OPTIMUM LOCATION OF MOTORWAY INTERCHANGES: CONCESSIONAIRES' PERSPECTIVE

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ABSTRACT

In this paper we present an optimization model aimed at assisting toll-motorway concessionaires when they deal with motorway interchange location problems. The decisions are assumed to be made from the toll-motorway concessionaires' perspective, with the objective of maximizing profit. The model is based on existing hub location models. Road users choose to travel through the existing road network or recurring to some motorway segments according to the routes' attractiveness. Routes' attractiveness is measured by travel costs, which are calculated based on a model which comprises four cost components: vehicle operating cost, accident costs, user time costs and tolling costs. A route choice model is used to predict the traffic flow travelling through the motorway. The long-term and hardly reversible nature of road design decisions justifies the inclusion of a section dedicated to a stochastic approach. The usefulness of the model is illustrated through two case studies.

Keywords: Motorway interchanges location, Hub location, Optimization models, Route choice model, Stochastic models

INTRODUCTION

In many cases the construction of a motorway takes place within the framework of built-operate-transfer (BOT) contracts. The government (through the Department of Transportation) defines the outline of the motorway and the company who wins the contract defines the detailed design for the motorway that it will operate for the number of years specified in the contract. For a motorway concessionaire the design decisions regarding the location of interchanges - that is, the points where drivers can enter or exit the motorway - are extremely important, because they strongly impact the amount of traffic that the motorway can capture from the existing road network.

In this article, we present a set of optimization models for assisting toll-motorway concessionaires in the analysis of the most profitable solutions for Motorway Interchange Location Problems, or MILP for brevity. The models apply to a region comprising several traffic generation centres, which can represent municipalities, cities, etc, where a new motorway will be built. The intersections of the motorway with the existing road network are the potential locations for the interchanges. The region crossed by the motorway is rural and the motorway is designed to ensure level of service A. Thus we assume that there is and will be no traffic congestion both in the existing road network and in the motorway. The models are generically designated as Motorway Interchange Location Models (MILM). We consider here a deterministic model and two stochastic models. The latter models deal with the uncertainty in traffic, vehicle operating costs, or any other parameters involved in motorways investments decisions, allowing to identify solutions that perform well under all possible realizations of the uncertain parameters, though not necessarily optimal in any of them (Snyder, Daskin & Teo, 2007). For a recent state-of-the-art on facility location under uncertainty, the reader is referred to Snyder (2006).

The MILM can be seen as a particular case of the p-hub median model (Campbell, 1994). Hub location problems arise in the literature in several transportation systems and telecommunication (Campbell et al., 2002). Hub location optimization models allocate demand centres to hubs such that traffic is routed at minimum cost, taking advantage of an exogenously determined discount of the inter-hub links. For a recent state-of-the-art review on hub location models, the reader is referred to Alumur and Kara (2008).

More specifically, the MILM can be classified as a non-strict multiple-allocation p-hub median model (Aykin, 1995). It is non-strict because, after the introduction of the motorway, trips can be made through the existing road network or recurring to some motorway segments (interhub links). Users will choose a motorway segment if the corresponding route is more cost efficient than the existing road network and according to a route choice model. It is multiple-allocation because trips with origin in a given traffic generation centre can be routed through different motorway interchanges according to their final destination. Ebery et al. (2000) provides detailed information on this kind of model.

Routes' attractiveness may be measured by the costs borne by users. Defining a cost function that comprises most of the cost components incurred by users is then of major importance to simulate the users' choices. The great majority of road user cost models consider three main components: vehicle operating costs, accident costs and value of time. A forth component related to tolling costs is also mentioned several times. Routes' travel cost will be calculated using the road user cost model in Santos (2007), which considers the four road user cost components mentioned above.

In general the probability of users choosing a given route is a function of the route's relative attractiveness (travel costs). In the MILP the choice is made between travelling through the existing road network and travelling through the best available route which uses one or more motorway segments. We assume that users will only use the motorway if it is less costly than going through the existing road network. The opposite is not true. There are factors other

than travel cost that may affect the choice. Empirical evidence suggests that habit plays an important role in travel choice behaviour (Ramming, 2002; Handy et. al., 2003). Some users may travel through the existing road network even when the corresponding routes are less cost efficient than using a new motorway. This article presents a route choice model to predict the traffic flow travelling through routes which recur to some motorway segment.
The remainder of the paper is organized as follows. In the next section we describe the road user cost model. Then, we present a route choice model based on logit theory. Afterwards we present the deterministic MILM and two stochastic versions of MILM. Then, we apply the models to case studies and discuss the results. Finally, we provide some concluding remarks and identify directions for future research.

