A MODEL OF INDIVIDUAL TRANSACTIONS IN A HOUSING MARKET FOR LAND-USE MICRO-SIMULATION

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ABSTRACT

Housing market has a lot of unique characteristics. Especially, variety of houses is one of the important characteristics of housing market. On the other hand, households who search houses are also various and their preferences for houses depend on attributes of the household. Therefore, housing market can be considered as a matching problem between various types of households and various types of houses. However, existing land-use micro-simulations have not expressly treated matching and individual transaction process of housings. The objective of this study is to develop a model of matching and transaction process of housing market for the land-use micro-simulation model. Gale and Shapley Algorithm which was developed for the marriage matching problem in 1962 is applied to the matching problem between households and houses in the local housing markets for the land-use micro-simulation. This paper presents theoretical framework of our housing market model and trial simple simulation with a set of hypothetical data.

Keywords: land-use micro-simulation, matching, housing market

1. INTRODUCTION

Several types of land-use micro-simulation models such as UrbanSim (e.g.Waddell(2002)), ILUTE(Miller and Salvini (2001)), LOCSIM (Oskamp(1997)) have been developed so far. Land-use micro-simulation models can forecast relocation, location choices, housing choice and so on by households with a set of micro-data. This study focuses on the modelling of the housing market for land-use microsimulation.

Housing market has unique characteristics such as asymmetric information, existence of disequilibrium (generation of vacant houses), which are different from that of other market (e.g. Walrasian market). Especially, variety of houses is one of the important characteristics of housing market. Housings are differentiated from each other by forms of ownership and selling, time passed from construction and those varieties of housings generate various markets and local direct bargaining. On the other hand, households who search houses are also various. The attributes of households such as number of people in a household, age of the head of a household and so on would have an influence on choice of houses. Therefore, housing market can be considered as a matching problem between various types of households and various types of houses.

However, matching and bargaining mechanism on local housing market has been scarcely modelled in the existing land-use micro-simulation models. The objective of this study is to develop a model of matching and bargaining process of housing market for the land-use micro-simulation model. The main contribution of this model is that matching mechanism is incorporated into the housing market modelling of the land-use micro-simulation. Gale and Shapley (1962) proposed the algorism for the marriage matching problem and proved that this algorithm leads to the stable state. Gale and Shapley Algorithm is applied to the matching problem between households and vacant houses in the local housing markets in our model, that makes possible to consider various household's preferences for various houses that cannot be expressed by the random matching. Thus, this model can give a theoretical consistency with the principle of economics.

This paper consists of six sections. The first is introduction. The second section shows characteristics of the real housing market and how those have been treated in the existing land-use micro-simulation model. In the third section, existing matching mechanisms and algorithms, especially Gale and Shapley algorithm for marriage problem are explained. In the fourth section, the housing market model proposed in this study is presented. In the fifth section, trial simulation of our model is presented with a set of hypothetic micro-data. Conclusion and further works are presented in the last section.

2. CHARACTERISTICS OF HOUSING MARKET AND EXISTING MODELS

2.1 Characteristics of housing market

Actual housing market has various intrinsic features that are different from other goods markets: e.g., costly search, imperfect capital markets, moving cost, and housing vacancies (Arnott(1989)). Especially, diversity of housing is important for the modelling of housing market. Each house stands on a inherent site, has unique room arrangement, floor area, neighbouring environment and so on. Because of such a variety of housing characteristics, housings are differentiated from each other and those varieties of housing form various markets. Housing market is differentiated by forms of ownership and selling such as a house for rent, a condominium, a custom-built house. Furthermore, time passed from construction also divides housing markets such as new-built house or used house.

Variety of housing market relates not only characteristics of housing but also those of households, e.g. number of people within a household influences on their choice of housing type. Single households have tendencies to want houses for rent. On the other hands, when they would be married or have some children, they would want to live houses that have some rooms or larger than ever. Age of head or income level of household are also related to the rate of home ownership.

Thus, it is critical to consider the variety of housing market caused by individuality and diversity of housing, heterogeneity of households' preferences caused by the characteristics of households. So, we consider housing market as a matching problem between various households who have heterogeneous preferences and various housings which they need. This study constructs a matching model of housing matching market in the following sections.

2.2 Modelling of housing market in existing models

Existing land-use models whether micro-simulation or not have scarcely treated unique characteristics of housing market originated in the individuality and diversity of housing and heterogeneous needs of households. However, we expect that micro-simulation could do realistic modelling of location and housing choices of households in the housing market and policy analysis with it since it is able to express dynamics of micro agents' behaviour in detail and variety of their preferences.

