SPATIAL LOCATION OF INLAND TERMINALS: AN EXTENDED FREE ECONOMIC ENERGY APPROACH

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ABSTRACT

Freight transport systems are generally characterised by economies of scale and thus, generating multiple market equilibria. In order to map the dynamic interactions on customers' and producers sides, the Free Economic Energy (FEE) model has been developed. It conciliates the concepts of distribution (entropy), market forms (monopolistic competition) and costs to map network effects.

In this paper the FEE model is extended to geographical analysis of freight transport. The expansion focuses on the location patterns of logistics nodes such as hinterland terminals. This approach is presented in this paper progressively, starting with the enlargement to one dimension and following with the accession to two dimensions. In both dimensions, two possible market forms are combined, namely, monopolistic competition and cost minimisation as borderline situations for the producers and customers. The model explains the distance between terminals, the equilibrium number of remaining terminals and the equilibrium size of terminals. All achieved solutions are expressed in function of fixed and variable cost and a heterogeneity parameter.

The Spatial FEE extension has been validated with data on the German inland terminals. Using regional data, the heterogeneity parameter determining the transport customers' choices and the characteristics of the market equilibria has been estimated. An application case identifies those regions with lacking or oversupply of intermodal facilities and services as well as the market situation that the region is facing. Based on the empirical study, conclusions are given concerning the calibration process, the model extensions and further model applications.

Keywords: free economic energy, meso-structures, dynamic states, product diversity, freight transport, hinterland, port, economic choices, cluster formation, entropy, monopolistic competition

1 INTRODUCTION

The role of ports within the world-wide container flow systems has dramatically changed during the past decades. With the drastic growth of transport flows, new forms of transport businesses have emerged and the role of logistics and economic activities only within the ports have become less important. Especially after ports' deregulation, the markets on both sides, maritime and hinterland, have been forcing ports to adapt quickly to the global changes and needs. The maritime network developments began influencing ports to entry into the ever growing container market while the hinterland confrontation exerts pressure to serve the spots of demand representing the future problem of global transportation. Today, the success of ports does rather depend on their connections to the transport networks than on their strategic positions to serve the cargo market. However, two aspects are important for ports to retain and expand their share in the market, namely, to follow the choice variety of (potential) clients and to develop a sound network in hinterlands (e.g. hub and spoke).

The main driver that moves freight-involved companies to permanently attract cargo volumes is the fact that greater savings can be achieved via economies of scale. Instead of obtaining proportional profits, with returns of scale their profit curve can be steep due to the incurred cost reductions. This statement makes even entrepreneurs to support negative-profit businesses at the start-up period. Thus, it is not possible to control this type of systems since they are self-organised. The problem of some policy-makers is that they want to control or at least to conduct the evolution of transport networks such as ports or even hinterlands. However, a common pattern can not be constructed since ports want always to develop their hinterland networks, but in reality they have either under- or overdeveloped hinterlands. For these purposes, an approach called "Free Economic Energy" (FEE) was built to describe the structures that interact in the core of the freight transport systems. On the one hand this approach describes the emergence and the development of transport systems with economies of scale and product heterogeneity. On the other hand, it indicates over- and undersupply of intermodal connections as some kind of decision support for policy makers, entrepreneurs and port developing agencies.

In order to introduce the methodology for the analysis of spatial location of inland terminals, this study has been structured in six sections. After the present introduction, the second section introduces the theoretical framework based on the dynamic equilibria induced by economies of scale and the formation of the Free Economic Energy model. In section three, the extension of the model to the spatial context is progressively developed for one and two dimensions. Section five explains and develops the real validation and case application of the spatial FEE. Finally, the conclusions of the present study are found in section six including the main problems for the formation and calibration of the model as well as some insights obtained with the results of the application case.

2 THEORETICAL FRAMEWORK

The dynamism, economies of scale, heterogeneity of service providers, customers and services render the freight transport system to complex. In order to present the main results for describing this type of systems, an investigation has been directed on the freight and logistics stakeholders, their strategies and decision-making process as well as the possibilities to model the characteristics of this system.

2.1 Economies of scale and dynamics in freight transport systems

Economies of scale are the benefits that a transport business acquires due to expansion. In other words, the producer's average cost per unit fall as scale is increased. If a service level for the customers is being increased for instance, the customers' willingness-to-pay also increases. Likewise in transportation, a service provider has enough interest to remain in the market if he expects to increase his demand. Hence, the success of some players in the market could result in new market entries. On the other hand, if a traditional provider is no longer able to produce an attractive service product, customers will buy or contract the competitors and the traditional provider must close his business. This situation is referred to as monopolistic competition. It is characterised by economies of scale and free market entries and exits.

Concerning real logistics markets, two other phenomena can also be observed: First, customers choose between different services. A perpetual search for something better or new is latent and the demand for suppliers is constantly changing according to the cease or the opening of alternatives where the suppliers also change. Thus, a dynamic behaviour is also induced in demand by combining different services. Secondly, logistics structures are rather complex structures characterised by horizontal and vertical collaboration and competition. Yet, many agents also perform as dealers or coordinators. Examples of the shippers' and logistics actors' decision making choosing all possible combination of services for their cargo are shown in Fig.1. Therefore, the range of possible interactions and combinations characterises this system as dynamic with multiple equilibriums.

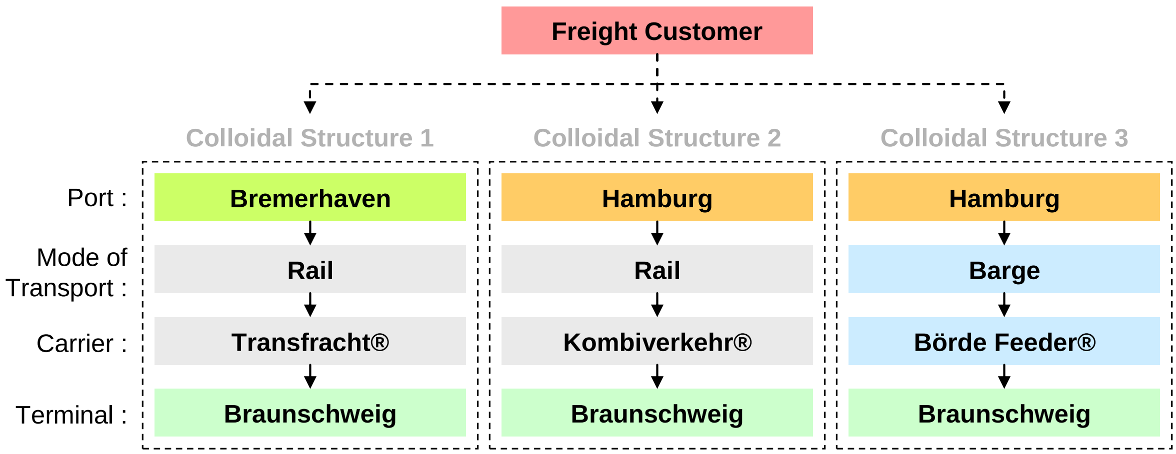

Figure 1. Dynamic interactions of colloidal structures

Given the complexity of logistics interaction systems, the notions of “colloidal structures” (McFadden, 2007) and “clusters” (Krugman, 1991) have been considered. These notions originating from chemistry and physics emphasize the dynamics and self-organisation aspects of such dynamic systems. In the remaining of this paper, clusters or colloidal structures describe any logistics sub-system characterised by economies of scale that in transport might be the combination of self-organised services including ports, shuttle train connections, terminals, etc.