USER COST MODEL

We used the road user cost model (the Portuguese case) presented in Santos (2007). The model incorporates four cost components: vehicle operating costs (*VOC*), accident costs (*AC*), user time costs (*UTC*) and tolling costs (*TC*). The general expression is as follows:

$$RUC = VOC + AC + UTC + TC \qquad (1)$$

where RUC designates road user costs. The units used for all components are €/vehicle/km.
The vehicle operating costs are based on the HDM-4 (Bennett and Greenwood, 2001), and they include fuel consumption, tyres, vehicle maintenance and vehicle depreciation costs; the accident costs are based on COBA (COBA, 2004) and HDM-4 (Bennett and Greenwood, 2001); the user time costs are based on the principles of HDM-4 (Bennett and Greenwood, 2001) and the formulation adopted by JAE (GEPA, 1995); and tolling costs are obtained from a fixed toll fee value.

The RUC is used to calculate route travel costs in the road network. The cost for travelling between two centres, *i* and *j*, through the existing road network is c_{ij}. The cost for travelling between a centre *i* and an interchange *m* through the existing road network is c_{im}. c'_{mn} is the cost for travelling in the motorway between two interchanges *m* and *n*. If the motorway is used between interchanges *m* and *n*, then the total route cost is $c_{ijmn} = c_{im} + c'_{mn} + c_{nj}$.

ROUTE CHOICE MODEL

Routes are chosen according to travel costs. There are two types of routes to consider when travelling between traffic generation centres i and j: routes through the existing road network (choice 1) and routes comprising some motorway segment (choice 2).

In this section we will develop a route choice model to simulate the choice between the two types of routes. The proportion of people using the motorway increases as the ratio c_{ijmn} / c_{ij} decreases. Additionally we assume that the new connections may generate additional traffic flows if travel costs decrease. Let q_{0ij} (q_{Fij}) be the initial (future) traffic flow between traffic generation centres i and j, given by a gravity model (Ortúzar and Willumsen, 2001).

$$q_{0_{ij}} = \alpha \frac{m_i m_j}{f(c_{0_{ij}})^{\beta}} \quad (2)$$

$$q_{F_{ij}} = \alpha \frac{m_i m_j}{f(c_{F_{ij}})^{\beta}} \quad (3)$$

where α is a proportionality constant and β is an attrition parameter; m_i is the mass (measured e.g. with population) of traffic generation centre *i*; $f(c_{0ij})$ is a generalized function of the initial travel costs and it is coincident with route choice 1 travel cost; $f(c_{2ij})$ is the generalized travel cost function of route choice 2 travel costs; and $f(c_{Fij})$ is the generalized function for future travel costs (knowing that users will split between both route choices). We can express $f(c_{Fij})$ as the weighted sum of $f(c_{0ij})$ and $f(c_{2ij})$as follows:

$$f(c_{F_{ij}}) = \frac{f(c_{0_{ij}})q_{1_{ij}} + f(c_{2_{ij}})q_{2ij}}{q_{F_{ij}}} \quad (4)$$

where q_{1ij}, q_{2ij} are, respectively, the traffic flow between traffic generation centres *i* and *j* that travels through route choice 1 and through route choice 2.

Future traffic flow between *i* and *j* is also given by the following equilibrium expression (5):

$$q_{F_{ij}} = q_{1_{ij}} + q_{2_{ij}} \quad (5)$$

Let us consider two travel cost functions: a power function, $f(c_{ij})=c_{ij}$, and an exponential function, $f(c_{ij})=exp(c_{ij})$. The proportion of trips travelling through choice 2 using the power and exponential travel cost functions is given, respectively, by the equations (6) and (7):

$$\frac{q_{2_{ij}}}{q_{F_{ij}}} = \frac{c_{2_{ij}}^{-\beta}}{c_{2_{ij}}^{-\beta} + c_{0_{ij}}^{-\beta}} \quad (6)$$

$$\frac{q_{2_{ij}}}{q_{F_{ij}}} = \frac{\exp(-\beta c_{2_{ij}})}{\exp(-\beta c_{2_{ij}}) + \exp(-\beta c_{0_{ij}})} \quad (7)$$

The functional form in (7) is known as *logit model*.

Combining expressions (2), (3), (4), (5) and (6) and the power travel cost function we get the following expressions for q_{2ij} and q_{Fij}:

$$q_{2_{ij}} = \frac{c_{0_{ij}}^{2\beta-\beta^2}}{(c_{0_{ij}}^{1-\beta} c_{2_{ij}}^{\beta} + c_{2ij})^{\beta} (c_{0_{ij}}^{\beta} + c_{2_{ij}}^{\beta})^{1-\beta}} q_{0_{ij}} \quad (8)$$

$$q_{F_{ij}} = \frac{c_{0_{ij}}^{\beta} + c_{2_{ij}}^{\beta}}{c_{0_{ij}}^{\beta}} q_{2_{ij}} \quad (9)$$

