THE EFFECT OF OD TRIP DISPERSION VERSUS CONCENTRATION IN EXPRESS SERVICE DESIGN

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ABSTRACT

In public transit systems with high demand levels, the use of express bus services that serve only a subset of stops along certain routes would seem to be a promising alternative given the benefits they offer to both users and operators. In actual practice, express services in systems such as Transmilenio (Bogota, Colombia), Transantiago (Santiago, Chile), and Metro Rapid (Los Angeles, CA) have proven to be highly appealing.

This raises the question about when express services are a reasonable option. Previous work has focused on how some characteristics of the demand structure of a corridor affects the benefits that express services can yield, showing that the load profile shape and the average trip length are crucial. This work presents some evidence that also the dispersion of the demand among different OD pairs (keeping the load profile and the total number of trips constant) affects the potential benefits of express services. As expected the more concentrated the demand into few OD pairs, the more cost savings that can be obtained.

To answer this question we developed a methodology to generate OD matrices that share all relevant attributes but differ in variability among OD flows. Thirteen matrices were generated and their optimal sets of bus services with their respective frequencies were obtained. Using the coefficient of variation as a measure of matrix variability, we confirmed that more demand variability (i.e. more flow concentration) gives room to more express services and lower social costs.

Keywords: Express Services, Bus Rapid Transit (BRT), Network Design, Route Design, Public Transport.

INTRODUCTION

In public transit systems with high demand levels, the use of express bus services that serve only a subset of stops along certain routes would seem to be a promising alternative given the benefits they offer to both users and operators. For users, express buses mean improved service levels in the form of lower travel times due to fewer stops and higher between-stop speeds, while for system operators they enable demand to be met with fewer vehicles thanks to shorter bus cycles. In actual practice, express services in systems such as Transmilenio (Bogota, Colombia), Transantiago (Santiago, Chile), and Metro Rapid (Los Angeles, CA) have proven to be highly appealing. The type of express services operated in different cities are different, in Transmilenio tend to be stop skipping services, where as in Transantiago are normally non-stop services connecting terminals at different extremes of the city.

Leiva *et al.* (2010) proposed an optimization method for designing express services that minimizes the social costs of a segregated bus corridor assuming a known trip demand matrix. The social costs considered include wait time, travel time and the costs facing the operator. For each possible service suggested, the model determines its frequency (eventually null) and bus sizes to be used.

Based on this methodology Larrain *et al.* (2010) studied which express services to offer on a bus corridor depending on some characteristics of its demand. The authors show the impact of trip length, load profile shape, volume and structure in the attractiveness of express services for the full system. The authors also mention that the more concentrated within few OD pairs the demand is, the more beneficial express services should be. However, they do not provide any evidence for this claim. The main objective of this paper is to determine the impact of demand variability within an OD matrix in express services design. In the context of this paper we will understand demand variability as how concentrated or spread are the trips within the OD pairs of a corridor assuming that the boarding and alighting in each bus stop is fixed and known. For instance if the destination of trips boarding at a specific stop are spread across many bus stops, then flow variability will be low in comparison to a case where most of those trips have a common destination. According to Larrain *et al.* (2010) express services would be more attractive in corridors where the OD demand matrix presents high variability.

The remainder of this paper is organized as follows. Section 2 presents previous literature in express services design focusing on the previous work that was directly used to reach our objectives. Section 3 introduces a performance indicator for demand variability and presents a methodology to develop OD matrices that share most structural attributes but show different levels of variability. In section 4 we implement the methodology to develop different OD matrix scenarios that are later evaluated in terms of their attractiveness for express services. Finally, section 5 completes the paper with some concluding remarks and suggestions for future extensions.

EXPRESS SERVICE DESIGN MODELS FOR BUS CORRIDORS

Literature Review

In Furth and Day (1985), three different established planning strategies for high-demand corridors are discussed:

1. *Short turning*. Some buses serving a route make shorter cycles in order to concentrate on areas of greater demand. This is useful when it is desired to bolster capacity along a given stretch of the route.

2. *Deadheading*. Empty vehicles return to the route starting point in the low-demand direction in order to begin another run as quickly as possible in the high-demand direction, thus increasing the latter's frequencies. This is advantageous when demand along the corridor is imbalanced.

3. *Express services*. Services that visit only a subset of the stops on a route.

Although various works in the literature focus on the first two approaches, such as Furth and Day (1987) on short turning and Ceder and Stern (1981) on deadheading, there appears to be no published research that explores optimization techniques for designing express services on high demand corridors and evaluating their benefits.

