is a function of the metering rate given. Mathematically, eqn. (2) and eqn. (3) could be rewritten into:

$$\mathbf{f}(t,\mathbf{Z}) = \left(f_k^{rs}(t,\mathbf{Z}) : k \in \hat{K}_{rs}, r \in R,\ s \in S\right) \tag{17}$$

$$\mathbf{f} = \left(\mathbf{f}(t,\mathbf{Z}) : t = 0,1,2,\cdots,T-1\right) \tag{18}$$

The objective of the single level ramp metering optimization model is to find an optimal solution, $\mathbf{Z}$ which can minimize the total travel time of the study area. Since the actual route travel time can be obtained using eqn. (10) for each route of each OD pairs, the total travel time for all the drivers in the study area can be computed. Essentially, the total travel time of the drivers in the study area is the summation of all the routes used for all OD pairs during the time horizon. Mathematically, it is expressed as:

$$F(\mathbf{Z}) = \sum_{\forall rs}\sum_{\forall k}\sum_{\forall t}\eta_k^{rs}(t,\mathbf{f})f_k^{rs}(t,\mathbf{Z}) \tag{19}$$

To ensure stability of traffic flow at a metered on-ramp, the ramp metering rate may vary after multiple consecutive time intervals, but not at every time interval. These multiple consecutive periods form a ramp metering period. More specifically, the discretized time horizon $\left[0,\delta,2\delta,\cdots,(T-1)\delta\right]$ is completely partitioned into N disjoint ramp metering periods, where N is a positive integer, named by $\left(\Delta_{l-1},\Delta_l\right]$, $l=0,1,2,\cdots,N$, in which $\Delta_0=0$ and $\Delta_N=(T-1)\delta$. Having defined these ramp metering periods, the period-dependent ramp metering rate pattern is obtained.

To imitate the traffic responsive type of ramp metering operation, the ramp metering rate should be adjusted according to the average queuing length of the on-ramp in the previous metering period, expressed by,

$$Z_a(t)=\rho_a\times H_a\left(\Delta_{l-1}\right),\quad \forall t\in\left(\Delta_{l-1},\Delta_l\right],l=1,2,\cdots,N,a\in A_{on}^{+} \tag{20}$$

where $0\le\rho_a\le1$. ρ_a is referred to as the ramp metering ratio which is the decision variable to be determined. $H_a\left(\Delta_{l-1}\right)$ is the average volume of vehicle at the on-ramp in the previous metering period $\left(\Delta_{l-2},\Delta_{l-1}\right]$ that can be calculated by

$$H_a\left(\Delta_{l-1}\right)=\frac{\sum_{t\in\left(\Delta_{l-1},\Delta_l\right]}\sum_{i=1}^{I_a}\left[n_{a_i}(t)-\sum_{h_m\in\Gamma^-(a_i)}y_{a_ih_m}(t)\right]}{\Delta_{l-1}-\Delta_l} \tag{21}$$

where $n_{a_i}(t)$ is the vehicle volume in cell (a,i) of arc $a\in A$ at time t and $y_{a_ih_m}(t)$ is the vehicle flow on link $\left(a_i,h_m\right)$ of the cell-based network, which connects cell (a,i) of arc $a\in A$ to cell (b,i) of arc $b\in A$, from clock tick t to $t+1$. To some extent, eqn. (20) gives a dynamic feedback control strategy of traffic flow.

It is now ready to present the single level optimization model for the ramp metering operation:

$$\min F(\mathbf{Z})=\sum_{\forall rs}\sum_{\forall k}\sum_{\forall t}\eta_k^{rs}(t,\mathbf{f})f_k^{rs}(t,\mathbf{Z}) \tag{22}$$

subject to

$$Z_a(t)=\rho_a\times H_a\left(\Delta_{l-1}\right),\quad \forall t\in\left(\Delta_{l-1},\Delta_l\right],l=1,2,\cdots,N,a\in A_{on}^{+} \tag{23}$$

$$f_k^{rs}(t,\mathbf{Z})=p_k^{rs}\left(\mathbf{C}^{rs}(t,\mathbf{f})\right)q_{rs}(t),k\in\hat{K}_{rs},r\in R,s\in S,t=0,1,2,\cdots,T-1 \tag{24}$$

As a brief explanation, eqn. (22) denotes the objective function of the optimization model is to minimize the total travel time of the study area with the ramp metering rate as the decision variables. Eqn. (23) expresses the ramp metering rate constraint, which can reflect the dynamic feedback control strategy of the traffic responsive type ramp metering rate modeled in this study. Eqn. (24) determines the tolerance-based DSUO flow condition.