The classical tools in freight transport modelling – namely discrete choice theory and network modelling – are based on the assumptions of having a given set of choice alternatives and convexity. Since these models can neither represent multiple equilibriums nor the cease and/or emergence of new alternatives (Carrillo Murillo and Liedtke, 2008), a new approach for finding dynamic equilibriums based on the Master equation is introduced. The proposed model comes from thermodynamics and statistical physics and has been adapted to be applied for social systems (Weidlich, 2000). It can be shown that the attractors of the Master equation for individual systems can be determined as the solution of an optimisation problem – namely the minimum of the Free Economic Energy.

2.2 Dynamic equilibrium in Master equation

The master function maps the dynamic transitions of individual decision makers’ states (economic agents). In our case, such transition expresses a change of an economic choice. Designed as a meso model, the master function does not map the state of each economic actor and its development. Instead, it maps the interaction of actors on choice options.

Simulating a dynamic freight transport markets, a fixed set of customers choosing between a variable set of choice alternatives is given. The alternatives might be terminals or transport chains and thus, they are examples of colloidal structures in freight transport. In order to map a dynamic market equilibrium, the number of actors leaving each cluster (without loss of generality, cluster 1 is considered) per time interval equals the number of actors entering into a different cluster[1]:

$$\sum_{i \neq 1} n_1 \exp\left(\frac{1}{2} \cdot \alpha(c_1 - c_i)\right) = \sum_{i \neq 1} n_i \exp\left(\frac{1}{2} \cdot \alpha(c_i - c_1)\right) \tag{3}$$

where:

n_1: number of actors choosing alternative 1

n_i: number of actors choosing alternatives *i*

c_1: user cost of alternative 1.

c_i: user cost of alternatives *i*

[1] The detailed mathematical deductions have been carried out in Carrillo Murillo and Liedtke (2008). It is an oline publication, for more information please refer to the bibliography.

α : parameter for the sensitivity on cost differences.

Extracting and grouping terms at both sides, it is obtained:

$$n_1 \exp(\alpha \cdot c_1) = \sum_{i \neq 1} n_i \exp(\alpha \cdot c_i) \quad (4)$$

The left-hand side can be interpreted as the potentials of alternative 1 to loose or gain members, while the right-hand side relates to the same potential, but referring to the whole system. Note that the stable number of alternatives in the dynamic market can be obtained by dividing the total number of actors N by n*. If the potential of a certain alternative is higher than the potential of other alternatives, they will loose their customers and the possibility of emergence of a new alternative will be created. Thus, a more expensive alternative leads to a higher possibility of loosing customers. Contrarily, a high number of customers increases the tendency of some customers to try other alternatives. Since the objective is to determine the equilibrium number of clusters, one can use any of the terms in (4) for minimising the transition of actors among clusters. Accordingly, the average number of actors choosing each alternative in the equilibrium $n*$ is determined by the resulting expression (5).

$$n* = -\frac{1}{\alpha \cdot C'(n*)} \quad (5)$$

The expression (5) is the unique information that can be obtained from the Master equation. As such, its stability conditions open the discussion on the potential information that can be subtracted from this result and the resulting structures (e.g. clusters of freight services).

2.3 Free Economic Energy

Since the aim is to obtain the resulted structures after a dynamic interaction of customers and suppliers in a freight transportation system, an alternative approach is developed to describe the dynamic market equilibria provided by the master function. The following step is to prove the possibility to build an alternative methodology for obtaining expression (5).

> ***Proposition***: The dynamic equilibria of the Master equation can be entirely reproduced with a completely different approach in form of an optimisation problem.

The potential of alternatives to loose costumers is similar to the potential of free and bound energy of a system in thermodynamics. In thermodynamics, free energy in a system refers to energy that is available to us for producing some mechanical work, while bound energy make allusion to energy that is dispersed in the system and we can no longer use for the same purpose. Free energy consists of two terms, namely, the internal energy U and the entropy S where entropy is influenced by the temperature of the system T and a constant k.

$$H = U - kTS \quad (6)$$

The internal energy U expresses the potential energy of all particles in a system. If there are repulsive forces between the particles, the internal energy is being increased. If there is attraction between the particles, they begin to associate into clusters and the internal energy is being reduced. In economic terms U could be interpreted as costs in an economic system since they behave the same way when returns of scale apply. The more the chaos in an economic system, the less efficient it is. Consequently, the stronger the positive effects from collaboration, the higher would be the cost savings. This analogy can be expressed as:

$$C \approx U \tag{7}$$

In thermodynamics, the temperature T is related to the speed or motion of the particles in a system and once the temperature is the same in the whole system, the particles stop moving or oscillating. In economic terms, T is similar to the curiosity or the wish to have autonomy and to act independently from others since it induces freight customers to constantly change the service providers. Curiosity could result from lacking information on the freight services offered on the market (product diversity) while autonomy is linked with exclusivity as shown in (8).

$$kT \approx \frac{1}{\alpha} \tag{8}$$

The transferability of S is defined as the natural logarithm of the number of micro states in the distribution of choice making entities on alternatives (Wilson, 1970). It is defined by

$$S = \ln \frac{N!}{\prod n_i!} \tag{9}$$

where:

N : total number of decision-makers

n : number of decision makers choosing option *i*

Due to the application of the thermodynamic concept and its further application in economics, the transferred expression introducing (7), (8) and (9) into (6) conforms the Free Economic Energy and is determined by:

$$FEE = C - \frac{1}{\alpha} \cdot S \tag{10}$$

Concerning the definition, there is a perfect congruence between the thermodynamic and the economic understanding. The conformed Free Economic Energy involves a peculiar assumption to which no attention has been paid in literature. It relates the cost of alternatives counterbalanced with the induced increase in cost caused by the distribution of agents choosing among freight services. Thus, two aspects find different equilibria with this model, namely, product diversity (force induced by demand) and economic efficiency (force induced by supply). Accordingly, the total cost of a transport market can be composed by:

$$C = N \cdot AC(n) \tag{11}$$

where $AC(n)$ denotes the average cost (or user cost c) depending on the number of users choosing that cluster by means of scale economies.