A similar deduction using the exponential travel cost function and expressions (2), (3), (4), (5) and (7) leads to the following expression for q_{2ij} and q_{Fij}:

$$q_{2_{ij}} = \frac{\exp(-\beta \frac{c_{2_{ij}} - c_{0_{ij}}}{1+\exp(\beta(c_{2_{ij}} - c_{0_{ij}})})}{1+\exp(\beta(c_{2_{ij}} - c_{0_{ij}})} q_{0_{ij}} \quad (10)$$

$$q_{F_{ij}} = \exp(-\beta \frac{c_{2_{ij}} - c_{0_{ij}}}{1+\exp(\beta(c_{2_{ij}} - c_{0_{ij}})} q_{0_{ij}} \qquad (11)$$

Despite lacking theoretical support, the power formulation of the gravity model simulates better the reality than the exponential one. The power formulation splits traffic flow between the two route choices according to the travel costs relation, c_2/c_0, while the exponential formulation splits traffic flow according to the absolute difference of travel costs, c_2-c_0. Let us considerer two cases, A and B, to illustrate the drawback to the exponential formulation. For a given pair *ij*, Let q_{0ij} be equal to 10000 vehicles/day. β takes the value 1.2 and 0.06, respectively, for the power and exponential formulations. Table 1 shows the results obtained for route choice 2 using formulations (8) and (10).

Table I – Comparison of results between the power and exponential route choice models

Case	c_{0ij}	c_{2ij}	c_{2ij}/c_{0ij}	Power		Exponential	
				q_{2ij}	q_{Fij}	q_{2ij}	q_{Fij}
A	20	18	0.9	5676	10677	5648	10657
B	4	2	0.5	11648	16718	5648	10657

Travel costs decrease the same absolute value in both cases, however, the relative reduction is five times larger in case B. Using the exponential formulation, the value of trips for choice 2 is the same in both cases, which is unrealistic. Furthermore, the results show that the travel cost reduction generates more traffic (q_{Fij}>q_{0ij}). Once again the power formulation simulates better reality. While using the exponential formulation the additional traffic is the same in both cases, whereas in the power formulation the additional traffic in case B is larger than in case A (which is consistent with a relative reduction in travel costs).

DETERMINISTIC MOTORWAY INTERCHANGE LOCATION MODEL

Consider an existing road network, a new motorway, the set of traffic generation centres of a region, *J*, and a set of candidate motorway interchanges, *M*. Motorway interchanges are uncapacitated facilities. Trips are made through the least cost route according to the power formulation for the route choice model presented in the previous section. d_{mn} is the distance for travelling in the motorway between interchanges *m* and *n*. a_{ijmn} is the constant by which the initial traffic flow, q_{0ij}, must be multiplied to obtain the traffic flow travelling through routes containing a motorway segment (choice 2 of the route choice model). Knowing that c_{0ij} is equal to c_{ij} and c_{2ij} is equal to c_{ijmn}, a_{ijmn} is given by q_{2ij}/q_{0ij} if c_{ijmn}< c_{ij} and is equal to zero otherwise. Given a certain toll fee value per kilometre, *t*, a fixed daily cost for installing an interchange, *f*, and a fixed daily cost regarding the motorway construction, *w*, we wish to determine the number and location of motorway interchanges and the traffic assignments network, so that profit is maximized. The mathematical formulation of MILM is as follows:

$$Max \quad \pi = 2\sum_{i \in J} \sum_{j \in J: i<j} \sum_{m \in M} \sum_{n \in M: n \neq m \text{ and } a_{ijmn} \neq 0} t a_{ijmn} q_{0_{ij}} d_{mn} x_{ijmn} - \sum_{m \in M} f y_m - w \qquad (12)$$

s.t.

$$\sum_{m \in M} \sum_{n \in M: m \neq n \text{ and } a_{ijmn} \neq 0} x_{ijmn} \leq 1 \quad \forall i, j \in J : i < j \tag{13}$$

$$x_{ijmn} = 0 \quad \forall i, j \in J, i < j, \forall m, n \in M : a_{ijmn} = 0 \tag{14}$$

$$\sum_{i \in J} \sum_{j \in J: i < j} \sum_{n \in M: a_{ijmn} \neq 0} x_{ijmn} \leq g_m^a y_m \quad \forall m \in M \tag{15a}$$

$$\sum_{i \in J} \sum_{j \in J: i < j} \sum_{m \in M: a_{ijmn} \neq 0} x_{ijmn} \leq g_n^e y_n \quad \forall n \in M \tag{15b}$$

$$\sum_{v \in R_{ijmn}} \sum_{b \in R_{ijmn}} x_{ijvb} + y_m + y_n \leq 2 \quad \forall i, j \in J, \forall m, n \in M : a_{ijmn} \neq 0 \tag{16}$$

$$y_1 = 1 \tag{17a}$$

$$y_M = 1 \tag{17b}$$

$$x_{ijmn} \geq 0 \quad \forall i, j \in J, m, n \in M \tag{18}$$

$$y_m \in \{0,1\} \quad \forall m \in M \tag{19}$$

where y_m is a binary variable that takes the value of 1 if a motorway interchange is located at the candidate interchange *m*, and zero otherwise; x_{ijmn} is the fraction of traffic from origin *i* to destination *j* routed via the motorway interchanges *m* and *n* in this order; g^a_m (g^e_n) is an upper limit on the number of trips that may use interchange *m* (*n*) as a motorway access (exit); and $R_{ijmn}=\{v,b|a_{ijvb}>a_{ijmn}\}$ is the set of potential routes between traffic generation center *i* and *j* that cost more than the route through motorway segment *mn*.