SOLUTION ALGORITM

A meta-heuristic method is adopted to solve the proposed single level optimization model of the ramp metering operations. Solving the DSUO flow using MSA yields approximated solutions. In view of this, meta-heuristics algorithms could be adopted to solve the optimization model since these algorithms only require the objective function values to judge the solution, without the necessary exact relationship or solution. Genetic algorithm (GA) is chosen as the solution method in light of its wide application in the engineering related problems (Gen and Cheng, 1996).

A dynamic ramp metering rate solution $\mathbf{Z}$ is encoded into a binary string, of which a specified portion represents a metering rate for a particular on-ramp at time t. More specifically, the metering ratio, $\{\rho_a, a \in A_{on}^{+}\}$ for the period-dependent ramp metering rate schemes shown in eqn. (20) is encoded as the binary strings. The strings then are decoded to find the corresponding dynamic ramp metering rate solution by means of eqn. (20). This is then embedded into MCTM to compute the corresponding actual route travel time.

The step-by-step procedure is shown as follows:

Step 0. (Initialization) Randomly generate a population of B strings.

Step 1. (Decoding) For each string, decode the string to get the dynamic time dependent ramp metering rate for the on-ramps of the expressway-arterial network, $\mathbf{Z}$.

Step 2. (Call MSA) Decode each string b in the population and perform the DSUO assignment by implementing the procedures shown earlier.

Step 3. (Fitness function calculation) Compute the value of fitness function defined by eqn. (19) for each string.

Step 4. (Generation of a new population) Repeat the following four sub-steps until the new population is completed.

- Step 4.1. (Selection) According to the fitness functions value evaluated in Step 2, use the rank selection method to choose two parent strings from the population.
- Step 4.2. (Crossover) With a crossover probability, denoted by p_c, cross over the parents to form a new offspring according to the one point crossover method. If no crossover is performed, offspring is the exact copy of the parents.
- Step 4.3. (Mutation) With a mutation probability, denoted by p_m, mutate new offspring at selected position in string.

Step 5. (Stopping criterion) If a stopping criterion is fulfilled, terminate the algorithm. Output the best solution from the population. Otherwise, go to Step 2.

There are two termination criteria that can be adopted in Step 4 of the GA. The GA can be terminated when it achieves maximum number of generations specified or if there is no improvement in the fitness function value for more than the number of generations specified. Note that the performance of the GA may rely on the size of population and crossover and mutation probabilities.

NUMERICAL EXAMPLE

An illustrative expressway-ramp-arterial network system shown in Figure 2 is used to test the applicability of the proposed methodology. It is a hypothetical network based on the I210W expressway-ramp network in Pasadena, California (Munoz et al. 2004). It is 22-km long and consisted of 21 on-ramps and 18 off-ramps. From the figure, it is observed that the expressway mainline is connected to the arterial roads through the on-ramps and off-ramps. The arterial roads serve as an alternative road to the expressway-ramp system. The origins and destinations nodes are labeled in the figure. In addition, dummy links are created to load the demand from these origins into the network. The cell-based network created for MCTM is shown in Figure 3. It is shown that the origin cells serve as the big parking lots that store all the vehicles that will be loaded into the network system. The vehicles are loaded into the network through dummy links/cells. Note that the length of the arterial roads in Figure 3 is not to the scale.

The input parameters for the MCTM is such as follows - the free flow speed for expressway mainline section, ramps, and arterial roads are 100km/hr, 60km/hr, and 60km/hr respectively. The backward shock wave speed is 28km/hr, and the jam densities are 17 veh/km/lane for the expressways, ramps, and arterial roads. The size of a cell on the expressway mainline segments, ramps, and arterial roads are 0.28km, 0.17km, and 0.17km respectively. The time interval δ is 10 seconds and the time dependent ramp metering period is 5 minutes. The capacity of the expressway is set to be 2200 veh/hr/lane while the arterial roads capacity is 2000 veh/hr/lane. The ramp capacity is 2000 veh/hr for one-lane ramp with speed of 60km/hr and 2400 veh/hr for two-lane ramp. This is obtained by referring to the Highway Capacity Manual (2000).