Using the Stirling's approximation the total entropy S results in:

$$S = N \cdot \ln N - N - \frac{N}{n} \cdot \left(n \cdot \ln n - n \right) = N \ln N - N \cdot \ln n \tag{12}$$

Substituting (11) and (12) into (10) we end-up with the extended form of the Free Economic Energy:

$$FEE = N \cdot \left(AC(n) - \frac{1}{\alpha} + \frac{1}{\alpha} \cdot \ln(n) \right) \tag{13}$$

Since the aim is to consolidate the decisions, the FEE must be minimised and is represented by

$$\frac{\mathrm{d}(FEE)}{\mathrm{d}n} = \frac{\mathrm{d}AC(n)}{\mathrm{d}n} + \frac{1}{\alpha} \cdot \frac{1}{n} = 0 \tag{14}$$

Proof: Giving a stable size of cluster $n*$

$$n* = -\frac{1}{\alpha \cdot C'(n*)} \tag{15}$$

The resulted expression (15) corresponds exactly with the result of (5) that comes from the master equation approach. Therefore, *a connection between the dynamic equilibria produced by the master equation and the concept of free economic energy has been proven*. However, this information lacks of dimension and distribution of the members in a market environment. For this purposes a brief introduction on the market forms used for extending the FEE to the spatial dimension concludes the theoretical framework.

2.4 Market forms

Two forms of market have been investigated for their crucial involvement in the spatial expansion of the FEE model. The first refers to the force induced by the supply side on a market. Hence, the market form that tends to minimise costs by the concentration of demand into the most cost-efficient entity of the supply such as a monopole. In contrast, the second market form relates to the force generated by the demand side in a market by means of diversity. For modelling the latter situation, a monopolistic competition can be assumed for the hinterland transportation market since the actors involved in this system are highly competitive since price differentiation in freight is minimal. Monopolistic competition is a market form including many competing producers selling products that are differentiated - products are substitutes, but not exactly alike- from another existent product (Chamberlin, 1934). Chamberlin expressed: "*this market structure . . . might be put as efficiency versus diversity*". Thus, the market equilibrium between these two forces exists. This equilibrium concept has been applied to describe optimum product diversity (Dixit and Stiglitz, 1975). Using constant and variable elasticity substitution models to express consumers' preferences, Dixit and Stiglitz concluded that the market solution would be characterised by too few firms in a monopolistically competitive sector while a subsidised sector enables many firms. Furthermore, in the latter sector the total output is greater than that determined by the social optimum.

An extended version of Dixit-Stiglitz equilibrium was applied to multi-product firms (Raubitschek, 1987). Raubitschek's main findings state that as the number of firms increase, the number of products increase and the profits fall down. However, in her research the number of firms is given exogenously and her results are useless for dynamic markets. A variant of market equilibrium with multi-product firms but applying a discrete choice- based model to generate demand has been developed, too (Anderson and de Palma 1991). Regarding this oligopolistic market, the authors affirm that the market equilibrium involves an excessive number of firms and each firm provides a small number of products. Nonetheless, this paper focuses on markets under economies of scale with the hardly task of firms to break even and hence, invalidating Anderson and de Palma's statements.

Since both extensions of the research of Dixit and Stiglitz go towards the right direction, but are not applicable to our concern, it should be better to continue considering the fundamental findings of Dixit and Stiglitz as latent for the assumptions on the market forms in freight transportation with one and two dimensions.

3 SPATIAL EXTENSION OF THE FREE ECONOMIC ENERGY

The association among suppliers and the decision-making of freight customers discussed in Section 2 has led to the extension of the Free Economic Energy (FEE) approach to spatial applications. In fact, it is intended to know how colloidal structures / clusters are distributed over an area. The expansion covers the decision making process of logistics actors when choosing a colloidal structure that enforces terminals to become hubs in a hinterland system (Fig. 2).

3.1 Problem statement

Assume a number of shippers in a cell n_c choosing on a set of k terminals (clusters, alternatives or colloidal structures) spatially distributed on a river (for barge) or on a track (for rail). The average distance between alternatives δ characterises this distribution. Fig. 1 shows the graphical representation of the interacting decision-making processes as an example for the one-dimensional case of inland waterways.

The number of actors n_k^c in a demand cell c choosing a certain colloidal structure k is distributed by the FEE as a part of the total demand of that cell n_c. The nearest terminal will have a n_1^c demand from cell c, the second nearest terminal will have a n_2^c demand of the same cell c and so on. In the equilibrium there is a stable average distance between the clusters. On the left hand side of Figure 2 there a too few clusters contrarily to the demand oversupply on the right hand side. Therefore, each colloidal structure has customers from several demand cells.

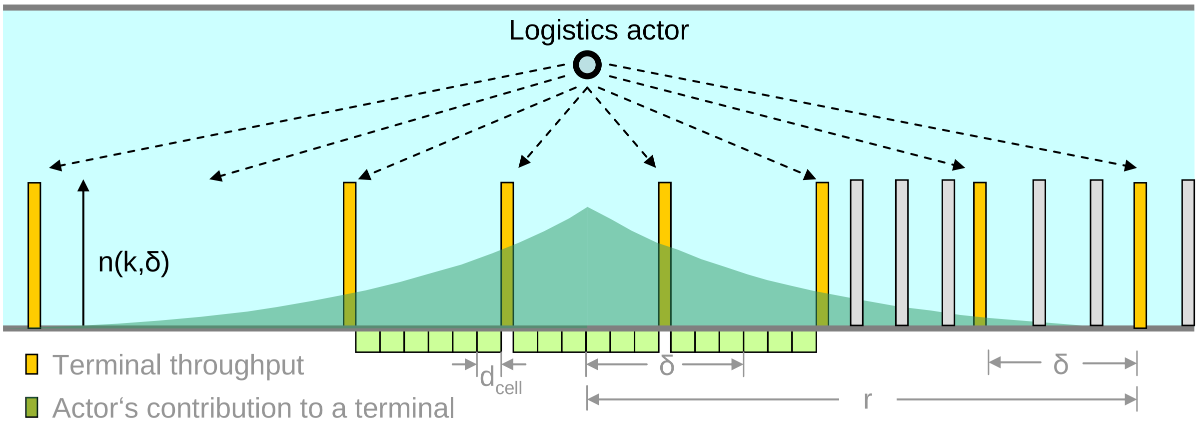

Figure 2. Spatial distributions

Source: Carrillo Murillo (2010)

In the case of an infinitively long linear chain or very large area, the number of customers of each cluster can be deduced as follows:

Number of actors choosing a cluster $n_k = \delta \cdot \rho$ for one dimension

$n_k = Area(\delta) \cdot \rho$ for two dimensions (16)

In order to determine the average stable distance δ, the FEE must be extended. This will be done by adding transport cost to the FEE expression which will be minimised in a further step. Prior to the deduction of each FEE item, a correct expression for the Entropy *S* must be found since we are now dealing with different demand cells in a spatial market.