The literature has a considerable amount of work on the transit-network design problem in which given a network and trip demand Origin-Destination matrix, a set of routes and their respective frequencies have to be determined by minimizing a sum of users and transit operator costs (see, for example, Ceder and Wilson, 1986, Leblanc, 1988, Baaj and Mahmassani, 1992 and 1995, Fan and Machemehi, 2006, and Mauttone and Urquhart, 2009). Recent reviews are presented in Ceder (2003), and in Desaulniers and Hickman (2007). One of the main limitations of these previous efforts is that capacity constraints are not considered in the assignment representation. In that case the system optimum and user equilibrium are equivalent. However, when capacity constraints are relevant user behavior should be explicitly considered in the service design problem. Fernández *et al.* (2003, 2008) present a bi-level approach to solve this problem. To the best of our knowledge, the first model that allows for service design in a corridor with capacity constraints in a single model, incorporating user behavior, was presented in Leiva *et al.* (2010). Therefore, this is the model used for express service design in the analysis performed in this article. This model is presented in the next subsection.

To better understand what conditions make a corridor favorable to the implementation of express services and other kind of strategies, Furth and Day (1985) suggested three indicators: (i) the ratio between the peak volume of the load profile versus the number of passengers that are boarding the bus before this point (denoted as PV/UB ratio), (ii) the corridor length, and (iii) the ambient speed on transit route. Among these three indicators the only one that describes the demand type of the corridor is the first one.

Model Used for Express Service Design

Leiva *et al.* (2010) propose a design model for express services in public transit corridors with capacity constraints that minimizes social costs. It assumes first of all that the network topology representing the corridor is known and fixed, implying that we know a set of stops $i \in P$ (where $n = |P|$) and have some notion of the between-stop distances so as to estimate the corresponding travel times. A second assumption of the model is that there exists an exogenous trip matrix for the corridor that is also known. Thus, $w \in W$ is defined as the set of network origin-destination pairs and T_w is the flow of passengers for pair w in a given period. For simplicity, this value is assumed to be fixed, and its effect on modal share of express services will be neglected. Finally, it is assumed that the set of possible corridor services from which the services to be used will be selected are known *a priori*. These services are defined by the set of stops they serve. This means that the planner has a basic intuition of what kind of lines would be of interest to take into account. This may leave some feasible solutions out of the analysis, but makes the problem considerably less complex. For each of these services the travel times for each pair of visited stops are also assumed known. Finally, the model requests passengers traveling on any OD pair to choose the route that minimizes their expected travel time.

To consider that passengers may transfer within services in the corridor, the mathematical model must allow trips to be divided into trip legs to reach their destinations. Thus, a set of route sections S (corresponding to trip legs that passengers may take, for details see De Cea and Fernández, (1993)) is defined and a set of flow variables, V_s^w with $s \in S$ and $w \in W$ representing the number of trips in pair w that will take route section s. Also, the following sets of variables must be defined: f_l, the frequency of line l; f_l^s, an auxiliary variable corresponding to the effective frequency of line l for passengers taking trip leg in route section s (it will be zero for passengers considering the line not attractive and will be f_l for those considering it attractive). This variable is needed to incorporate the rational travel behavior proposed by Chriqui and Robillard (1975) as introduced in Leiva *et al.* (2010). The proponed model is as follows:

$$\min SC = C_{Op} + C_{TT} + C_{WT} + C_{Tr} \qquad (1)$$

Subject to:

$$0 \leq f_l^s \leq f_l, \ \forall l \in L, \ \forall s \in S \qquad (2)$$

$$\sum_{s \in S_i^+} V_s^w - \sum_{s \in S_i^-} V_s^w = \begin{cases} T_w & \text{if } i = O(w) \\ -T_w & \text{if } i = D(w) \\ 0 & \text{otherwise,} \end{cases} \qquad \forall i \in P, \forall w \in W \qquad (3)$$

Where:

$$C_{Op} = \sum_{l \in L} c_o \cdot f_l \qquad (4)$$

$$C_{TT} = \theta_{TT} \cdot \sum_{w \in W} \sum_{l \in L} V_s^w \cdot \frac{\sum_{l \in L} tt_l^s \cdot f_l^s}{\sum_{l \in L} f_l^s} \qquad (5)$$

$$C_{WT} = \theta_{WT} \cdot \sum_{w \in W} \sum_{l \in L} V_s^w \cdot \frac{k}{\sum_{l \in L} f_l^s} \qquad (6)$$

$$C_{Tr} = \theta_{Tr} \cdot \left(\sum_{w \in W} \sum_{l \in L} V_s^w - \sum_{w \in W} T_w\right) \qquad (7)$$

In this model, the objective function is to minimize the social costs corresponding to the sum of operator costs, in vehicle trip times, waiting times and transfers. Decision variables are flows, V_s^w, and frequencies, f_l and f_l^s. The first set of constraints (2) requires the effective frequency of a line for a route section *s* to be non-negative and not to exceed f_l. The second set of constraints (3) guarantees flow continuity in the network, where S_i^+ and S_i^- stand for the sets of route sections that start and end on node i, and functions $O(w)$ and $D(w)$ indicate the origin and destination node of pair w.