Several types of land-use micro-simulation models for practical use such as UrbanSim, ILUTE have been ever developed. UrbanSim (Waddell(2002), Waddell et.al.(2004)) that is one of the most popular model ever build uses land prices as the indicator of the match between demand and supply of land at different locations and with different development types, and of the relative market valuations for attributes of housing, non-residential space, and location. Land prices are modelled using a hedonic regression of land value on attributes of the land and its environment. The estimated hedonic regression is adjusted based on the relative position of the current and structural vacancy rate, as follows:

$$P_{ilt} = \alpha + \delta V_{it}^{c} + \beta X_{ilt}$$

Where:

P_{ilt} is the price of land per acre of development type *i*, at location *l* in time *t*

V_{it}^{c} is the current vacancy rate at time *t*, weighting local and regional vacancy

X_{ilt} is a vector of locational and site attributes

α and β are estimated parameters

δ is set by the user based on sensitivity testing

This equation for the land price in UrbanSim can be considered to express the state of disequilibrium with vacancy rate. However, it does not bring out the best in strengths of micro-simulation in the modelling of housing or land market, in concrete, does not express the bargaining process between various households who is demander of houses and supplier of housings. This study focuses on the bargaining process of housing market and makes a model that explicitly expresses the matching mechanism between various households who have own preferences for houses and various houses.

3. THEORIES OF MATCHING MECHANISM

The theory of matching problem which has been applied to broad fields, labour market, college admissions problem, marriage problem, hospital assignment problem and so on has achieved remarkable development. In the field of housing market, Wheaton(1990) tried to do modelling for the housing market in which households searches housing they prefer and matching. Matching mechanism that Wheaton modelled assumes random matching between households and houses which they search. Shapley and Scarf (1974) modelled exchange of indivisible good (house swapping market) and introduced top trading cycle algorithm that gives an allocation in the core of the market with indivisible goods. Abdulkadiroğlu et.al.(2002) proposes an auction mechanism for room assignment-rent division problems which mimics the market mechanism.

Matching mechanism can be roughly divided into random matching and non-random matching. The former assumes that matching occurs randomly, the latter assumes that matching is caused by some algorism based on the preferences of players in the model. As far as housing market, various types of household choices houses based their own preferences for housing. Therefore, random matching does not suit the modelling of housing market. So, we adopt matching mechanism based on the preferences of household for the modelling of housing market of land-use micro-simulation. Especially, matching algorithm developed by Gale and Shapley (1962) could be the simple and potent way of modelling matching and bargaining process in the housing market. In the next sub-section, Gale and Shapley model is introduced as a potential algorithm of matching.

3.1 Gale-Shapley Model

Gale and Shapley (1962) first proposed the stable matching model for marriage problem in which there are two-sided players: *n* men and *n* women. In this model, each person ranks those of the opposite sex in accordance with his or her preferences for a marriage partner. They described a simple algorithm that always finds a stable matching. Here, definition of "stable" in the context of marriage problem is that a set of matching is called unstable if under it there are a man and a woman who are not married to each other but prefer each other to their actual mates. The algorithm that Gale and Shapley proposed is as follows,

> Step1. Each boy proposes to his favourite girl.
>
> Step2. Each girl who receives more than one proposal rejects all but her favourite from among those who have proposed to her. However, she does not accept him yet, but keeps him on a string to allow for the possibility that someone better may come along later.
>
> Step3. Those boys who were rejected now propose to their second choices.
>
> Step4. Each girl receiving proposals chooses her favourite from the group consisting of the new proposers and the boy on her string, if any. She rejects all the rest and again keeps the favourite in suspense.
>
> Step3 and Step4 are repeated in the same manner until everyone gets married.

Gale and Shapley represented this algorithm guarantees everyone gets married and this set of marriages is stable. They also represented the condition when the number of boys and girls is not same. If there are b boys and g girls with $b<g$, then the procedure terminates as soon as b girls have been proposed to. If $b>g$ the procedure ends when every boy is either on the same girl's string or has been rejected by all of the girls. In either case the set of marriages that results is stable.

4. MODELING OF HOUSING MARKET

In this section, modelling of housing market in the land-use micro-simulation we are developing is described. Especially, matching mechanism and bargaining process of housing market to which Gale-Shapley matching model are applied are presented.