> ***Proposition****:* A Sub-Categorisation of homogeneous demand segments does not affect Entropy variations.

The homogeneity refers to the distribution of sub-segments on colloid structures. The demand segments *A* and *B* in Fig. 2 can be illustrated as cities for instances.

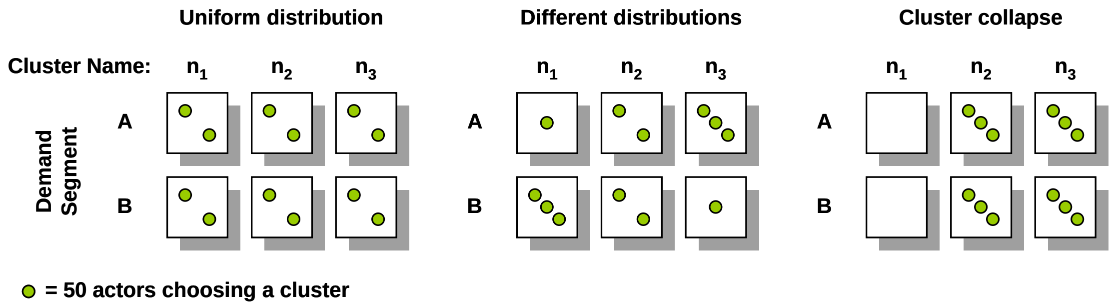

Figure 3. Graphical states of demand segments

Source: Carrillo Murillo (2010)

Three examples are drawn in Fig. 2. The first is the case of a uniform distribution which means that every colloidal structure at every demand segment has the same quantity of demand. The second example denotes every possible combination of states for every colloidal structure and demand segment. The third example refers to a cluster collapse which means a demand segment of no demand. In the latter case, costs increase and the colloidal structure will end up in a total loss to finally shut down the activity. All examples will help to better understand the possible states and to highlight the entropy decoupling property.

Numerical proof*:* The examples shown in Fig. 2 were used to compute the entropy behaviour and the results are shown in Table 2.

Table 2. Breakdown of demand segments

Elements per demand segment	Cluster name				Partial entropy			Total Entropy	
	n_1	n_2	n_3	N	$n_1 \ln(n_1) - n_1$	$n_2 \ln(n_2) - n_2$	$n_3 \ln(n_3) - n_3$	N ln(N) - N	- Entropy
A	100	100	100	300	360,5	360,5	360,5	1411,1	329,6
B	100	100	100	300	360,5	360,5	360,5	1411,1	329,6
Total	200	200	200	600	859,7	859,7	859,7	3238,2	659,2
A	50	100	150	300	145,6	360,5	601,6	1411,1	303,4
B	150	100	50	300	601,6	360,5	145,6	1411,1	303,4
Total	200	200	200	600	859,7	859,7	859,7	3238,2	659,2
A	0	150	150	300	0,0	601,6	601,6	1411,1	207,9
B	0	150	150	300	0,0	601,6	601,6	1411,1	207,9
Total	0	300	300	600	0,0	1411,1	1411,1	3238,2	415,9

The results in Table 2 illustrate three properties:

1. When two demand groups have a uniform distribution on clusters (rows 1 to 3), they can be arbitrary aggregated or subdivided and the demand segments A and B could be shown together: The aggregated Entropy is proportional to the sum of the individual Entropies. This property eliminates novelty in aggregation.

2. If the demand segments have different distributions (rows 4 to 6), there is a difference between the group-specific entropies and the aggregated Entropy.

3. In the event of cluster collapse (rows 7 to 9), the sum of Entropy decreases regardless of the sub-categorisation of segments.

Therefore, it is possible to express *S* as the sum of entropies relating to different demand cells. This allows us to express the FEE minimisation problem as a FEE minimisation problem for an individual (representative) demand cell. This representative demand cell causes its specific entropy cost. It is responsible for a certain proportion of the terminals' fixed cost. In addition transport activities to the terminals cause certain transport cost. Thus, the cell-specific FEE results in:

$$FEE = Pfc + Vc + \beta \cdot Sc \tag{17}$$

with: $\beta = -\frac{1}{\alpha}$

Pfc : proportion of fixed costs
Vc : variable cost or generated transport cost
β : heterogeneity parameter
Sc : entropy

The items of expression (17) must be treated one by one but, because of the nature of deductions, they are distinguished according to the dimensional expansion.

3.2 Extension to one dimension

According to the construction of the FEE, the model is based on an optimisation problem conciliating the cost and the distribution of freight customers among services for cargo and the equilibrium is reached as soon as these two items are minimised. Thus, a situation of customers or demand cells linearly distributed among colloidal structures (fig. 1) will find an equilibrium if and only if:

$$\min\{FEE\} = \min\{Pfc + n_1 c_1 + \beta(n_1 \ln(n_1) - n_1) + n_2 c_2 + \beta(n_2 \ln(n_2) - n_2) + ...\}$$
$$\text{s.t.} \sum_{k=0} n_k = n \qquad (18)$$

where:

n : total cells' demand

n_k : demand of a cell choosing cluster *k*

c_i : *transport cost from demand cell to cluster i.*

1st step – Deduction of the proportion of fixed costs

The first element to deduce is the proportion of fixed costs (Pfc). Let each cell has a diameter $dcell$ and the average distance between each cluster is δ, then the ratio $\delta / dcell$ determines the demand for each cluster as graphed in Fig. 1. The Pfc per colloidal structure depends on the number of demand cells distributed along the average distance δ as shown in (18).

$$\text{Proportion of fixed costs} \qquad Pfc = c_{fix} \frac{dcell}{\delta} \qquad (19)$$

To perform the minimisation, Pfc must be introduced into the extended form (18). By using the Lagrange multipliers the solution of the demand of a cell choosing a cluster, say i = 1, was calculated (20).

$$n_1 = \lambda \cdot \exp\left(-\frac{c_1}{\beta}\right) = n \frac{\exp\left(-\frac{c_1}{\beta}\right)}{\sum \exp\left(-\frac{c_1}{\beta}\right)} \approx n \frac{\exp\left(-\frac{c_1}{\beta}\right)}{f} \qquad (20)$$

where:

f : normalisation factor

Equation (20) indicates the FEE minimum assuming a given and fixed distance between clusters (i.e. the result of the short-term FEE minimisation). For the purpose of calculating the long-term FEE minimum it is still necessary to determine δ that minimises the whole FEE.