The operational costs on each line are assumed proportional to their frequencies; this is ensured by equation (4). Average in-vehicle trip times on each route section correspond to an average of trip times of all attractive lines for that section weighted by their frequencies, as presented in equation (5). For this calculation, the travel time for each route section on each line, tt_l^s, is assumed to be known and fixed. Equation (6) states that waiting times on each route section are inversely proportional to the effective frequency on the section. In this equation, parameter k could be replaced by k_l to address differences on headway variances between different lines. In this work, for simplicity we considered the same for all lines. Finally, the total number of transfers can be computed as the sum of all the trip legs minus the sum of all the trips in the network (7). In these expressions, θ_{TT}, θ_{WT}, and θ_{Tr} represent monetary costs of in-vehicle travel times, waiting times and transfers. Notice that the parameter θ_{Tr} is assumed to be the same regardless of the stop in which the transfer takes place, and that it does not include waiting times (just transfer times and a penalty due to the inconvenience of the transfer) because expression (6) takes into account the waiting time of every trip section.

The user behavior that underlies this model assumes that passengers do not have information about the departure times and that the distribution of bus headways is captured by k in expression (6): for example, values for k of 0.5 and 1 represent the cases where the headways are regular, and when they follow an exponential distribution. Under these conditions, users choose the itinerary that minimizes their total expected travel time. These assumptions are appropriate when service frequencies are high and operated without schedule, as in the case of the city of Santiago. However, in some cases it would be of interest to consider other type of behavior like the one that occurs when frequencies are low and the services are scheduled, that also may have implications regarding the itinerary choice (see for example Nuzzolo (2003)).

Demand structure effect

Larrain *et al.* (2010) used the model proposed in Leiva *et al.* (2010) to determine in which corridors would be more convenient to use express services according to their demand attributes. The O/D matrix attributes considered were:

1. Load profile shape: The corridor load profile gives the hourly number of passengers traveling between each pair of consecutive stops (i.e. boarding somewhere upstream and alighting somewhere downstream). Three shapes for the load profile were tested: decreasing, single peaked, and double peaked. These three shapes are depicted in figure 1.

2. Scale of demand: This attribute corresponds to the total trip volume in the corridor. It's implemented via a scaling factor. This is of particular interest in the case when vehicle capacity is considered.

3. Demand imbalance: Given that the corridors tested function in both directions, both outbound and inbound profiles are needed. Larrain *et al.* (2010) assumed that the latter is just a mirror image of the former, but scaled down by an imbalance factor.

4. Average trip length: It can be calculated as the total distance traveled by all trips (obtained from the load profile) divided by the number of trips. It does not depend directly from the load profile, because a single load profile can be obtained from few long trips or a lot of small trips (for example, it is possible to divide one long trip into two short ones without altering the load profile, and clearly shortening the average trip length).

For each one of these attributes, three levels were defined, as can be seen in figure 1. This yielded a total of 81 studied scenarios.

