TRANSPORT SUPPLY AND DEMAND MANAGEMENT STRATEGIES WITH RECOURSE CONSIDERATIONS

Xiaosu Ma, Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong,
Hong K Lo, Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong, E-mail: cehklo@ust.hk

ABSTRACT

The importance of developing sustainable transport and land use systems is widely recognized, when cities have to accommodate the ever increasing housing and travel demands. Meanwhile, the global economy and population growth are subject to change and uncertainty. The planning process, hence, is tasked to make recommendations on long-term large investments and demand management, such as pricing, while having little control over the external environment that the plan must proceed with. To this end, formulating flexible and adaptive transport supply and demand management (TS-DM) strategies is crucial for ensuring sustainable urban development. In this study, the approach of recourse considerations is incorporated in the planning process, which allows for implementation of the TS-DM strategies adaptively over time as the population growth uncertainty is gradually revealed. The formulation takes the form of a multi-stage stochastic program with equilibrium constraints, with the planning horizon being divided into several stages. A numerical example is constructed to illustrate and evaluate the additional benefits of this probabilistic approach under demand uncertainty, as compared with the traditional deterministic approach.

Keywords: bid-rent process; transport supply and demand management; recourse planning.

1. INTRODUCTION

Sustainability has been recognized as a key issue in recent urban studies, especially for developing megacities with large urban growth potentials but limited infrastructure funding. As problems associated with urban development, such as traffic congestion and shortage of housing supply, emerge as by-products of economic growth, it is important to incorporate the perspective of sustainable urban transport and land use systems in the planning process in advance. It is generally recognized that any transport supply or demand strategy will not only affect residents' travel behaviors, but also their locating decisions, leading to eventual changes in the land use patterns, and hence changes in land value, due to the well-known land use and transport interaction (Alonso, 1964; Wilson, 1967; McFadden, 1978). To this end, formulating integrated transport supply and demand management (TS-DM) strategies, e.g. highway infrastructure investments and road pricing schemes, is crucial for ensuring sustainable urban development. Typically, TS-DM strategies, such as transport infrastructure programs and real estate property development to accommodate the growing population, involve large investments and financing over a long period of time. Meanwhile, the global economy, economic and population growths of the region, etc. are subject to change and uncertainty. The planning process, hence, is tasked to make recommendations on long-term large investments, while having little control over the external environment that the plan must proceed with. Needless to say, building flexibility and adaptability within the TS-DM strategy to be implemented is a key to its success.

The land use and transport interaction has been studied for decades. Researchers developed land-use or more recently integrated land-use and transport models based on different micro-economic theories, e.g., the random utility theory (Mattsson, 1983; Anderstig and Mattsson, 1991, 1998; Boyce and Mattsson, 1999; Eliasson and Mattsson, 2001; Jonsson, 2003, 2007; Mattsson, 2007), and the bid-rent models associated with the hedonic theory (Rosen, 1974; Mackett, 1991; Martínez, 1992, 1995; Cheshire and Sheppard, 1995; Martínez and Araya, 2000; Chang and Mackett, 2006; Martínez and Henriquez, 2007; Briceño *et al.*, 2008; Bravo *et al.,* 2009). Meanwhile, in addition to developing these sophisticated integrated transport and land use models, policy studies on the development of transport supply and demand management strategies over time also attracted much attention, due to fiscal and equity concerns of transport infrastructure projects. Lo and Szeto (2004, 2009), Szeto and Lo (2005, 2006, 2008) and Siu and Lo (2009) introduced the time-dependent treatment of transport supply and demand management strategies, individually and jointly, to the continuous network design problem so as to improve the overall system performance. In our previous study (Ma and Lo, 2009), we developed a modeling framework to study the impacts of time-dependent TS-DM strategies on both the transport system performance and residents' location and travel choices, which is formulated as a mathematical program with equilibrium constraints. The model optimizes the total discounted system performance, e.g. social welfare or transport related cost, over the planning horizon, in which highway link expansions and road pricing for each discrete time period are jointly determined. As is customary, the approach taken there is deterministic, assuming that the total locating demand for each time period is fixed.

In this paper, we will extend this modeling framework with the consideration of recourse planning, focusing on demand uncertainty. Recourse planning is a widely used methodology for engineering design problems under uncertainty, stemming from dynamic programming

(Bertsekas, 1995). Basically the approach incorporates the likely future scenarios in a probabilistic manner into the formulation, so that the design will cater for them even at the early stage of implementation. Ukkusuri and Patil (2009) developed a multi-period transportation network design problem under demand uncertainty via a flexible network design formulation (FNDP). The solution takes the form of an open-loop control, resulting in a deterministic time-dependent investment plan. Our formulation here will treat the demand uncertainty probabilistically, allowing for the implementation of TS-DM strategies adaptively over time as the population growth uncertainty is gradually revealed, with the planning horizon being divided into several stages. The resultant solution takes the form of an optimal policy, which prescribes the planning decisions to be taken based on the system states (e.g. demand level and network provision) revealed in the future. Section 2 describes the overall modeling framework. Section 3 provides a numerical example to illustrate and evaluate this more flexible approach under demand uncertainty. Section 4 provides some concluding remarks.