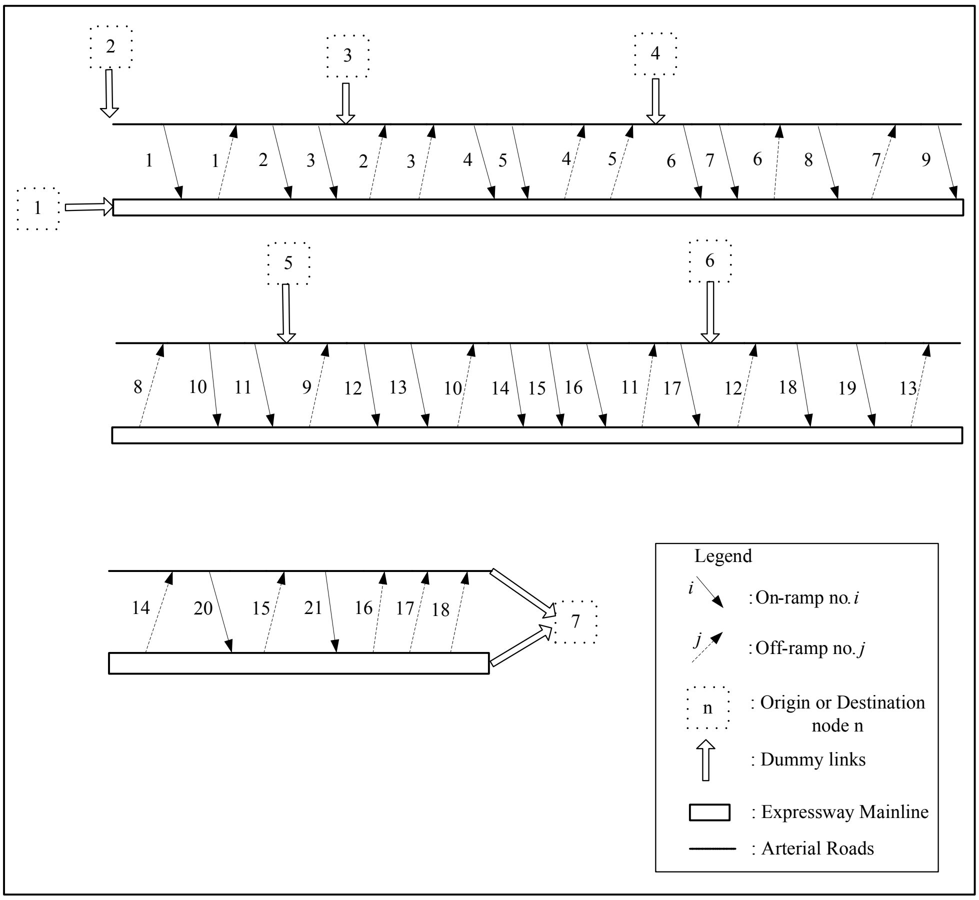

Figure 2 The hypothetical expressway-arterial network system

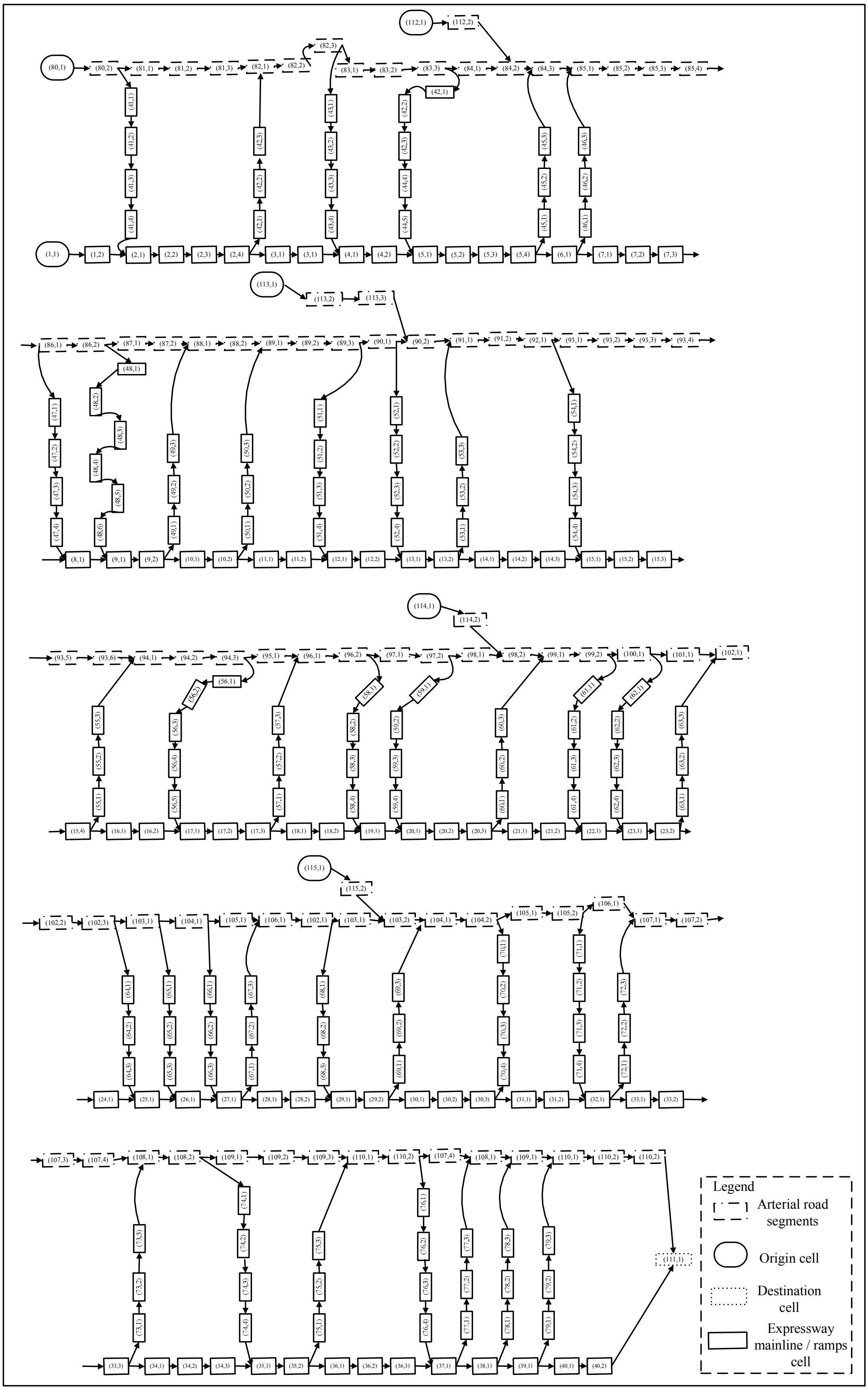

Figure 3 The cell-based network for Figure 2

Assume that there are 6 pairs of origin-destination (OD) trips in the network system with assumed demand value. These OD pairs and their demand are presented in Table 1. The departure time of the drivers is fixed and is carried out during the first 10 time step of the assignment period. The simulation is run until all the vehicles arrived at the destinations, which takes about 40 minutes of the simulation time. The k-shortest route between each OD pair is found using the algorithm earlier. The shortest route size is shown in Table 1 using $\hat{K}_{rs}$ -shortest route algorithm. The value for the parameters is set such as follows: $O_p = 0.5$ and $\alpha = 2$.

For the genetic algorithm setting, each ramp metering ratio in eqn.(20), $0 \le \rho_a \le 1, a \in A_{on}^{+}$, is encoded by a 7-bit binary chromosome. With a total of 21 on-ramps, the length of a chromosome is 147 genes in the GA embedding with the MCTM. In addition, the population size is 50 and the algorithm is terminated when the objective function value does not change for 3 generations, and the crossover probability and the mutation probability are 0.7 and 0.03, respectively.

For the Monte Carlo sampling, the value for $\theta_{kl}^{rs}(t)$ shown in eqn. (9) is taken as 10 and is identical for all overlapped route of all OD pairs at all departure time. The sampling process is run for 200 iterations before the probability is computed.

Table 1 The OD pair and the route number

OD Pair	Origin Node	Destination Node	Route Number	Demand (veh/hr)
1	1	7	20	750
2	2	7	20	500
3	3	7	12	200
4	4	7	14	200
5	5	7	10	155
6	6	7	10	195

Results

Using the proposed strategy, the result for the hypothetical network is shown in Figure 4. It can be seen that GA terminates at the 19th generation when the objective function values does not change for 5 subsequent generations. It is found that there is about 11.25% of improvement in the total travel time if compared to the initial solution. The resulted ramp metering rate obtained is shown in Figure 5. It is found that the percentage of total travel time improvement is sensitive to the OD demand level used. The benefit gained will be smaller if lighter demand is loaded.