4.1 Preference of household for housing

The matching model we are developing is composed of a set of households I and a set of houses H. A set of I and H are formulated as follows,

$I = \{i_1, i_2 \square, i_m\}$ is a set of households

$H = \{h_1, h_2 \square, h_n\}$ is a set of houses

where number of households *m* and houses *n* are not necessarily same. A set of households is a group of those who have an intension to change their houses and move to new ones. A set of houses include vacant and newly constructed ones at that point.

Preferences of households for houses are derived from the utility function. Given utility optimization of a household under budget constraint, indirect utility function of household *i* u_i can be expressed as a function of attributes of households, attributes of houses λ_{ik} and price of house as follows,

$$u_i(\xi_{ik}, \lambda_{jl}, r_j)$$

where $\xi_{ik} = \{\xi_{i1}, \xi_{i2}, \square\}$ is a set of attributes of household *i*
$\lambda_{jl} = \{\lambda_{j1}, \lambda_{j2}, \square\}$ is a set of attributes of house *j*
$r_j = \{r_1, r_2, \square, r_n\}$ is a price of house *j*

Variables in a set of attributes of household could include number of family members, number of children, age of family members, etc. Variables in a set of attributes of house could include dwelling size, accessibility or transport availability, neighbourhood environment, etc. Households rank houses not only they want to move but also they live now in accordance with their own utility because they could take option not to move this term if utility to move to a new house would be lower than that to live in present house.

4.2 Preference of housing supplier for household

Bid price of household *i* is defined as a maximum value of price for the house *j* under the level of indirect utility $u_i(\xi_{ik}, \lambda_{jl}, r_j)$ is $\bar{u}$. Bid price of household *i* for house *j* is expressed as follows,

$$r_{ij}^* = r(\xi_{ik}, \lambda_{jl}, \bar{u}_i)$$

Preferences of housing suppliers for households who propose to their housing are assumed to depend on the bid price that households present. Therefore, orders of preferences of housing suppliers for households are not decided until they receive proposal of households who want their housings ,that is different situation to Gale-Shapley model. The market prices of houses are decided through the negotiated transaction between households and housing suppliers. Rubinstain (1982,1985) has theoretically clarified price determination in bargaining process between demander and supplier. Our model for housing market treats bargaining process between households and suppliers of houses as bid price competition.

4.3 Matching algorithm for housing market

As mentioned above, random matching is not appropriate to the problem of housing market, because there is a strong relation between the types of households and houses which they prefer. So, we proposes such a concept of the model of housing market for land-use micro-simulation that applies the algorithm of Gale-Shapley model to the matching problem of housing market. However, Gale-Shapley model does not include price in matching process since it concerns marriage problem. But, price must be included in the model of housing market as an adjustment factor in the market. Revised algorithm in order to express the matching process between households and houses in term *t* can be expressed as follows,

> Step1. Each household proposes to their favourite house.
>
> Step2. Each housing supplier who receives more than one proposal rejects all but a household who presents the highest bid price from among those who have proposed to the house. However, the supplier does not accept the household yet, but keeps them on a string to allow for the possibility that someone better may come along later.
>
> Step3. Those households who were rejected now propose to their second choices.
>
> Step4. Each housing supplier receiving proposals chooses their favourite from the group consisting of the new proposers and the households on their string, if any. They reject all the rest and again keep the favourite in suspense.
>
> Step3 and Step4 are repeated in the same manner until every household would get new houses or give moving up as their proposal to all houses which they can accept would have been rejected by housing suppliers.

As Gale and Shapley represented, this algorithm guarantees every households gets houses and this set of houses is stable even if the number of households and houses is not same. However, if the number of houses are larger than the number of households $m<n$, the houses to which no households propose are left over, that is vacant houses in term *t*. The vacant houses will go on sale with the down price in term *t+1* based on the result in the last term. On the other hand, if the number of households who have a wish to move are larger than the number of houses $m>n$, the households are left over. The households who are left over cannot move in term *t* and will have to propose to houses they want to live again under the revised preferences.