2nd step – Deduction of the variable costs and entropy

In the 2nd step, the result from equation (20) is introduced into (18). For the sake simplicity, two sub-cases are distinguished. The 1st involves a monopolistic competition market meaning that shippers choose among a high number of clusters ($\delta << \beta c_v$). The 2nd case is a cost minimisation market meaning that heterogeneity benefits are small and thus, shippers always choose their closest cluster ($\delta >> \beta c_v$).

2nd step – Sub-case monopolistic competition

The first market form is assumed to be monopolistic competition (Chamberlin, 1934). Yet, the market flows are mature enough that specialised services are offered. Accordingly, the customers choose among a high number of terminals and thus, the spatial distribution can be approximated using a continuous function. The sum of transport cost and the customers' distribution (entropy) can be expressed as follows:

$$Vc + \beta \cdot Sc = \int_{r=-\infty}^{\infty} \left(n_r \cdot \left|r_{k,\delta}\right| \cdot c_v + \beta\left(n_r \ln(n_r) - n_r\right)\right) dr \tag{21}$$

where:

n_r: distribution of decision makers on the clusters (result from (20)).

c_v: unitary variable costs or the cost per unit of distance (e.g. km or mille).

r: spatial position

As such, Vc is annulated since the variable costs are no relevant for choosing a terminal (Carrillo Murillo, 2010). Introducing (20) into (21) and solving the integral, it is obtained:

$$Vc + \beta Sc = \beta c_v + \beta \cdot n\left(\ln\left(n\frac{\delta \cdot c_v}{\beta}\right) - 2\right) \tag{22}$$

Yet, the overall expression is composed by (22) and the proportion of fixed cost of (19). Minimising the resulted overall expression, the resultant δ is shown in (23).

$$\delta = \frac{c_{fix}}{\beta \cdot \rho} \tag{23}$$

From (23) the FEE model under the assumption of high specialised services demonstrates that :

- The *higher* the industrial density, the *smaller* the distance between clusters.
- The *higher* the fixed cost, the *higher* the distance between clusters.
- The *higher* the β coefficient (i.e. the sensitivity to cost differences or "degree of homogeneity"), the *lower* the distance between clusters.

2nd step – Sub-case cost minimisation

The assumed market form considers that actors in each demand cell always choose the closest cluster. For a demand cell located directly at the position of a cluster (standpoint 1), the expected variable cost will be null since no distance need to be covered (<r> = 0). On the other hand, the demand cell located just in the middle of two potential clusters (standpoint 2), requires a distance < r >= 1/2 δ for reaching the nearest cluster. Thus, the expected average distance for the overall demand is represented by just the mean of both standpoints giving a < r > of 1/4 δ. Solving Vc for this market form, all elements were integrated (including the annulated Sc due to the presence of only one cluster) and the resulted expression of δ that minimises the FEE of the second sub-case is presented in (24).

$$\delta = 2\sqrt{\frac{c_{fix}}{c_v \cdot \rho}} \tag{24}$$

The solution of the FEE under the assumption of cost minimisation in behaves as follows:

- The *higher* the industrial density, the *smaller* the distance between clusters.
- The *higher* the fixed cost, the *higher* the distance between clusters.
- The *higher* the variable cost, the *smaller* the distance between clusters.

3rd step – Integration of one dimensional results

The results of the two sub-cases at the step 2 are integrated into an equation able to provide each result in function of β. If β is low, the result of cost minimisation is obtained but, if β is high the result of the monopolistic competition appears. By means of numerical experiments the function that suited the best took a power form. The resulted expressions for the expansion to one dimension are:

Distance between clusters
$$\delta = \frac{1}{\rho\sqrt[3]{\left(\frac{1}{2}\sqrt{\frac{c_v}{c_{fix}\cdot\rho}}\right)^3 + \left(\frac{\beta}{c_{fix}}\right)^3}} \tag{25}$$

Optimal number of clusters
$$\frac{N}{n^*} = N\sqrt[3]{\left(\frac{1}{2}\sqrt{\frac{c_v}{c_{fix}\cdot\rho}}\right)^3 + \left(\frac{\beta}{c_{fix}}\right)^3} \tag{26}$$

Optimal size of clusters
$$n^* = \frac{1}{\sqrt[3]{\left(\frac{1}{2}\sqrt{\frac{c_v}{c_{fix}\cdot\rho}}\right)^3 + \left(\frac{\beta}{c_{fix}}\right)^3}} \tag{27}$$

The results achieved reduced the process to analyse for which the enlargement of the model to two dimensions could enjoy straightforward.

3.3 Accesion to two dimensions

Again, Pfc is deducted in the same manner as for the one dimensional case. However, for two dimensions, the catchment area of a cluster is based on the approach of (Lösch 1940) and (Christaller 1933). Lösch based his research from a bottom-up composition of the market, allowing competition at the spatial context. Christaller, however, introduced his approach from a top-down formation of the market. Even if it seems that both approaches are completely against each other, both of them agree in the geographical formation for analysing a market on a geographical area. Accordingly, the assumed catchment area takes the form of a hexagon while the assumed demand cells take a quadratic form (Fig. 3).

1st step – Deduction of the proportion of fixed costs

Let each cell has a diameter $dcell$ and take the form of a square and the average distance between each cluster is δ as shown in Fig. 3.

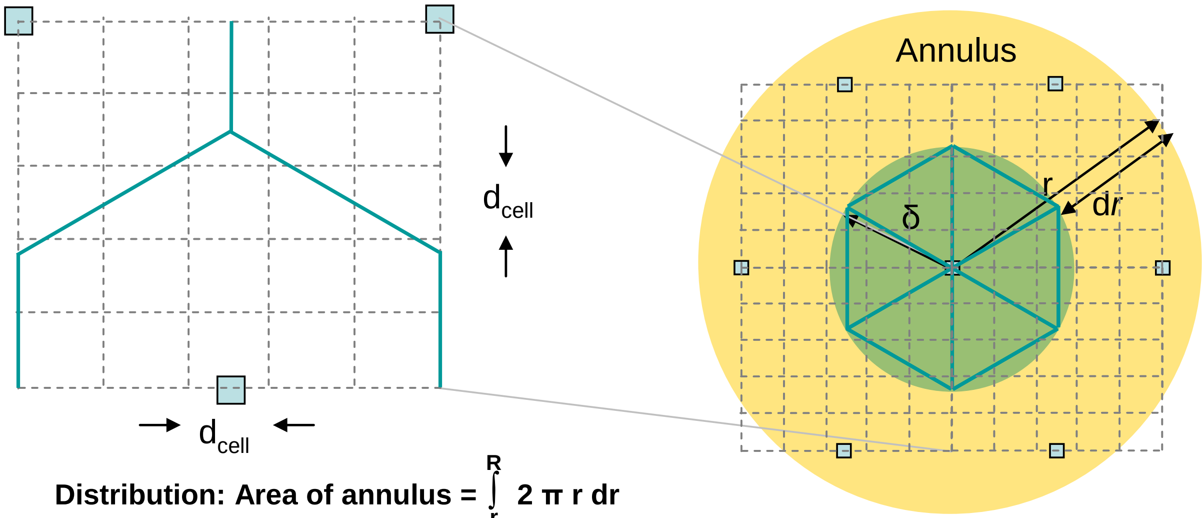

Figure 4. Two-dimensional distribution of a cluster

Source: Carrillo Murillo (2010)

The catchment area is formed by an approximated hexagon. Using the area of a hexagon and calculating the demand cells in that hexagon, Pfc was deducted as presented in (28).