The objective function (12) expresses the profit, π, for the concessionaire, given as the difference between total toll fee revenue (multiplied by two to consider both directions) and fixed charges for installing and operating the interchanges and constructing the motorway. The assignment constraints (13) guarantee that each trip is assigned to no more than one route. If the route is composed by some motorway segment, *mn*, then x_{ijmn} = 1. If the trip is made only through the existing road network then x_{ijmn} = 0. Since we are dealing with a multiple-allocation *p*-hub median model, *x* may be fractional if for the same *ij* pair there is more than one route with the same travel cost (see Lemma 1 in Ernst and Krishnamoorthy, 1998). Constraints (14) ensure that all non cost efficient routes, under route choice 2 ($c_{ij}<c_{ijmn}$), are eliminated from the optimal solution. Constraints (15a) and (15b) ensure that trips are only assigned to motorway segments limited by two motorway interchanges. Constraints (16) were adapted from Wagner and Falkson (1975) closest assignment constraints. They work together with binary constraints (19) to ensure that each trip is assigned to the least cost route available (within route choice 2). If candidate motorway interchanges *m* and *n* are chosen (y_m=1 and y_n=1), then no trips from *i* to *j* can be assigned to routes belonging to R_{ijmn}. The omission of constraints (16) would make the model assign trips to routes with longer motorway segments (producing higher profit for the concessionaire) but which are not necessarily the most cost efficient routes available for users (within route choice 2). Further information on closest assignment constraints is available in Gerrard and Church (1996). Constraints (17a) and (17b) locate interchanges by default at the extremities of the motorway. Constraints (18) ensure that the assignment decision variables are nonnegative. Constraints (19) guarantee that the location decision variables are binary. The model takes advantage of the symmetric characteristic of all traffic (*q*) and travel cost (*c*) matrices by only using their upper triangle ($i<j$). In order to eliminate

superfluous constraints and variables, the constraints (13), (15) and (16) and the objective function (12) only consider an assignment variable x_{ijmn} if $a_{ijmn}>0$.

STOCHASTIC MOTORWAY INTERCHANGE LOCATION MODELS

Deterministic models are based on the assumption that all parameters are known and unchangeable. However, during the motorway lifespan the parameters involved in the decision process may fluctuate widely. Considering a stochastic model for the MILP allows to hedge against forecast errors or changes in parameters over time (Snyder, Daskin & Teo, 2007). The parameters evolution trends or potential changes are represented by a finite set of scenarios (scenario planning). The stochastic models seek the solution which is expected to behave best across all scenarios (Louveaux & Peeters, 1991).

We present two stochastic versions for the MILM. The first deals with traffic flow uncertainty and is based on the stochastic optimization model in Weaver and Church (1983). We designate it as SMILM. The second deals with fuel costs uncertainty and is based on the Stochastic *r*-robust Uncapacitated Fixed-charge Location Problem (*r*-SUFLP) addressed in Snyder and Daskin (2006). It is designated as *r*-SMILM. Below we provide detailed information on model formulation.

SMILM

Traffic flows are assumed to be discrete random variables, yielding a finite number of scenarios. Travel costs are the same across all scenarios and therefore the traffic assignment solution is common to all of them. Thus the SMILM is a one-stage model, in that strategic (interchange locations) and tactical (assignments network) decisions are made at once.

We define *S* as the set of scenarios. The initial traffic flow is now defined in the context of scenario planning, q_{0ijs}, where p_s is the probability that a given scenario *s* occurs. Given these definitions, the mathematical formulation of SMILM is as follows:

$$Max \quad \pi = 2\sum_{s\in S}\sum_{i\in J}\sum_{j\in J:i<j}\sum_{m\in M}\sum_{n\in M:n\neq m \text{ and } a_{ijmn}\neq 0} tp_s a_{ijmn} q_{0_{ijs}} d_{mn} x_{ijmn} - \sum_{m\in M} f y_m \qquad (20)$$

s.t. (13), (14), (15), (16), (17), (18) and (19)

The objective function (20) expresses the expected profit for the toll-motorway concessionaire over all scenarios, considering the corresponding probabilities.

r-SMILM

Fuel costs directly influences travel costs. By allowing this parameter to vary across scenarios the assignment solution may be different between each scenario. Therefore, and contrary to the SMILM, the *r*-SMILM is a two-stage model, in that strategic (interchange locations) decisions are made at once, before knowing which scenario will prevail, while tactical (assignments) decisions are made in the future, after the uncertainty has been resolved.