4.4 Flow of the housing market model

Simulation flow for the matching model of housing market we are developing is shown in Figure 1. It is roughly divided into two parts, one of which in the left side shows behaviour of households and the other of which in the right side shows behaviour of housing supplier. In the left side, utility function can be estimated with explanatory variables, attributes of households ξ_{ik}, attributes of housing λ_{hl} and selling price of houses. Preference of each household for houses is derived from utility function as mentioned above. In the right side, selling price of houses is set up based on the market price of same type of housing in the last term $\hat{r}^{t-1}(\lambda_h)$. Matching process between households and housings was designed in the last section. When every household will get new houses, simulation ends. If houses remain unsold, the properties are carried over to the next term and will cut prices.

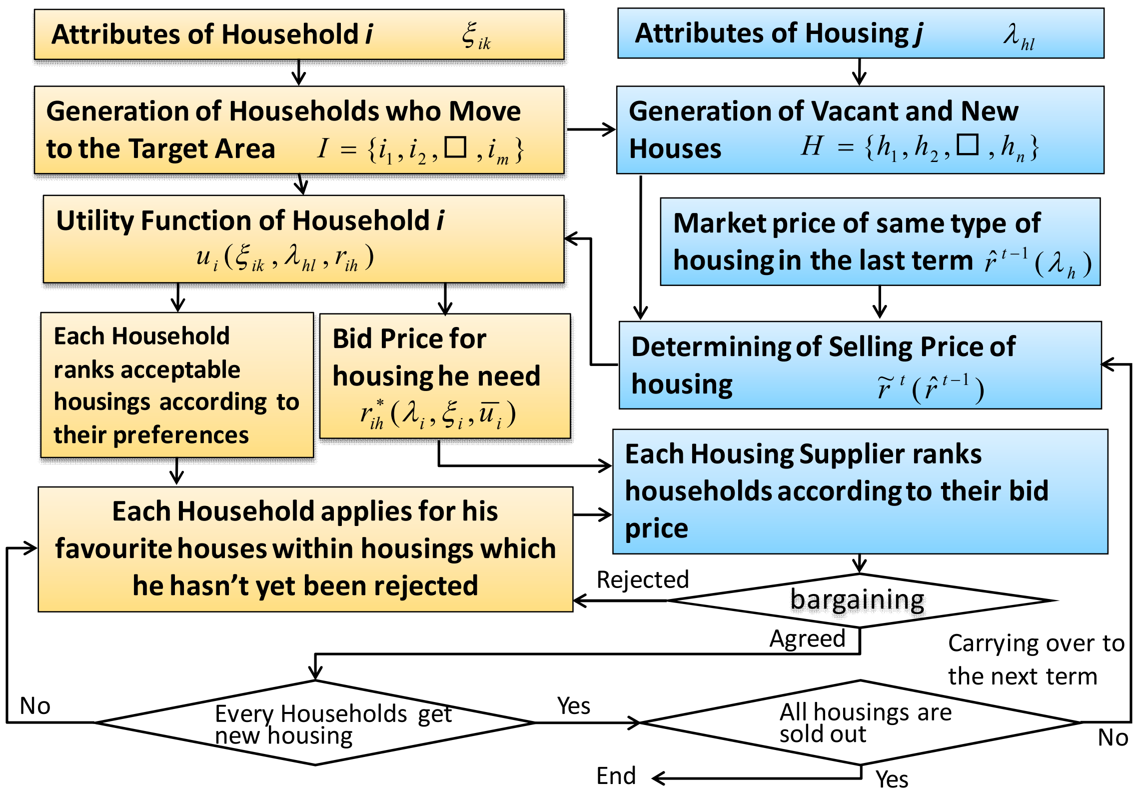

Figure 1. Simulation Flow for matching model of housing market

5. TRIAL SIMULATION OF THE HOUSING MARKET MODEL

In the last chapter, the concept and structure of matching model of housing market for the land-use micro-simulation is explained. This chapter expresses the trial simple simulation of our matching model for housing market with a hypothetical data set of ten households I and ten houses H

, where

$I = \{i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8, i_9, i_{10}\}$ is a set of households who wish to change residence

$H = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}\}$ is a set of houses that households can choose

Table 1 shows the preferences of households for houses. The numbers in this table expresses the order of houses by each household. That is, first choice of household i_1 is h_7, second choice is h_1, third choice is h_8 and so on. Here, h_0 denotes the house in which each household live at present. By including households' current houses in the order, this model could express the possibility of generating vacant houses even if the number of households is more than that of houses because households would not move to the new houses which they rank in lower than their current house. Properly, the order of houses is derived from the indirect utilities of households, but these are given by generating random numbers with a computer in this hypothetical trial simulation.