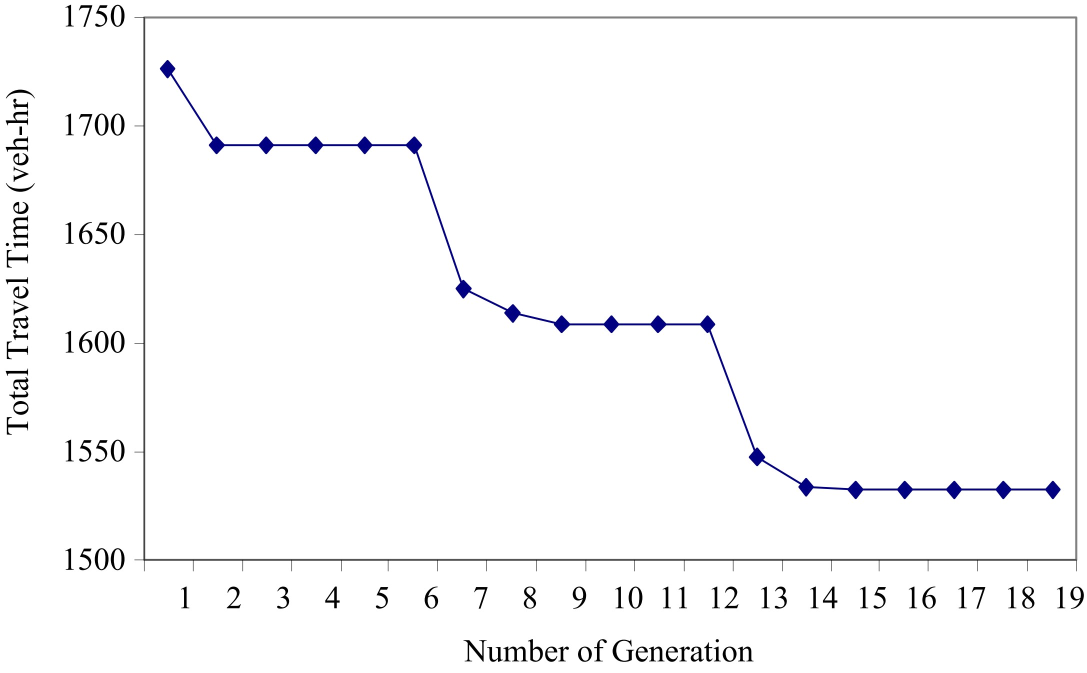

Figure 4 The convergence trend of the GA-DSUO

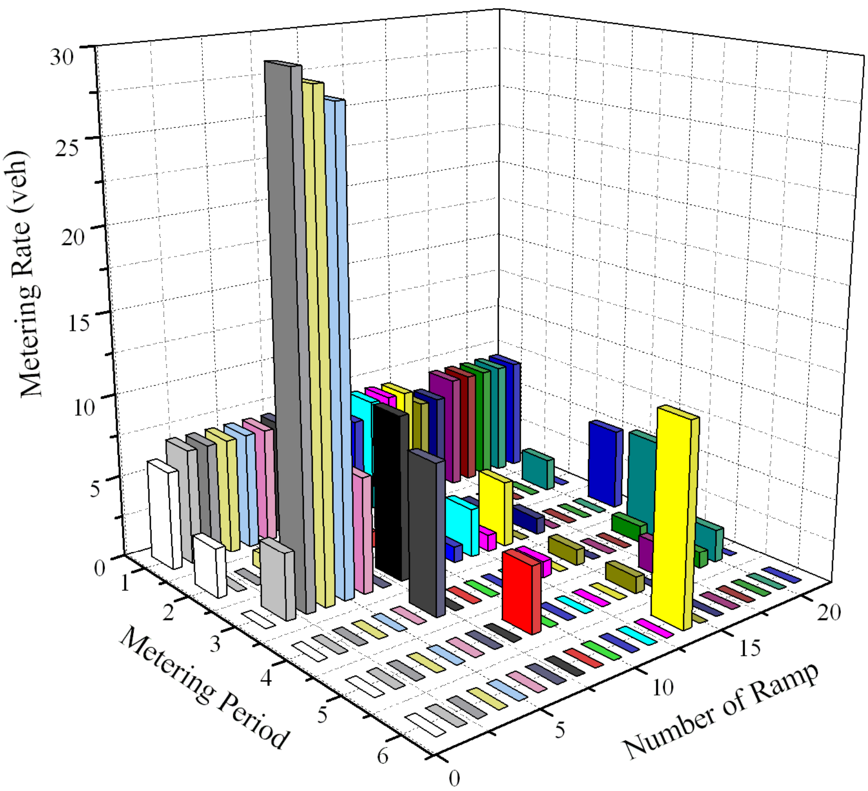

Figure 5 Metering rate solutions

MSA Results

Using the best time-dependent ramp metering rate, i.e. the metering rate that gives the lowest total travel time, the DSUO flow is computed by performing the MSA as proposed earlier. When the average absolute error of the algorithm, ∇ does not change for 3 subsequent iterations, the algorithm is terminated and the results are collected. The average absolute error shown in the figure is 1 vehicle. The trend of MSA over iterations is shown in Figure 6. It is worth noting that, the maximum absolute error observed is 22 vehicles, which occurs for the vehicles loaded during the last departure time.