Like the *r*-SUFLP in Snyder and Daskin (2006) the *r*-SMILM takes advantages of both the stochastic and robust optimization approaches. The model searches the solution that maximizes expected profit while bounding the relative regret in each scenario (relative regret in each scenario is constrained to be no more than *r)*. The regret associated with scenario *s* is the difference between the objective function value that results from having to locate at the compromise locations, and the best objective function that could be obtained, V_s, if scenario *s* was optimized alone. *r* is the desirable robustness level, $r \geq 0$ and is defined by the decision-maker. Let us also redefine the notation previously introduced in the context of scenario planning, where c_{ijs} is the cost for travelling between two centres, *i* and *j*, through the existing road network under scenario *s* and c'_{mns} is the cost for travelling in the motorway between two interchanges *m* and *n* under scenario *s.* If the motorway is used between interchanges *m* and *n* under scenario *s*, then the total route cost is $c_{ijmns} = c_{ims} + c'_{mns} + c_{njs}$. R_{ijmns} is the set of potential routes between traffic generation centres *i* and *j* that cost more than the route through the motorway segment *mn* under scenario *s*. a_{ijmns} is the constant by which initial traffic flow, q_{0ijs}, must be multiplied to obtain the traffic flow travelling through routes containing a motorway segment (choice 2) under scenario *s*. a_{ijmns} is given by q_{2ijs}/q_{0ijs} if $c_{ijmns} < c_{ijs}$ and is equal to zero otherwise. The *r*-SMILM seeks not only the number and location of motorway interchanges, but also the assignment network for each scenario that maximizes the expected profit. Given these definitions, the mathematical formulation of *r*-SMILM is as follows:

$$Max \quad \pi = 2\sum_{s\in S}\sum_{i\in J}\sum_{j\in J:i<j}\sum_{m\in M}\sum_{n\in M:n\neq m \text{ and } a_{ijmns}\neq 0} tp_s a_{ijmns} q_{0_{ijs}} d_{mn} x_{ijmns} - \sum_{m\in M} f y_m - w \qquad (21)$$

s.t. (17), (19) and

$$\sum_{m\in M}\sum_{n\in M:m\neq n \text{ and } a_{ijmns}\neq 0} x_{ijmns} \leq 1 \quad \forall i,j\in J, \forall s\in S: i<j \qquad (22)$$

$$x_{ijmns} = 0 \quad \forall i,j\in J, \forall m,n\in M, \forall s\in S: a_{ijmns} = 0 \qquad (23)$$

$$\sum_{v\in K_{ijmns}}\sum_{b\in K_{ijmns}} x_{ijvbs} + y_m + y_n \leq 2 \quad \forall i,j\in J, \forall m,n\in M, \forall s\in S: a_{ijmns} \neq 0 \qquad (24)$$

$$\sum_{i\in J}\sum_{j\in J:i<j}\sum_{n\in M:a_{ijmns}\neq 0} x_{ijmns} \leq g^a_{ms} y_m \quad \forall m\in M, \forall s\in S \qquad (25a)$$

$$\sum_{i\in J}\sum_{j\in J:i<j}\sum_{m\in M:a_{ijmns}\neq 0} x_{ijmns} \leq g^e_{ns} y_n \quad \forall n\in M, \forall s\in S \qquad (25b)$$

$$2\sum_{s\in S}\sum_{i\in J}\sum_{j\in J:i<j}\sum_{m\in M}\sum_{n\in M:n\neq m \text{ and } a_{ijmns}\neq 0} q_{ij} a_{ijmns} x_{ijmns} d_{mn} t - \sum_{m\in M} y_m f \geq (1-r) V_s \quad \forall s\in S \qquad (26)$$

$$x_{ijmns} \geq 0 \quad \forall i,j\in J, m,n\in M, s\in S \qquad (27)$$

where x_{ijmns} is the fraction of traffic from origin *i* to destination *j* routed via the motorway interchanges *m* and *n,* in this order, under scenario *s*; and g^a_{ms} (g^e_{ns}) is the maximum number of trips that may use interchange *m* (*n*) as a motorway access (exit) under scenario *s*.

The meaning of the objective function and constraints is the same presented in the previous models but analysed under the context of scenario planning. Constraint (26) ensures that the objective function value for each scenario under the compromise locations is not more than r percent worst than the best objective function that could be obtained for each scenario alone, i.e., it enforces the r-robustness condition.

If we define $r \rightarrow \infty$ constraint (26) tends to become inactive and we get a purely stochastic optimization problem like the one studied by Weaver and Church (1983) and Mirchandani et al. (1985). On the contrary, small values for r may turn the problem unfeasible.

CASE STUDY RESULTS

In this section we present the results obtained for two case study applications, dataset A and dataset B. Though both applications are academic, the data used is real. Both use the Portuguese traffic network. The motorway considered for both applications is the same. It intersects the existing road network in 33 places. The traffic generation centres are Portuguese municipalities located in the region crossed by the motorway. Dataset A comprises 73 traffic generation centres. Dataset B is a subset of dataset A with only 54 traffic generation centres.