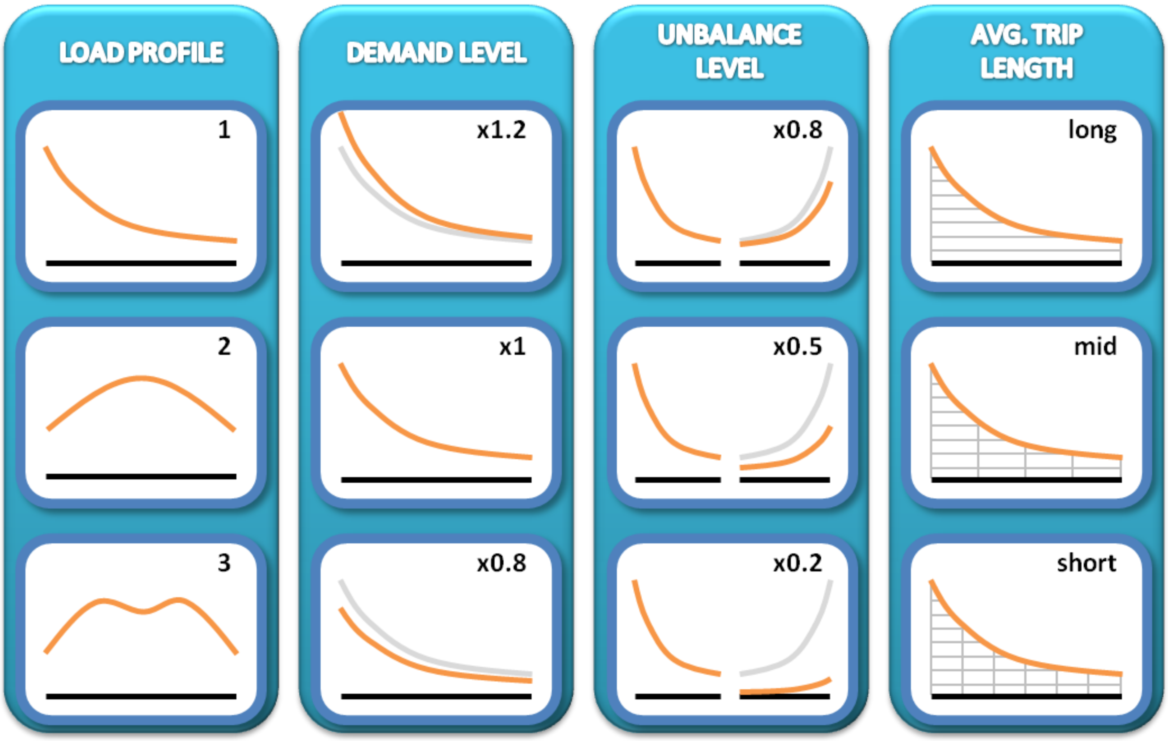

Figure 1 – Three levels are described for each of the four attributes considered.

To judge the significance and potential of the express services under the different scenarios on the basis of this information, the following three indicators were defined:

1. Optimal express service participation. This indicator is calculated as the percentage of the total fleet assigned to non all-stop services in a given scenario.

2. Number of different services: Refers to the number of different services that coexist within the optimal design of a given scenario. In the case where there exist no express runs, only an all-stop service, this indicator takes the value of 1.

3. Reduction of social costs: Expresses the percentage reduction of the objective function value compared to the value for an all-stop service operating at its optimal frequency.

One of the main conclusions of Larrain *et al.* (2010) is that the potential benefits of express services grow with the average trip length along the corridor. It also noticed that the demand matrices used in that work were "soft" (due to the entropy maximization model used to generate them) in the sense that trips are somehow evenly distributed across OD pairs, whereas real trip matrices have more singularities (that is, isolated pairs that for various reasons capture significantly more flow than others). That work affirmed that these singularities should provide more opportunities for applying express services.

METHODOLOGY

With regard to measure how matrix variability affects express service design, we need to determine how demand variability will be measured and develop a methodology to build different scenarios where most demand characteristics are shared, but variability. Thus, the matrices will be constructed keeping load profiles and average trip lengths constant, in order to isolate the effect of demand variability.

Measuring matrix variability

To compare the demand variability among different scenarios, we will use the coefficient of variation (CV) among OD trips. This index is defined as the ratio of the standard deviation to the mean of a series of data. This is a dimensionless value.

OD matrix generation

As mentioned earlier, in order to isolate the effect of demand variability we propose a method for generating a set of OD matrices sharing the load profile and average trip length but with different levels of variability. To do so, we observed that given the origin and destination vectors O_i and D_j of any OD matrix, the total passenger load carried on any consecutive section of bus stops (p, p+1) can be computed as:

$$Load_{p,p+1} = \sum_{i=1}^{p}(O_i - D_i) \quad (8)$$

In simpler terms, this equation states that in any given section of the corridor the number of passengers riding the buses is the total of passengers that have boarded before this section, minus the passengers that have already alighted before this point. On the other hand, the average trip length can be calculated as follows:

$$atl = \frac{\sum_{i=1}^{n} Load_{i,i+1} \cdot lenght_{i,i+1}}{\sum_{i=1}^{n} O_i} \qquad (9)$$

Equations (8) and (9) imply that if two matrices have the same origin and destination vectors, then their load profiles and their average trip lengths will be the same. Hence, one way of creating demand matrices with different variability without altering these attributes would be to take an initial feasible OD matrix and alter its flows without varying its origin and destination vectors. If we call $T_{i1,j1}$ the flow on an arbitrary pair, a way to increase this value without changing these vectors would be to increase $T_{i1,j1}$ and an auxiliary pair $T_{i2,j2}$ (such that $i_1<j_2$ and $i_2<j_1$) by the same amount, and compensate these increments by reducing the flows $T_{i1,j2}$ and $T_{i2,j1}$ accordingly. An example of this type of permutation can be seen on figure 3 where Δ corresponds to the amount changed in each of the four cells. Notice that the origin and destination vectors remain unchanged. Based on this idea, the following procedure generates an OD matrix that preserves the profile and trip length of an initial matrix, but differs in variability:

1. Take a feasible OD matrix, consistent with the requested origin and destination vectors.

2. List all feasible permutations; i.e. all sets i_1, j_1, i_2, j_2 such that $i_1<i_2$, $j_1 \neq j_2$, $i_1<j_1$, $i_2<j_2$, $i_1<j_2$ and $i_2<j_1$.