Proportion of fixed cost $$Pfc = \frac{8}{3\sqrt{3}} \frac{c_{fix} \cdot dcell^2}{\delta^2} \qquad (28)$$

As well, the minimisation for one representative spatial cell was carried-on through the Lagrange multipliers. The same development and the same result were obtained as those for one-dimension since only the term which changes is Pfc and through the first derivation with respect to n_1 the mentioned term is swept out. Please refer to (20).

2nd step – Deduction of the variable costs and entropy

Similarly to the one dimensional case, we distinguish between two situations: monopolistic competition ($\delta << \beta c_v$) and cost minimization ($\delta >> \beta c_v$).

2nd step – Sub-case monopolistic competition

The demand was also transferred into continuous demand and the resulted expression is shown in (29).

$$Vc + \beta \cdot Sc = \frac{8\pi}{3\sqrt{3}\delta^2} \int_{r=0}^{\infty} \left(n_r \cdot r_{k,\delta} \cdot c_v + \beta\left(n_r \ln(n_r) - n_r\right)\right) dr \qquad (29)$$

Once (29) was solved, the proportion of fixed cost was added to the FEE. The result obtained from the FEE minimisation is presented in (30).

$$\delta = \frac{5}{4}\sqrt{\frac{c_{fix}}{\beta \cdot \rho}} \qquad (30)$$

2nd step – Sub-case cost minimisation

As well, for the assumed market similar to a monopoly case, the expected average distance was deduced based on Fig. 5.

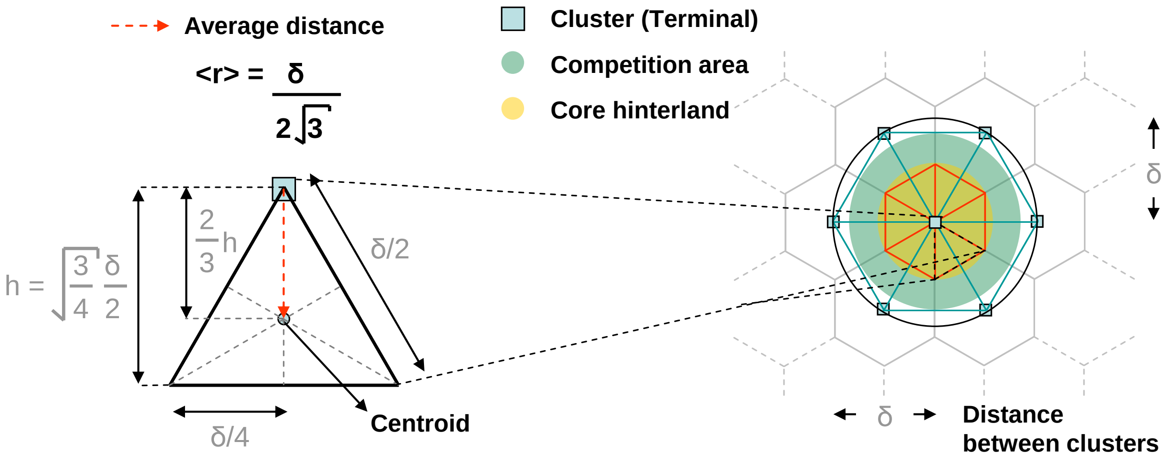

Figure 5. Average distance between clusters at two dimensions

Source: Carrillo Murillo (2010)

Once, the average expected cost were obtained, the minimisation of FEE was performed giving the resulted δ shown in (31).

$$\delta = \sqrt[3]{\frac{32 \cdot c_{fix}}{3 \cdot c_v \cdot \rho}} \qquad (31)$$

The cost minimisation behaviour of customers giving raise to δ in expression (31) for two dimensions behaves as follows:

- The *higher* the fixed cost, the *higher* the distance between clusters.
- The *higher* the cost per kilometre c_v, the *smaller* the distance between clusters.
- The *higher* the industrial density ρ, the *lower* the distance between clusters.

3rd step – Generalisation of two dimensional results

The generalised expressions obtained, with the results of the two dimensional expansion, are the following:

Distance between clusters
$$\delta = \frac{1}{\rho \cdot \sqrt[3]{\left(\frac{1}{\pi}\sqrt[3]{\frac{c_v^{\,2}}{c_{fix}^{\,2} \cdot \rho}}\right)^3 + \left(\frac{\beta}{c_{fix}}\right)^3}} \tag{25}$$

Optimal number of clusters
$$\frac{N}{n*} = N \cdot \sqrt[3]{\left(\frac{1}{\pi}\sqrt[3]{\frac{c_v^{\,2}}{c_{fix}^{\,2} \cdot \rho}}\right)^3 + \left(\frac{\beta}{c_{fix}}\right)^3} \tag{26}$$

Optimal size of clusters
$$n* = \frac{1}{\sqrt[3]{\left(\frac{1}{\pi}\sqrt[3]{\frac{c_v^{\,2}}{c_{fix}^{\,2} \cdot \rho}}\right)^3 + \left(\frac{\beta}{c_{fix}}\right)^3}} \tag{27}$$

4 REAL VALIDATION AND APPLICATION CASES TO THE GERMAN HINTERLAND

The present section demonstrates that the Spatial FEE model can be used and applied for real cases and also transferred for policy purposes. With the use of three variables (industrial density, fixed and variable costs) and one parameter (β) the FEE can describe the location of economic clusters in a private market such as the location of inland terminals in hinterlands. For this purpose, information was collected and computed with regards on the German hinterland in order to obtain the distance between terminals and the industrial density that characterise the real spatial location of inland terminals.

4.1 Data Sources

In total five sources were collected for the mentioned validation. The main contribution of existent sources relates to the data on the inland terminals in Germany which has been elaborated only for the present study. The rest of the data can be purchased or obtained from statistical agencies or other sources. For database sources, please refer to Carrillo Murillo (2010).