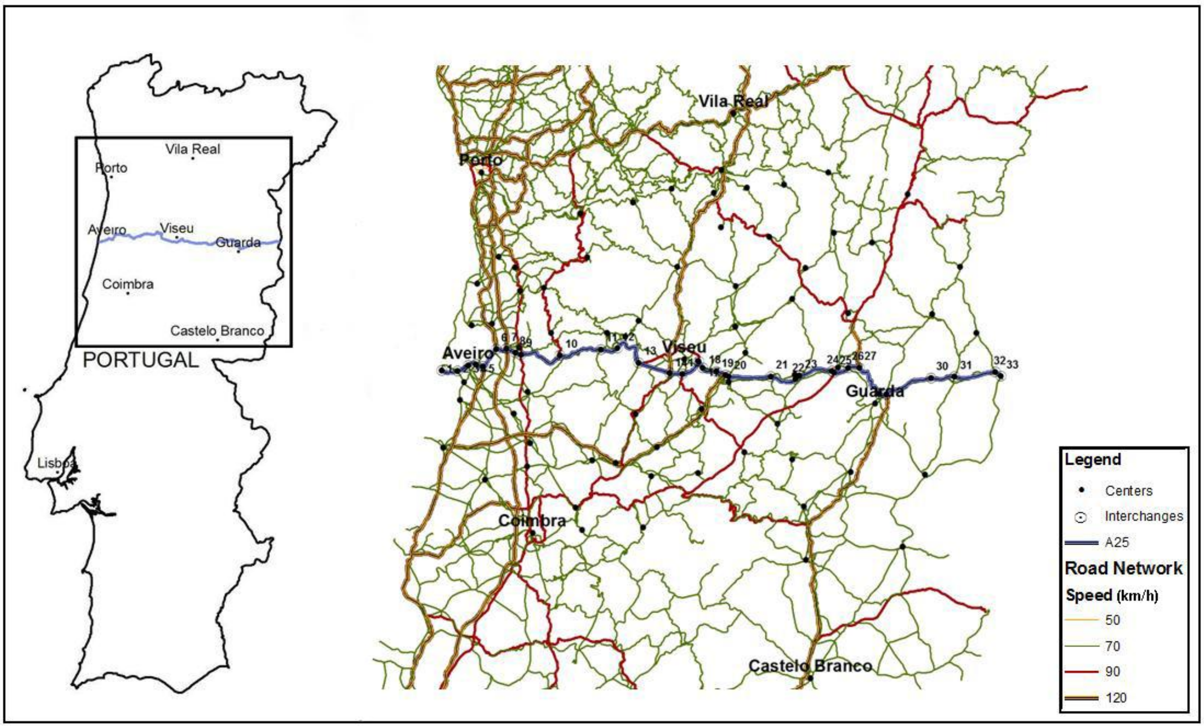

Figure 1 – Location and outline of the motorway and the municipalities in dataset A

The initial traffic flow, q_{0ij}, is given by the gravity model presented above, with β equal to 1.2 and α equal to 1.4. We considered four scenarios for fuel costs, which are shown in Table 2.

Table II – Fuel cost (€/litre) scenarios

Fuel type	SCN 1	SCN 2	SCN 3	SCN 4
Diesel	0.498	0.995	1.692	2.985
Gas	0.610	1.219	2.072	3.657

The road user cost model considers four vehicle classes: passenger cars (PC), light duty vehicles (LD), trucks (TRK) and buses (BUS). We used the results obtained in Santos (2007) for the Portuguese road user cost model, with updated values for the fuel cost. The percentage of each vehicle class in the Portuguese national car fleet is available in IMTT, 2006 (a) and 2006(b). Because we did not have enough information regarding accident costs we assumed that the accident cost value is the same in every road. The value used here was calculated in Santos (2007) for a similar motorway. Tolling costs are added when applicable (considering the toll fees currently being applied). Route travel costs are calculated using the road user costs for a passenger car unit (given as the weighted sum of the vehicle classes). The road user costs components values are summarized in table 3.

Table III – Road user costs

Vehicle class	% in the national car fleet	Vehicle operating costs (€/km/veh)							Accident costs (€/km/veh)	User time costs (€/hour/veh)
		Fuel				Tyres	Maintenance	Depreciation		
		SCN 1	SCN 2	SCN 3	SCN 4					
PC	76,3	0,030	0,060	0,102	0,180	0,007	0,008	0,083	0,01	7,50
LD	21,0	0,022	0,045	0,076	0,134	0,006	0,008	0,046	0,01	6,00
TRK	2,4	0,219	0,438	0,744	1,313	0,027	0,027	0,080	0,01	9,06
BUS	0,3	0,179	0,358	0,609	1,075	0,026	0,019	0,248	0,01	43,56

The fixed costs were estimated considering averages costs of 2 million Euros for each interchange and 1.75 million Euros per motorway kilometre and a lifespan of 20 years. We assumed that the daily cost of each interchange, *f*, and the daily motorway construction cost, *w*, is, respectively, 274€ and 69617€. The models were solved with an Intel Core 2 Duo Processor T7500 2.2 GHz computer with 2 GB of RAM and the Xpress-MP (Dash Optimization, 2008) optimizer.