3. Randomly select a permutation (i_1, j_1, i_2, j_2) from the list. Determine Δ_{max} as the maximum value that Δ can take for this permutation of the OD matrix as the minimum between $T_{i1,j2}$ and $T_{i2,j1}$. Set the step, Δ to be taken as a fraction α of Δ_{max}.

4. Repeat step 3 β times altering the OD matrix accordingly to each permutation and its associated Δ value.

5. Repeat steps 3 and 4 using different values for α and β in order to obtain matrices that are closer or further in variability from the initial matrix.

SCENARIOS DEFINITION AND RESULTS

Scenarios definition

The attractiveness of express services for the different OD matrices will be determined using the Leiva et al. (2010) methodology. Each scenario to be tested will be composed by a topology of the corridor, an OD matrix, a set of a priori attractive services and a set of parameters that will be kept constant over all the scenarios.

Network topology

In this experiment we used the same network for all scenarios. This network is the same that was used in Larrain *et al.* (2010), depicted in Figure 2. It represents a 3.3Km long bidirectional corridor with 10 stops in each direction. The arcs joining stops 1 through 10 constitute the outbound direction of the corridor while those uniting stops 10 through 19 describe the inbound direction. It is assumed that the stops are distributed equidistantly.

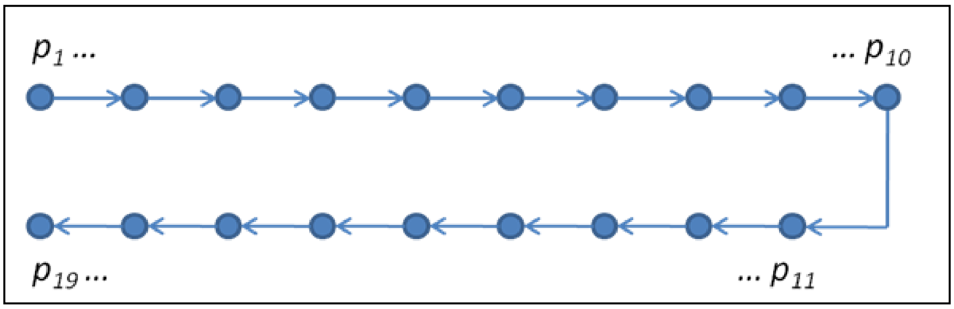

Figure 2 – Network topology.

OD Matrix generation

Each scenario will be fed with a different OD matrix. The set of matrices will be obtained using the methodology introduced in section 3. This method requires an initial feasible matrix from which all the other matrices will be derived. We chose as an initial OD matrix one with very low variability to be used as a benchmark from which other matrices will grow in variability. Thus, the initial matrix is generated with an entropy maximization model in order to assure a low variability level as is explained in Larrain *et al.* (2010).

The initial matrix for this experiment was also taken from Larrain *et al.* (2010), and corresponds to the scenario with a decreasing load profile, a scaling factor of 1 and an unbalance factor of 0.5. The average trip length of this scenario is of 4.27Km, considered long in the context of those experiments. The OD flows of the initial matrix for the outbound direction is shown in figure 3. This matrix allows 420 different feasible permutations (one of them is depicted in the figure).

Origin node (i) \ Destination node (j)	1	2	3	4	5	6	7	8	9	10	O_i
1		1889.8	1248.4	1134.5	613.7	561.6	364.0	192.6	181.3	2104.9	**8290.9**
2			71.9	61.6	32.3	**28.9 + Δ**	18.5	9.7	9.0	**104.4 - Δ**	**203.1**
3				72.4	35.8	31.1	19.4	10.0	9.3	106.3	284.3
4					43.1	35.3	21.4	10.8	9.9	111.7	**232.2**
5						**42.1 - Δ**	24.1	11.8	10.5	**117.6 + Δ**	**46.4**
6							28.9	13.4	11.5	126.4	**180.2**
7								15.7	12.8	136.0	**164.6**
8									14.5	144.9	**159.4**
9										154.3	**154.3**
D_j	**0.0**	**1889.8**	**1320.3**	**1268.5**	**724.9**	**627.9**	**476.3**	**264.1**	**258.9**	**2884.6**	**9715.3**

Figure 3 – Initial matrix. The highlighted flows constitute a feasible permutation.

We applied the methodology to generate twelve new OD matrices. Using the described network and this twelve matrices plus the initial one, we defined thirteen scenarios for optimization. Table 1 shows these scenarios and their respective indicators.