Table 1 Preferences of households for houses

	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}
i_1	6	2	10	5	7	9	8	1	3	4	11
i_2	3	7	8	10	5	4	6	2	1	11	9
i_3	7	9	2	10	11	3	6	4	5	1	8
i_4	7	3	10	4	8	11	9	6	2	1	5
i_5	5	8	3	9	4	2	11	10	6	1	7
i_6	8	5	7	4	11	6	2	9	10	3	1
i_7	5	3	9	6	10	1	8	2	11	4	7
i_8	11	10	5	9	2	3	1	8	4	6	7
i_9	6	11	1	2	5	3	4	10	7	8	9
i_{10}	3	4	10	5	2	7	6	1	11	8	9

Hence we will show the matching process with the algorithm that Gale-Shapley algorithm is applied. Table 2 shows the first step of algorithm mentioned in section 4.3 that expresses each household proposes to the most favourite houses, where number 1 denotes the houses each household ranks 1. Here, h_9 is proposed by three households, i_3, i_4 and i_5, h_7 is proposed by two households, i_1 and i_{10}. And houses i_2, i_5, i_6, i_8 and i_{10} are received a proposal of only one household. Houses i_1, i_3 and i_4 are received no proposal of household.

Table 2 First proposal to houses by households (Step1)

	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}
i_1	0	0	0	0	0	0	0	1	0	0	0
i_2	0	0	0	0	0	0	0	0	1	0	0
i_3	0	0	0	0	0	0	0	0	0	1	0
i_4	0	0	0	0	0	0	0	0	0	1	0
i_5	0	0	0	0	0	0	0	0	0	1	0
i_6	0	0	0	0	0	0	0	0	0	0	1
i_7	0	0	0	0	0	1	0	0	0	0	0
i_8	0	0	0	0	0	0	1	0	0	0	0
i_9	0	0	1	0	0	0	0	0	0	0	0
i_{10}	0	0	0	0	0	0	0	1	0	0	0

In step 2 of the algorithm, each housing supplier who receives more than one proposal rejects all but a household who presents the highest bid price from among those who have proposed to the house. Table 3 shows the result after housing suppliers who receive more than one proposal select a household who presents the highest bid price. In this case, order of bid price each household presents are assumed as follows,

$r^*_{i_1 h_7} > r^*_{i_{10} h_7}$ for house h_7

$r^*_{i_5 h_9} > r^*_{i_4 h_9} > r^*_{i_3 h_9}$ for house h_9

Therefore, the housing supplier h_7 would select household i_1 and the housing supplier h_9 would select household i_5. Temporary price of houses are decided at bid price of household who has proposed to the house and presented highest price among those who have proposed to the house.

Table 3 Choices of highest bidder by housing supplier (Step2)

	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}
i_1	0	0	0	0	0	0	0	1	0	0	0
i_2	0	0	0	0	0	0	0	0	1	0	0
i_3	0	0	0	0	0	0	0	0	0	0	0
i_4	0	0	0	0	0	0	0	0	0	0	0
i_5	0	0	0	0	0	0	0	0	0	1	0
i_6	0	0	0	0	0	0	0	0	0	0	1
i_7	0	0	0	0	0	1	0	0	0	0	0
i_8	0	0	0	0	0	0	1	0	0	0	0
i_9	0	0	1	0	0	0	0	0	0	0	0
i_{10}	0	0	0	0	0	0	0	0	0	0	0

On the 3rd Step, households who were rejected at the last step propose to their second choices. Table 4 shows the second proposal to houses by household i_3, i_4, i_{10} who have rejected at the first proposal.

Table 4 Second proposal to houses by households (Step3)

	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}
i_1	0	0	0	0	0	0	0	1	0	0	0
i_2	0	0	0	0	0	0	0	0	1	0	0
i_3	0	0	2	0	0	0	0	0	0	0	0
i_4	0	0	0	0	0	0	0	0	2	0	0
i_5	0	0	0	0	0	0	0	0	0	1	0
i_6	0	0	0	0	0	0	0	0	0	0	1
i_7	0	0	0	0	0	1	0	0	0	0	0
i_8	0	0	0	0	0	0	1	0	0	0	0
i_9	0	0	1	0	0	0	0	0	0	0	0
i_{10}	0	0	0	0	2	0	0	0	0	0	0

On the 4th Step, housing supplier of houses h_2 and h_8 receiving proposals chooses their favourite from the group consisting of the new proposers and the households on their string. Housing supplier h_2 and h_8 respectively reject all the rest and again keep the favourite household i_3 for h_2 and i_4 for h_8 in suspense.