MILM was tested using dataset A and fuel costs scenario SCN 2 (the closest to the current costs). The results for different values of toll fees are summarized in table 4.

Table IV – MILM results for dataset A and fuel cost scenario 2

Toll fee	π (€/day)	Routes	Interchanges location	CPU (sec)
0.030	3416	25337	1, 3, 4, 6, 9, 10, 11, 12, 13, 14, 15, 16, 20, 22, 24, 28, 29, 31, 33	468
0.040	20907	21561	1, 3, 4, 6, 7, 9, 10, 11, 12, 13, 14, 15, 17, 19, 20, 22, 23, 24, 28, 29, 31, 33	185
0.049	34662	18483	1, 3, 4, 6, 7, 9, 10, 11, 13, 14, 15, 17, 20, 22, 24, 26, 28, 29, 31, 33	149
0.050	35789	18125	1, 3, 4, 6, 7, 9, 10, 11, 13, 14, 15, 17, 19, 20, 22, 24, 26, 28, 31, 33	127
0.051	29363	17740	1, 3, 4, 5, 6, 7, 8, 10, 11, 13, 14, 15, 17, 19, 20, 22, 24, 26, 28, 29, 31, 33	179
0.055	33177	16544	1, 3, 4, 5, 6, 7, 8, 10, 11, 13, 14, 15, 18, 19, 20, 22, 24, 26, 28, 29, 31, 33	105
0.062	17239	14526	1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 15, 20, 22, 24, 26, 28, 29, 31, 33	70
0.070	17741	12352	1, 3, 5, 6, 7, 8, 9, 10, 11, 14, 15, 20, 24, 26, 28, 29, 31, 33	51
0.080	11727	10033	1, 3, 5, 6, 7, 8, 9, 10, 11, 15, 16, 17, 19, 20, 21, 24, 26, 28, 29, 31, 33	77
0.090	5863	8373	1, 3, 5, 6, 7, 8, 9, 10, 11, 15, 16, 19, 21, 24, 26, 28, 29, 31, 33	36

More than the number of traffic generation centres and interchanges, the CPU time needed to find the optimal solution depends on the number of cost efficient routes within route choice 2 (column Routes in Table 4). As the toll fee value increases the number of cost efficient routes decreases. When the toll fee is low there are more cost efficient routes and there will be more motorway users but the profit is also lower. Higher values of toll fees mean fewer cost efficient routes and therefore fewer motorway users, but paying more.

In general, the interchanges are located close to large traffic generation centres (interchange 3, 15, 28) or in the intersection of the motorway with other important roads (interchange 5, 6, 7, 9, 10, 14, 19, 20, 27 and 29). The optimal solution is found considering a trade-off between the number of motorway users and the toll fee value. For dataset A the maximum profit is reached applying a toll fee of 0.05€/km. This solution comprises 20 interchanges.

SMILM was applied to dataset A and fuel costs scenario SCN 2. We considered 50 scenarios for the traffic flow. The scenarios were randomly generated through a normal distribution and according to the traffic generation centres population tax variation in the last period. All scenarios have the same probability. Table 5 shows the results for several toll fee values.

Table V – SMILM results for dataset A and fuel cost scenario SCN 2

Toll fee	π (€/day)	Routes	Interchanges location	CPU (sec)
0.040	13043	21561	1, 3, 4, 6, 8, 9, 10, 11, 12, 13, 14, 15, 17, 19, 20, 22, 23, 24, 28, 29, 31, 33	147
0.049	26235	18483	1, 3, 4, 6, 7, 9, 10, 11, 13, 14, 15, 17, 20, 22, 24, 26, 28, 31, 33	122
0.050	27214	18125	1, 3, 4, 6, 7, 9, 10, 11, 13, 14, 15, 17, 19, 20, 22, 24, 26, 28, 31, 33	146
0.055	25040	16544	1, 3, 4, 5, 6, 7, 8, 10, 11, 13, 14, 15, 18, 19, 20, 22, 24, 26, 28, 31, 33	117
0.062	10201	14526	1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 15, 20, 22, 24, 26, 28, 31, 33	84
0.070	10597	12352	1, 3, 5, 6, 7, 8, 9, 10, 11, 14, 15, 20, 24, 26, 28, 29, 31, 33	48

Though we are considering 50 traffic flow scenarios the CPU time required to solve the model did not increase (for some toll fee values it decreased) when compared to the results obtained for the MILM. When the toll fee is equal to 0.049, 0.055 and 0.062€/km the SMILM optimal solutions have one less interchange (interchange 29) than the MILM. For the other values of toll fee tested the optimal location and number of interchanges is the same. The toll fee value that maximizes profit is 0.05€/km, which is the same as in MILM. The results obtained for both formulations do not differ much, which proves that the MILM is not excessively sensitive to traffic flow uncertainty.