Table 1 – Coefficient of variation for each scenario.

Scenario	CV
0	2.10801
1	2.11081
2	2.13780
3	2.14905
4	2.14986
5	2.15720
6	2.17346
7	2.18568
8	2.19630
9	2.29899
10	2.33180
11	2.34178
12	2.35243

A priori attractive services

The express services for these scenarios were designed using the model from Leiva *et al.* (2010). To apply this model, a set of *a priori* attractive services were constructed that included all the ones that were used in the solutions for the diminishing load profile scenarios in Larrain *et al.* (2010), adding extra services that focus on the pairs with highest flow among the thirteen generated matrices, and on the pairs that show a higher increment compared to the initial matrix. This set of 28 services over the 19-stop corridor is depicted in figure 4.

	Bus stop																		
Service ID	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
1	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●
2	●		●		●		●		●	●	●		●		●		●		●
3	●	●		●		●		●		●		●		●		●		●	●
4	●	●																●	●
5	●		●														●		●
6	●			●												●			●
7	●				●										●				●
8	●					●								●					●
9	●						●						●						●
10	●							●				●							●
11	●								●		●								●
12	●									●									●
13	●	●	●														●	●	●
14	●	●	●	●												●	●	●	●
15	●	●	●	●	●										●	●	●	●	●
16		●			●										●			●	
17		●								●								●	
18			●			●								●			●		
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24	●				●					●					●				●
25	●	●	●							●							●	●	●
26	●		●			●								●			●		●
27	●		●		●	●		●		●		●		●	●		●		●
28	●	●		●	●		●		●	●	●		●		●	●		●	●

Figure 4 – Set of *a priori* attractive services. A dot indicates a bus stop where the service stops, other bus stops are omitted by the service.

Results

The optimal design for the 13 scenarios considered only 6 lines out of the 28 available. Table 2 displays for each scenario the optimal frequencies obtained for each of the six lines, the percentage of the fleet devoted to express services, the number of chosen services and the social cost reduction in comparison to the single all-stop service performance.

Table 2 – Scenario optimization results.

Scenario	CV	f_1	f_2	f_3	f_{12}	f_{21}	f_{23}	Express participation	No. of services	Cost reduction
0	2.108	172.2	0.0	0.0	58.8	0.0	0.0	25.4%	2	6.5%
1	2.111	171.9	0.0	0.0	58.8	0.0	0.0	25.5%	2	6.5%
2	2.138	156.3	0.0	0.0	0.0	74.5	0.0	32.3%	2	6.6%
3	2.149	169.9	0.0	0.0	60.9	0.0	0.0	26.4%	2	6.8%
4	2.150	169.6	0.0	0.0	61.0	0.0	0.0	26.5%	2	6.9%
5	2.157	168.6	0.0	0.0	62.3	0.0	0.0	27.0%	2	7.0%
6	2.173	11.7	87.7	134.4	0.0	0.0	0.0	95.0%	3	6.9%
7	2.186	167.1	0.0	0.0	63.7	0.0	0.0	27.6%	2	7.2%
8	2.196	166.8	0.0	0.0	64.4	0.0	0.0	27.9%	2	7.3%
9	2.299	125.4	0.0	0.0	0.0	0.0	105.9	45.8%	2	8.7%
10	2.332	160.2	0.0	0.0	70.7	0.0	0.0	30.6%	2	8.3%
11	2.342	156.3	0.0	0.0	74.3	0.0	0.0	32.2%	2	9.0%
12	2.352	159.6	0.0	0.0	71.4	0.0	0.0	30.9%	2	8.4%

Scenario 0 in this table corresponds to the initial matrix with the lowest CV. Frequencies are presented in vehicles per hour. These are very high frequencies (approximately three buses every minute) but this result is consistent with the actual operation conditions of the real corridor from which the original data in Leiva *et al.* (2010) was taken.

It can be observed from this results that in most cases an optimum configuration consist of operating an all-stop service (line 1) in combination with a single express service that connects the most important pairs of the corridor. 10 out of the 13 scenarios use express service 12 that connects directly the two extreme nodes of the corridor. Scenarios 2 and 9 use instead a slight variation of this service, in which a single busy stop is also visited. Scenario 6 uses a configuration where the main services serve bus stops in an alternate fashion, reducing travel times for longer trips, emulating the AB railway operation scheme (currently in use in the Metro of Santiago).