The data collected for the German hinterland contains information about 184 terminal operators distributed over 139 inland terminals. The present database includes information such as NUTS-3 (Nomenclature of Territorial Units for Statistics) classification, region, district name, name of terminal, terminal operator, turnover in TEU, type and frequency of services, latitude, longitude and source. A peculiarity of this database is that not every terminal has only one operator, so it can be more than one operator per terminal. The data collected was obtained from terminal websites (e.g. DUSS, Deutsche Bahn, eurogate, mct), EU-Projects (e.g. ITIP and PROTRANS) and the Hamburg port authority.

The data on the number of employees per district has been acquired through the Federal Employment Office of Germany (Bundesagentur für Arbeit, 2007). It includes the number of employees per sector and district. The sectors included in the database follow the WZ2003 classification with a total of 210 sector categories.

The information for import and export trades for Germany was acquired through the German Statistics Agency (Bundesamt, 2009). It includes the total trade of Germany including internal and external trade in 2008. Two types of data have been collected, the overall German trade and the German trade outside Europe.

The production per sector in Germany has been obtained in two sources (Friedrich, 2003) and (Liedtke, 2006). It refers to the allocation of production at district and production sector level. The allocation has been initially constructed by Friedrich (2003) and further improved by Liedtke (2006). The database is based on the aggregation of units per production sector and further distributed into districts.

Finally, the area and population of every district in Germany can be found in Eurostat (2010) through the regional statistics compilation.

4.2 Processing of databases

For obtaining the distance between terminals, only the database on inland terminals was processed. It has been calculated by counting the nearest five terminals and obtaining their average distance. (see also Carrillo Murillo (2010)).

For the industrial density, the export volume was obtained at a sector level. Then, to disaggregate the data into district level, the database on the number of employees per sector was used. The import volume of containers was obtained in a similar manner as for the export. In order to distribute the import volume to a district level, the database on the population and area was processed. The distribution was based on the assumption that goods in containers are destined to the final clients and hence, to the inhabitants of every district. The total trade was obtained by summing-up both results at a district level and transformed into industrial density. The latter was computed through the total trade per district divided by the district's area of the regional statistic database. The resulted container volume was transformed into container units by assuming a cargo load of 15 tonnes per container.

4.3 Real case validation of the spatial extension

Once the key variables of the spatial FEE model have been computed, it remains to prove the accuracy of the model. All German terminals were graphed according to their distance between clusters (δ) against their industrial density (ρ). The terminals show a clear decreasing trend from the smallest value of industrial density until 500 TEU/km² approximately and then, it seems that the tendency becomes flatter to reach the terminals with the highest industrial density (fig. 6).

In order to test the results of the Spatial FEE, the generalised function over two dimensions on the distance between clusters was implied (25). The model was evaluated in function of the industrial density (ρ) considering the following values: Variable cost or cost per km (c_v) = 1 euro/km and fixed cost of a colloidal structure (c_{fix}) = 15 000 euro/day including the full cost of a terminal and the cost of the rail haulage. The values are assumed to be realistic and a research has been launched on this issue. With all the values, δ depends only on the β -parameter. A sensibility test has been carried-on for observing the trend of the function in function of β .

In statistics, the Mean Absolute Error or MAE (e.g. Diebold, (2004)) measures how close forecasts or predictions are to the eventual outcomes. This measure has been used to calculate the optimal statistical value of β because the errors are not scaled. However, the MAE on the distance between clusters was complemented with the industrial density in order to catch the errors induced by both variables in the FEE.

After analysing the database on terminals, it turns out that 159 operators located in 120 terminals are located inside a town independent of the county. Thus, there is incentive to correct the database calculation by spreading the turnover into the closest terminals considered. In this case, the industrial density in study divided by n+1 terminals. For the current case we divide the industrial density by six. The results of the last two paragraphs were graphed in fig. 6.

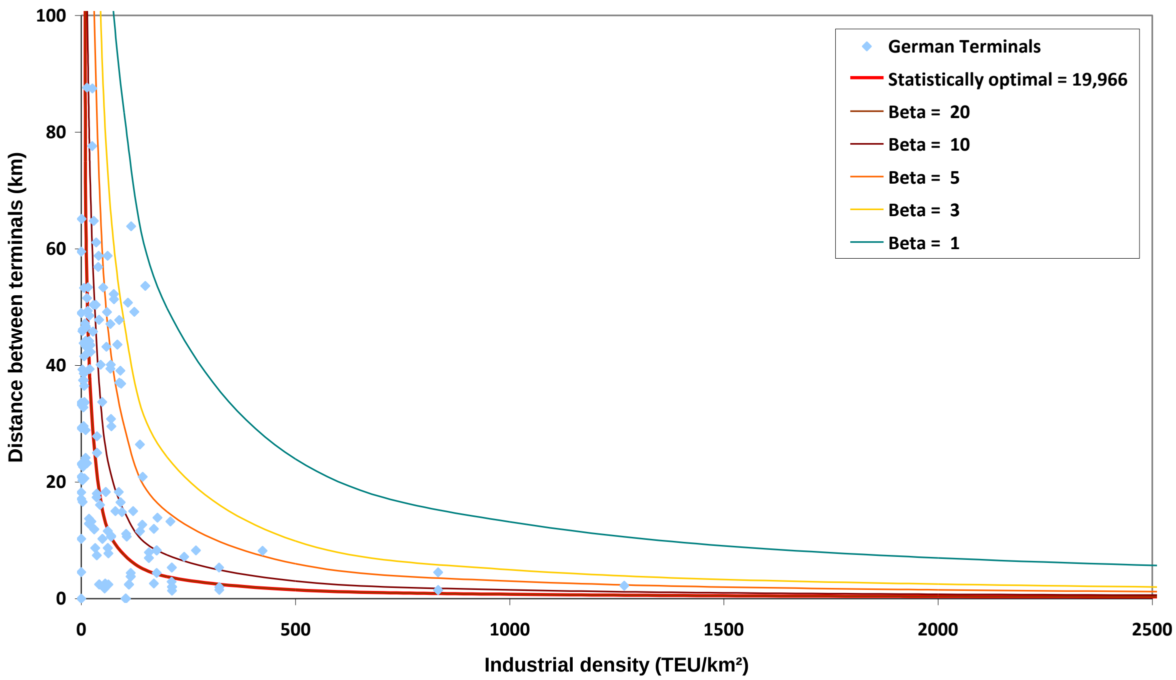

Figure 6. Real validation of the Spatial FEE

Source: Carrillo Murillo, 2010

The FEE describes completely the spatial patterns of the German hinterland as shown in Fig. 6. The advantage of the spatial FEE with the integrated solutions (Step 3) is the identification of both market forms at once. Indeed, the partial solutions would mean that the function at the very left side of Fig. 6 (descending curve of low industrial density and distance > 20 km), only describes the terminals facing cost minimisation. Yet the curve with high industrial density and distance less than 20 km, describes only the terminals facing monopolistic competition (right side of the curves at Fig. 6). Accordingly, the condition separating both market forms is the distance obtained with the statistically optimal β parameter. The deductions were developed considering that β equals δ that fulfils the condition characterising both market forms. For the German case, δ is the distance resulted for the social optimum since it is the same as the statistically optimal value of the FEE according to the results of Dixit and Sitglitz (1975).