The *r*-SMILM model was tested using dataset B, the four fuel cost scenarios with the toll fee value set equal to 0.08€/km. The optimal solution value, V_s, for each scenario *s* is obtained applying the MILM. The results are presented in Table 6.

Table VI – MILM results for dataset B and all fuel cost scenarios

Fuel	π (€/day) (Vs)	Routes	Interchange location	CPU (sec)
SCN 1	-33021	4525	1, 3, 5, 6, 7, 8, 9, 10, 11, 13, 15, 16, 17, 19, 20, 24, 26, 28, 33	12
SCN 2	-36391	5381	1, 3, 5, 6, 7, 8, 10, 11, 13, 15, 17, 19, 20, 21, 24, 26, 28, 33	27
SCN 3	-39731	6092	1, 3, 5, 6, 7, 8, 9, 10, 11, 13, 15, 16, 17, 19, 20, 24, 26, 28, 31, 33	19
SCN 4	-44676	7074	1, 3, 5, 6, 7, 8, 9, 10, 11, 13, 15, 17, 18, 19, 20, 24, 26, 28, 31, 33	30

As fuel costs increases there are more cost efficient routes through route choice 2, because the motorway often provides shorter itineraries (less fuel consumption). Thus, the optimal solutions under scenarios 3 and 4 have twenty interchanges instead of eighteen as in scenarios 1 and 2. However, higher travelling costs make the traffic flow decrease (see expressions (2) and (3)). Consequently, the concessionaire profit decreases from scenario 1 to scenario 4.

The results show that in the conditions of dataset B the motorway is not profitable for the concessionaire. However, results are deeply influenced by two decisions we have made: first we are only analysing a subset of the municipalities (only 54 in a universe of 73); second we are considering a toll fee value of 0.08€/km instead of 0.05€/km (the one that provides the most profitable solutions for all scenarios). These decisions were made to reduce the size of the problem. In order to analyse the results we would get from *r*-SMILM, we will consider, hereafter, that the daily motorway construction cost, *w*, is equal to 20000€ instead of 69617€. Thus, the optimal solution, V_s, for the four scenarios is, respectively, 16596€, 13227€, 9886€ and 4941€.

Let us consider the following occurrence probability set, *P*=[0.10, 0.45, 0.35, 0.10] where the fuel cost scenarios SCN 1 and SCN 4 are unlikely to happen. Table 7 presents the *r*-SMILM results for three *r*-robustness' levels: ∞, 0.020 and 0.015.

Table VII – *r*-SMILM results for dataset B and occurrence probability set *P*

r (%)	π (€/day)	Interchange location	CPU (sec)
∞	11200	1, 3, 5, 6, 7, 8, 9, 10, 11, 13, 15, 17, 19, 20, 24, 26, 28, 31, 33	314
2.0	11127	1, 3, 5, 6, 7, 8, 9, 10, 11, 13, 15, 17, 19, 20, 24, 26, 28, 29, 31, 33	610
1.5	Infeasible	-	328

The optimal solution for the probability set *P,* with *r*→∞, is the one that maximizes expected profit, according to the scenarios probabilities. Enforcing the *r*-robustness measure provides solutions with lower regret without large decreases in the expected profit. The solution found, with *r*=0.02, guarantees that the relative regret in each scenario is no more than 2%, and the profit losses are only 0.65%.

The CPU time required to solve the model increased considerably because the model seeks the optimal solution considering all cost efficient routes (within route choice 2) across all scenarios (dataset B has 23072 cost efficient routes).

CONCLUSION

In this paper we presented three optimization models applicable to motorway interchange location problems, one deterministic and two stochastic. Despite the wide variety of transport engineering fields where hub location problems have been applied, to the best of our knowledge they have never been used to assist decisions on motorway interchanges locations.

The motorway interchange location problem was dealt from the concessionaires' perspective, which we believe to be the most relevant in real-world applications. Moreover the route choice model presented turns the motorway travel demand elastic and thus more approximated to users' choices. The models presented are therefore useful in toll-motorway concessionaires' cost-benefit analysis.

However we can identify several drawbacks. The models assume that roads are uncongested. Also, model solutions are highly dependent on the road user costs function parameters and the toll fee value. Changes in the toll fee value or the user time value affect the motorway attractiveness.

Though traffic flow and fuel costs uncertainty were dealt separately in the stochastic models, it is possible to integrate them into the same model and even add some other uncertainty parameters. The resulting model would be larger and more complex, which would increase the difficulty or even make it impossible to solve with the commercial software available. In the upcoming months we plan to concentrate our efforts in developing a heuristic to solve larger problems and apply the models to a real case-study.

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