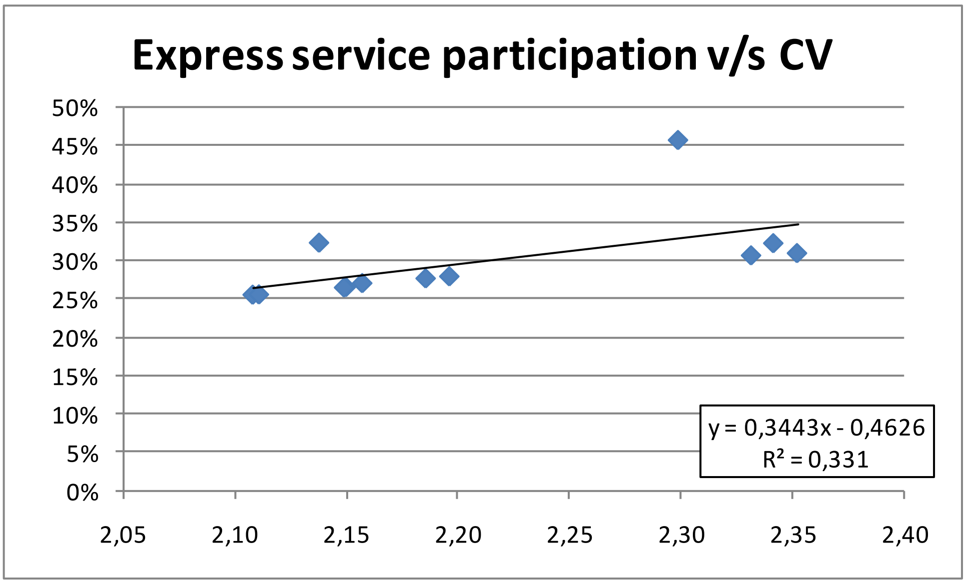

Figure 5 – Express service participation for different CV levels.

The relation between CV and express service participation is shown in figure 5. As expected, it seems that when the variability is higher, participation of express services is also higher. Scenario 6 was not considered for the regression on express service participation because the suggested operation is based on the AB scheme which is significantly different than the one suggested in the other 12 scenarios.

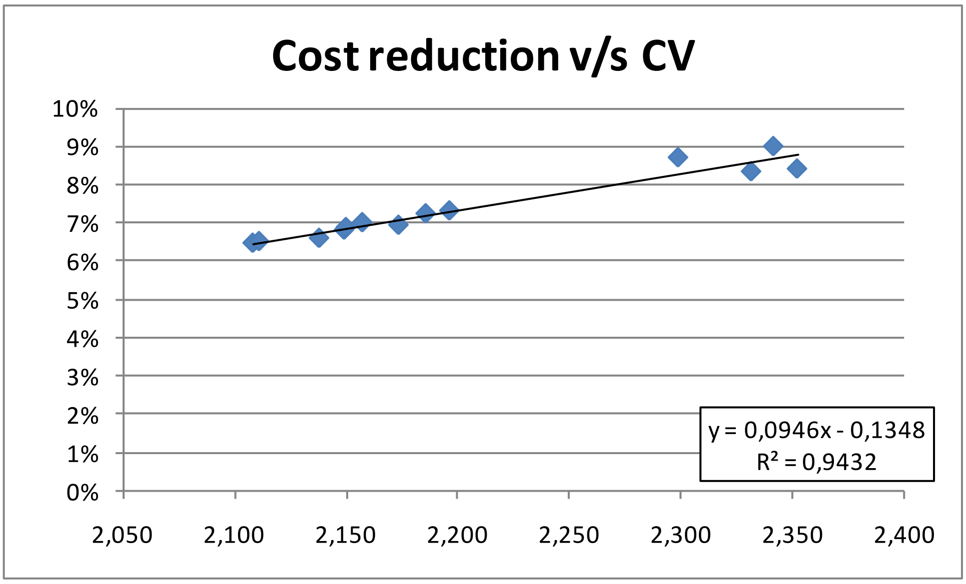

Figure 6 – Cost reduction for different CV levels.

Figure 6 relates express service cost reduction with the index CV. There appears to be a very strong correlation between cost reduction and CV, confirming that variability not only justifies the operation of more express services, it also gives room for a greater cost reduction. Notice that the social cost for the single all-stop service operation is constant across all scenarios.

CONCLUSIONS

This work confirms the claim presented in Larrain *et al.* (2010) that the more concentrated the OD matrix in few OD pairs, the more room for express services. Indeed, when the variability of a corridor's demand matrix is higher, more express services become attractive and their potential impact grows. This can be explained as the combination of two effects. On one hand higher variability means that more flow is concentrated in few pairs. Providing an express ride for these pairs benefits more passengers than in a demand disperse case. On the other hand, there are more pairs with low flows, opening an opportunity to skip some combinations of stops in prejudice of fewer users.