A closer investigation on the comparison of the DSUO flow obtained for the with-ramp metering case and the without-ramp metering case show that there is a shift of traffic flow from the expressway-ramp system to the arterial streets. Taking OD pair number 2 as an example, it has 20 routes for drivers who would like to travel from origin node 2, on the arterial road to node 7, on the expressway. For the 20 routes given, drivers have different choices of the on-ramps to choose from to enter the expressway. Table 2 presents the entry ramps for each route and the DSUO route flow obtained for the ‘with ramp metering’ and ‘without ramp metering’ case. The values shown in columns 3 and 4 of Table 2 are the summation of the vehicles departing from node 2 using respective route over the period of time. Note that although some of the routes have the same on-ramp entry points, the routes are different. For example, once drivers enter on-ramp 1 using route 1, they will travel using the expressway mainline to the destinations. Nevertheless, for route 4, drivers will exit the expressway mainline using the off-ramps located at the downstream of the network system, and continue their journey to the destination using the arterial roads. Similarly, drivers using route 4 exit the expressways through different off-ramp compared to route 6. The same explanation also goes to other routes. Besides, an interesting point to note from Table 2 is that, no drivers choose to use route 2. This is because route 2 is an arterial road that connects between 2-7. No drivers perceived it as the shortest route at any of departure time because the design speed of the arterial roads are lower than that set for the expressways.

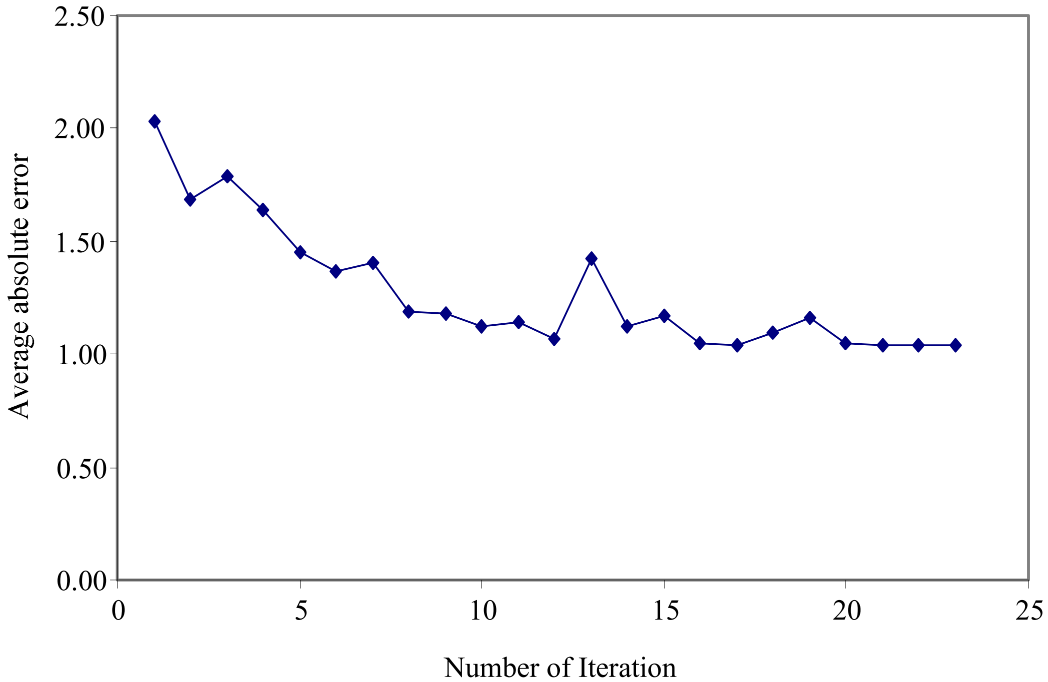

Figure 4 The approximated converging pattern for MSA

From Table 2, it could be observed that before the implementation of ramp metering, about 60% of the drivers choose to use on-ramps 1 and 2. However, when the ramp metering is applied, it could be seen that, a significant number of drivers have shifted to other routes. A 5% increment in the number of drivers could be observed for routes 7 and 8, while other routes have slight increment in users. This increment is accompanied by a 15% drop in the number of drivers using route 1 and 3. This shows that ramp metering has influenced the route choice decision of drivers. The results found are consistent with Wu (2001) findings. When it is applied, some of the drivers choose other routes, or other on-ramps to enter the expressway mainline. This has confirmed the argument made earlier in this study to support the importance of the incorporation of route choice model (DSUO) in the ramp metering study. The total travel delay before the ramp metering implementation is 109.1 veh-hr, while the best ramp metering solution yields a total travel time of 61.7 veh-hr. This is a saving of 76% in total travel time.