4.4 Application case: Spatial policy assessment of intermodal terminals

This section provides a deeper view on how the spatial FEE approach could be applied to transport policy and the private sector. Accordingly, an excess of terminals in an area might indicate that there have been too many measures in the past to foster intermodality. Regions with an excess in supply can no longer provide opportunities for private investments. In contrast, an under developed area in terms of terminals might indicate opportunities for investors to develop and support freight infrastructure as well as awareness to policy-makers over the weaknesses of the ports disserving that area. On the other hand, the market form can also be identified in order to give a view on the market situation that every terminal is facing. Combining the market form and the supply situation, four markets can be identified:

1. Monopolistic competition with more terminals than required
2. Monopolistic competition with less terminals than the social optimum
3. Cost minimisation with more terminals than the social optimum
4. Cost minimisation with fewer terminals than required

The situation evaluated for this application does not consider the presence of subsidies in order to test whether the already colloidal structures can cover their incurred costs with their actual markets. Therefore, expression (26) is tested considering the heterogeneity parameter β describing the spatial location of terminal operators in Germany (fig. 6) as well as the variable cost or cost per kilometre (1 euro/km) and the average distance between clusters for each terminal operator including the nearest terminals. The obtained results are listed in Table 3.

Since at first sight, the results described in Table 3 seem outrageous, it should be stated the fact that they apply only to the intermodal competition or competition between terminals, not to the modal competition. With regards to modal competition, it is true that intermodal services need subsidies in order to reach an equilibrated and fair competition against pure road-haulage.

Table 3. Subsidies harming competition among intermodal services

	Monopolistic Competition	Cost minimisation	More clusters than social optimum	Less clusters than social optimum
Baden-Württemberg	19	5	16	8
Bayern	15	17	16	16
Berlin	3	1	4	0
Brandenburg	1	10	8	3
Bremen	0	1	1	0
Hamburg	0	1	1	0
Hessen	7	3	3	7
Mecklenburg-Vorpommern	0	5	5	0
Niedersachsen	0	15	14	1
Nordrhein-Westfalen	32	8	22	18
Rheinland-Pfalz	8	6	5	9
Saarland	0	2	2	0
Sachsen	3	6	9	0
Sachsen-Anhalt	5	6	11	0
Schleswig-Holstein	0	3	3	0
Thüringen	0	2	2	0
	93	91	122	62

Source: Carrillo Murillo (2010)

Two thirds of all German terminals are located in markets with more clusters than the optimal number for a sustainable market while a third of the terminals is competing in markets with less clusters than the required by the demand.

Out of the German analysis of terminals, 19 per cent of the terminals are facing high competition either for the market form (monopolistic competition) as well as for the excess of terminals according to the existent demand. Therefore, approximately a fifth of the German terminals are operating on a market mature enough concerning intermodal services.

Almost a fourth, of all terminals, is confronting a situation with competition but still the market is not mature enough and less terminals than the expected by the demand are present. It is in this market where possibilities of creating new freight structures become interesting for investors to serve and to develop.

Approximately the half of the German terminals operates in a market of cost minimisation scheme with an over presence of supply actors. This market is well served in terms of specialisation but too much developed in terms of efficiency. If subsidies are given to these intermodal structures, the market will be harmed in terms of intermodal competition allowing the emergence of too much supply for the existent demand.

Finally, a rate of 10,9 per cent is working on a cost minimisation based market with less supply than the socially optimum required. Indeed, it is this market that should be pushed-up by both, the private and the public sector, either because there is a lack of terminals and the customers need to reach their nearest terminal. Anyway, there is still room for the development of terminals in this area to which attention has not been paid on. Investors have an opportunity to serve these markets and subsidies are justified from the part of the public sector.

5 CONCLUSIONS

In the present study, the confrontation of (i) aggregated logistics and transport interactions and (ii) the location of intermodal infrastructure have been resumed in a model and a new perception of freight activities has been applied to the core of the hinterland transportation system. Based on the behavioural decision of logistics actors when constructing transportation chains, the approach describes the multiple equilibriums out of the dynamic interactions between demand and supply giving raise to the emergence or cease of intermodal offers through the use of terminals.

The spatial extension of the FEE model has been progressively constructed. Starting with the expansion to one dimension, the market forms were deducted for obtaining the distance between terminals. The resulted expressions were integrated through a mathematical approximation consisting of three spatial variables. Concerning the extension to two dimensions, the linear deductions could not be applied with the same approach. Instead, assumptions on the catchment area of terminals and the distribution patterns were adapted for both market forms. As well, the consistency of the results has been corroborated for each market form. Once the integration of the market forms was achieved, the resulted expressions of the three spatial variables were also consistent with the extension to one dimension but in a cubic form.

The principal barrier for calibrating the model lies on the availabilty of information on inland terminals. For the German case, the available data has been elaborated with information from operators, the port authority and several European research projects. After obtaining the number of terminals, the latitude and longitude were collected graphically. Then, they were connected with the corresponding district classification. Due to the availability of (i) production and number of employees per sector and district and (ii) regional data, the processing of databases provided the information on the industrial density and the distance between terminals (clusters). Both variables were graphed and confronted with the resulted Spatial Free Economic Energy showing the robustness of the model. However, a part of the graphed terminals seemed to be outliers, but after a detailed analysis on the database, all of them turned out to be towns independent of a county (kreisfrei Stadt) which are considered as dense cities, giving raise to the possible corrections on the accuracy of the data.

A more adapted solution for the correction of the data was found in the allocation of half of the demand generated in those dense cities to the local operators while the rest was attributed to the adjacent operators (terminals). The corrected accuracy of the data resulted in a better spatial description of terminals by the model.

Finally, results of the real case study draw the limits of transport policy to enable a fair competition among intermodal services. The strength of this results lies on the policy indication to opt for other types of policy that equilibrate also the modal competition such as the internalisation of external cost. In that case, the competition is balanced and subsidies can entry into the transportation system only for launching or sustain terminal services with low volumes of cargo but increasing welfare.

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