Table 5 Choices of highest bidder by housing supplier (Step 4)

	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}
i_1	0	0	0	0	0	0	0	1	0	0	0
i_2	0	0	0	0	0	0	0	0	0	0	0
i_3	0	0	2	0	0	0	0	0	0	0	0
i_4	0	0	0	0	0	0	0	0	2	0	0
i_5	0	0	0	0	0	0	0	0	0	1	0
i_6	0	0	0	0	0	0	0	0	0	0	1
i_7	0	0	0	0	0	1	0	0	0	0	0
i_8	0	0	0	0	0	0	1	0	0	0	0
i_9	0	0	0	0	0	0	0	0	0	0	0
i_{10}	0	0	0	0	2	0	0	0	0	0	0

Step3 and Step4 are repeated in the same manner until every household would get new houses or give moving up as their proposal to all houses which they can accept would have been rejected by housing suppliers. Table 6 shows the final assignment of houses to households. All but a household i_2 have reached agreement with housing suppliers in the matching and bargaining process. However, household i_2 abandoned moving since they have lost in competition of bid price for the houses which they ranked higher than their current house. This set of matching is stable from the definition of stability explained in section 3.1. Market prices of housings where new occupants have been determined are equal to the bid prices which households who will live have presented. A house h_1 has not been sold, so it would be carried over to the next term t+1 and the selling price of it would be reduced.

Table 6 Final assignment of houses to households

	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}
i_1	0	0	0	0	0	0	0	1	0	0	0
i_2	3	0	0	0	0	0	0	2	0	0	0
i_3	0	0	2	0	0	0	0	0	0	0	0
i_4	0	0	0	0	0	0	0	0	2	0	0
i_5	0	0	0	0	0	0	0	0	0	1	0
i_6	0	0	0	0	0	0	0	0	0	0	1
i_7	0	0	0	0	0	1	0	0	0	0	0
i_8	0	0	0	0	0	0	1	0	0	0	0
i_9	0	0	0	2	0	0	0	0	0	0	0
i_{10}	0	0	0	0	2	0	0	0	0	0	0

As the results of trial simulation, it is confirmed that this matching algorithm we proposed reached a matching solution and market prices of all soled houses were decided. This solution is unique and stable because there are no other pair of households and housing suppliers who are not matched to each other but prefer each other to their actual contractor. Here, this stable state is guaranteed under the condition that suppliers could keeps household who they once accept on a string for the possibility that someone better may come along later. However, if the market would be on a first-come, first served as is usual in rented housing market, this algorithm does not necessarily reach stable matching.

6. CONCLUSION

In this study, we presented the modelling of search and bargaining of housing market for a land-use micro-simulation model, which is modelled as a matching process by applying Gale-Shapley marriage matching model to housing market problem. Housing market has unique characteristics such as costly search, imperfect capital markets, moving cost, and housing vacancies, diversity of houses and so on. Especially, direct bargaining process between demander and supplier of houses is crucial for describing housing market because of diversities of houses and households who have various preferences which depend on their attributes of households. However, existing land-use models whether micro-simulation or not have scarcely treated these unique characteristics of housing market originated in the individuality and diversity of housing and heterogeneous needs of households.

Therefore, we have developed the model of housing market which focuses on the matching process between households and houses they prefer by applying Gale-Shapley matching model for marriage problem to the problem of housing market. The algorithm of matching process of housing market in which the price of housing is built as well as the structure of this model has been presented in chapter 4. In chapter 5, trial simple simulation of our matching model for housing market has been presented with a hypothetical data set of ten households and ten houses. As the results of this simulation, it could be confirmed that this algorithm we developed by applying Gale-Shapley algorithm has led to the stable matching. Our matching model successes to express a bargaining process of price competition in local housing market by various households who have various preferences that could not been treated in existing land-use micro-simulation models.

This paper presented the importance and availability of the model of housing market which can express matching process in the land-use micro-simulation. However, this paper has presented no more than simple simulation with hypothetical data set. We have further tasks to be addressed in the near future such as simulation with a set of realistic and large-scale data, verification for this model, incorporation into other part of the model we are developing (e.g.Sugiki et.al. (2009)). Furthermore, algorithms other than what we proposed in this paper should also be considered in accordance with types of housing market.

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