Table 2 Comparison of DSUO flow for the with and without ramp metering case

Route number [1]	Entry on-ramp [2]	Exit off-ramp [3]	DSUO flow without ramp metering [4] (veh)	DSUO flow after ramp metering [5] (veh)	Change [5]-[4] (veh)
1	1	N.A.	196	166	-30
2	N.A.	1	0	0	0
3	2	N.A.	98	60	-38
4	1	6	0	5	5
5	4	7	32	35	3
6	1	9	2	6	4
7	5	3	25	45	20
8	6	10	40	52	12
9	2	16	1	5	4
10	4	17	2	5	3
11	4	10	19	19	0
12	3	13	13	15	2
13	7	9	9	10	1
14	19	N.A.	3	9	6
15	3	11	11	11	0
16	5	15	2	4	2
17	17	N.A.	13	13	0
18	5	16	16	16	0
19	4	12	17	17	0
20	8	13	1	7	6
Total			500	500	

CONCLUSION

A ramp metering study that considers origin-destination trip information has been considered in this study. A non-linear single level optimization model is developed to minimize the total travel time of the expressway-ramp-arterial network system while searching for the optimal ramp metering rate. A probit-based dynamic ideal stochastic user optimal (DSUO) assignment model is adopted to describe the drivers' route choice decision behavior in response to the given ramp metering rate. The DSUO assignment model is formulated as a fixed point problem and is considered as one of the constraints of the optimization model. As such, the objectives of both parties, the system managers and the drivers, could be taken into account. Not all of the available routes for an origin-destination (OD) could be considered, especially in the case of a larger size network, where only partial routes are considered. In addition, the utilization of the MCTM model as the traffic flow model has posed some challenges in determining the exact solution of the probit-based ideal DSUO. As a result, the outcome from the MSA is an approximation. Hence, the optimization model is solved by meta-heuristics algorithm which only requires the objective function values without the need for exact relationship or solutions, be known.

The illustrative network example presented shows that MSA could solve the probit-based DSUO assignment model with sufficient accuracy. In addition, GA could solve the optimization model to give a better solution compared to the initial solution. A closer investigation revealed that DSUO could address the drivers' route choice decision well. A significant diversion of drivers from the expressway-ramp system to the arterial road could be observed. Results show that drivers would consider the queuing condition at the on-ramps and choose to enter the expressway-ramp system through the less congested on-ramps. It is therefore necessary for the system managers to consider the drivers route choice behavior during optimization of ramp metering operations. As the future work, the calibration and validation of the network should be carried out.

REFERENCES

Bellemans, T., B. De Schutter, and B. De Moor. (2003). Anticipative model predictive control for ramp metering in freeway networks. Proceedings of the American Control Conference, 5, 4077-4082.

Berinde, V. (2007). Iterative approximation of fixed points. 2nd. Edition. New York: Springer.

Bovy, P.H.L., and S.F. Catalano (2007). Stochastic route choice set generation: Behavioral and probabilistic foundations. Transportmetrica, 3 (3), 173-189.

Cambridge Systematics Inc. (2000). Twin Cities Ramp Metering Effectiveness Study: "Before" and "After" Qualitative Research with Travelers. Technical Report. <http://www.dot.state.mn.us/rampmeterstudy/reports.html >Assessed May 10, 2007.

Cascetta, E., F. Russo, and A. Vitetta. (1997). Stochastic user equilibrium assignment with explicit path enumeration: Comparison of models and algorithms. In M. Papageorgiou and A. Pouliezo, eds. Proceedings of the 8th International Federation of Automatic Control On Transportation Systems. Chania, Greece, 1078-1084.

Chang, T-H., and Z.Y. Li (2002). Optimization of mainline traffic via an adaptive co-ordinated ramp-metering control model with dynamic OD estimation. Transportation Research Part C, 10, 99-120.

Chu, L. Y., X.L. Henry, W. Recker, and H.M. Zhang (2004). Performance evaluation of adaptive ramp-metering algorithms using microscopic traffic simulation model. Journal of Transportation Engineering, 130(3), 330-338.

Dijkstra, E.W. 1959. A note on two problems in connection with graphs. Numerical Mathematics, 1, 269-271.

Gen, M., and R. Cheng (1996). Genetic algorithms and engineering design. New York: John Wiley.