For the analysis of the express service participation, the indicator proved to be somewhat unreliable, because it may overestimate the importance of express services in some cases, like the one aroused in scenario 6 of this experiments. This indicator should be refined; we propose to define for each service an indicator of how "express" it is (i.e. how different from an all-stop service it is), measured in terms of, for instance, how many stops are skipped or the travel time saved. For example, it would be reasonable to give a lower weight to a service that omits just a couple of stops and a higher one to a service that goes directly from one extreme of the corridor to the other.

The procedure proposed to generate OD matrices with different variability could be slightly modified to capture other type of variability. On this experiment, variability was spread among all OD pairs homogeneously. However, it might be interesting to study the case where the demand is intentionally concentrated in very few cells of the demand matrix. These matrices can be obtained using the same kind of permutations proposed in this work but selecting just the ones that have the desired effect on the selected cells.

The impact of express services has been somehow underestimated in this analysis since the origin and destination vectors have been considered fixed and given. Incorporating origin and destination stop choice depending on the services provided would be more realistic. This implies including the network access cost from the trip origin to the selected stop to begin the bus trip (and similar cost to finish it) in the model formulation. Such an extension of the model, in addition to being a more faithful representation of real world conditions, promises to generate increased benefits for express service patterns due to the greater flexibility of options for users.

Finally, the impact of dwell times and travel times, corridor length and bus stop density on limited-stop pattern design should be studied. Our intuition suggests that longer corridors, higher stop density and slower stops would provide a more fertile ground for implementing express services.

REFERENCES

Baaj, H. and H.S. Mahmassani (1992) AI-Based System Representation and Search Procedures for Transit Route Network Design. *Transportation Research Record* 1358, 67-70.

Baaj, M.H. and H.S. Mahmassani (1995), A Hybrid Route Generation Heuristic Algorithm for the Design of Transit Networks. *Transportation Research C,* Vol. 3C, No. 1, 31-50.

Ceder, A. (2003) Designing Public Transport Network and Routes. In Lam, W.H.K. and M.G.H. Bell (eds.) *Advanced Modeling for Transit Operations and Service Planning*. Pergamon, 59-92.

Ceder, A. and H.I. Stern (1981). Deficit Function Bus Scheduling with Deadheading Trip insertion for Fleet Size Reduction. *Transportation Science* 15 (4), 338-363.

Ceder, A. and N. Wilson (1986) Bus Network Design. *Transportation Research B* 20(4), 331–344.

Chriqui, C. and P. Robillard (1975). Common Bus Lines. *Transportation Science*, 9, 115-121.

De Cea, J. and J.E. Fernández (1993) Transit Assignment for Congested Public Transport Systems: An Equilibrium Model. *Transportation Science*, 27(2), 133-147.

Desaulniers, G. and M. Hickman (2007) Public Transit. In Barnhart, C., G. Laporte, editors. Handbooks in operations research and management science, volume 14, *Transportation*. Amsterdam, North-Holland, 69–128.

Fan, W. and R. Machemehl (2006) Using a simulated annealing algorithm to solve the transit route network design problem. *Journal of Transportation Engineering*. 132(2), 122–32.

Fernández, J.E., J. De Cea and I. Norambuena (2003) Una Metodología para el Diseño Topológico de Sistemas de Transporte Público Urbano de Pasajeros. In Proceedings of the XI Congreso Chileno de Ingeniería de Transporte, Santiago de Chile.

Fernández, J.E., J. De Cea and R. H. Malbran (2008) Demand responsive urban public transport system design: Methodology and application. *Transportation Research A,* 42, 951-972.

Furth, P.G. (1987) Short Turning on Transit Routes. *Transportation Research Record* 1108, 42–52.

Furth, P.G. and F.B. Day (1985) Transit Routing and Scheduling Strategies for Heavy Demand Corridors. *Transportation Research Record* 1011, 23-26.

Larrain, H., J.C. Muñoz, R. Giesen (2010) Choosing the Right Express Services for a Bus Corridor with Capacity Restrictions. Forthcoming in *Transportation Research Record*.

Leblanc, L. (1988) Transit System Network Design. *Transportation Research B*, 22B(5), 383-390.

Leiva, C., J.C. Muñoz, R. Giesen and H. Larrain (2010) Design of Limited-Stop for an Urban Bus Corridor with Capacity Constraints. *Transportation Research B*, doi:10.1016/j.trb.2010.01.003.

Mauttone, A. and M. E. Urquhart (2009). A route set construction algorithm for the transit network design problem. *Computers & Operations Research*, 36(8), 2440-2449.

Nuzzolo, A. (2003) Schedule-Based Transit Assignment Models. In Lam, W.H.K. and M.G.H. Bell (eds.) *Advanced Modeling for Transit Operations and Service Planning*. Pergamon, 125-163.