Hasan, M., M. Jha, and M. Ben-Akiva. (2002). Evaluation of ramp control algorithms using microscopic traffic simulation. Transportation Research Part C, 10, 229-256.

Hegyi, A., B. De Schutter, H. Hellendoorn and T. Van Den Boom. (2002). Optimal coordination of ramp metering and variable speed control-an MPC approach. In Proceedings of the American Control Conference. 3600-3605.

Hellinga, B. R., and M. Van-Aerde. (1995). Examining the potential of using ramp metering as a component of an ATMS. Transportation Research Record, 1494, 169-172.

Jacobson, L., K. Henry, and O. Mehyar. (1989). Real time metering algorithm for centralized control. Transportation Research Record, 1232, 17-26.

Kotsialos, A., and M. Papageorgiou. (2004). Nonlinear optimal control applied to coordinated ramp metering. IEEE Transactions on Control Systems Technology, 12(6), 920-933.

Kotsialos, A., M. Papageorgiou, M. Mangeas, and H. Hadj-Salem. (2002). Coordinated and integrated control of motorway networks via nonlinear optimal control. Transportation Research Part C, 10(1). 65-84.

Lahiti, S., A.C. Gan, and Q. Shen. (2002). Using simulation to estimate speed improvements from simple ramp metering at on-ramp junction. Paper presented in the 81st Transportation Research Board Annual Meeting. Washington, D.C.CD-ROM.

Lau, R. (1997). Ramp metering by zone: The Minnesota algorithm. Minnesota Dept. of Transportation. USA.

Lim, Y., and H. Kim. (2005). A shortest path algorithm for real road network based on path overlap. Journal of Eastern Asia Society for Transportation Studies, 6, 1426-1438.

Lo, H., and A. Chen. (2000). Traffic equilibrium problem with route-specific costs: Formulation and algorithms. Transportation Research Part B, 34, 493-513.

Lo, H., and W.Y. Szeto. (2002). A cell-based dynamic traffic assignment model: Formulation and properties. Mathematical and Computer Modeling, 35, 849-865.

Meng, Q., and H.L. Khoo (2009). A Pareto-optimization approach for a fair ramp metering. Transportation Research Part C. In Press. Doi:10.1016/j.trc.2009.10.001.

Munoz, L., X. Sun, D. Sun, G. Gomes, and R. Horowitz. (2004). Methodological calibration of the Cell Transmission Model. In Proceeding of the 2004 American Control Conference, Boston, Massachusetts, June 30-July 2, 2004. 798-803.

Papageorgiou, M. and A. Kotsialos. (2002). Freeway ramp metering: An overview. IEEE Transactions on Intelligent Transportation Systems, 3(4), 271-281.

Papageorgiou, M., J.M. Blosseville, and H.S. Habib. (1990). Modeling and real-time control of traffic flow on the southern part of Boulevard Peripherique in Paris: Part II: coordinated on-ramp metering. Transportation Research Part A, 24, 361-370.

Papageorgiou, M., H.S. Habib, and J.M. Blosseville. (1991). ALINEA: A local feedback control law for on-ramp metering. Transportation Research Record, 1320. 58-64.

Papageorgiou, M., H. Salem and F. Middelham. (1997). ALINEA local ramp metering: Summary of field results. Transportation Research Record, 1603. 90-98.

Sheffi, Y. (1985). Urban transportation networks: Equilibrium analysis with mathematical programming methods. New Jersey: Prentice Hall.

Szeto, W.Y. and H. Lo. (2005). Non-equilibrium dynamic traffic assignment. In Mahmassani, H.S., ed, Proceedings of the 16th International Symposium on Transportation and Traffic Theory. New York: Elsevier. 427-445.

Wattleworth, J.A. and D.S. Berry. (1965). Peak-period control of a freeway system-some theoretical investigations. Highway Research Record, 89(1). 1-25.

Wu, J. (2001). Traffic diversion resulting from ramp metering. Master Degree Thesis, The University of Wisconsin, Milwaukee.

Yang, H., and S. Yagar. (1994). Traffic assignment and traffic control in general freeway arterial corridor systems. Transportation Research Part B, 28 (6), 463-486.

Zhang, H., S.G., Ritchie and W.W., Recker. (1996). Some general results on the optimal ramp control problem. Transportation Research Part C, 4(2). 51-69.

Zhang, H.M., Ritchie, S.G., and R. Jayakrishnan. (2001). Coordinated traffic-responsive ramp control via nonlinear state feedback. Transportation Research Part C, 9. 337-352.