2. MODELING FRAMEWORK

In this paper, the objective is to model the transport and land use interaction, especially the impact of transport policies on the housing sub-market, and to determine TS-DM strategies adaptively so as to optimize the overall system performance under demand uncertainty over time. We formulate the problem as a mathematical program with equilibrium constraints (MPEC) expressed in a quasi-dynamic structure, in which the equilibrium conditions are extended to include the housing sub-market (Ma and Lo, 2009). Both residents' location and travel choices, i.e., the locator problem, and real estate developers' decisions on housing supply, i.e.. the supplier problem, are modeled as equilibrium constraints to this optimization program. To determine the adaptive optimal TS-DM policies over time under demand uncertainty, the optimization is formulated as a multi-stage stochastic program, where the approach of recourse planning is applied.

2.1 The Locator problem

2.1.1 Nested structure

Residents' choices including both location and corresponding travel choices are incorporated at the network equilibrium level. For this purpose, a nested multinomial logit choice structure is adopted, as shown schematically in Figure 1. Locators have flexibility to choose their residential locations (labeled as $r1,...,rr$) and workplaces (labeled $s1,...,ss$), as well as their travel modes and commute routes. The choice process in this framework is defined as the hierarchical structure as shown in Figure 1, which makes intuitive sense. At the highest level is the residence location choice, then modal choice, finally route choice. For simplicity, in this paper, the modal choice contains only the auto mode, with the public transport or metro mode being put there as a place holder for future extensions, which can be readily incorporated.

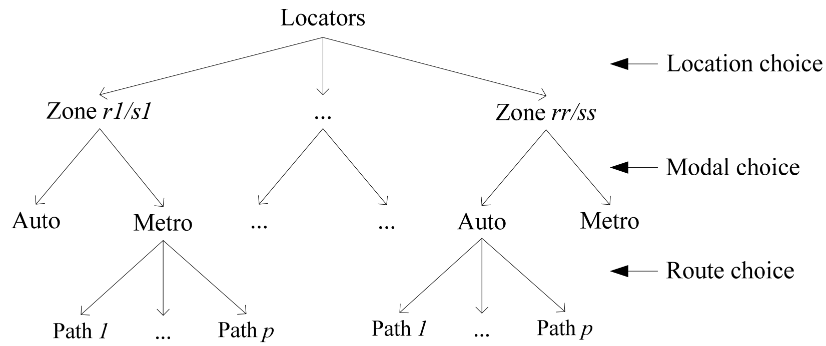

Figure 1 The nested multinomial logit choice structure adopted.

2.1.2 Travel cost

In this formulation, travel cost captures travel time, expressed in monetary terms, and other monetary costs, such as tolls or fares for different transport modes, if any. For the auto mode, the BPR link performance function is adopted:

$$t_a^{(\tau+n_1)} = t_a^{0(\tau+n_1)}\left(1+\alpha_1^{(\tau+n_1)}\left(\frac{x_a^{(\tau+n_1)}}{C_a^{(\tau+n_1)}}\right)^{\alpha_2^{(\tau+n_1)}}\right), \tag{1}$$

$$C_a^{(\tau+n_1)} = \begin{cases} C_a^{(0)}, \ 0 \le \tau < n_1 \\ C_a^{(\tau)} + y_a^{(\tau)}, \ \tau \ge n_1 \end{cases}. \tag{2}$$

$t_a^{0(\tau+n_1)}$ is the free-flow travel time on highway link a in period $\tau+n_1$, which is also used as a proxy for link length. $x_a^{(\tau+n_1)}$ is the flow on highway link a in period $\tau+n_1$; $\alpha_1^{(\tau+n_1)}, \alpha_2^{(\tau+n_1)}$ are coefficients of the BPR function. Most transport investments planned in period τ, e.g. highway link expansions, involve relatively longer planning and investment periods, denoted as n_1 in this formulation, before they are operational, as compared with residents' travel choice adaptation behaviors (Wegener, 1994). As a result, the future link capacity in period $\tau+n_1$, $C_a^{(\tau+n_1)}$, is the sum of link capacity $C_a^{(\tau)}$ in period τ plus the planned link expansion $y_a^{(\tau)}$ to be realized. It is further assumed that once a link is expanded, it will not be demolished. Link expansions will require construction costs, whereas all links, expanded or otherwise, will induce maintenance costs. Let $C_a^{(0)}$ be the initial link capacities in the first period; $r \in R$ denotes an origin node out of the origin set R; likewise, $s \in S$ denotes a destination node out of the destination set S. The auto travel cost for travellers in income group k between OD pair rs through path p in period τ, notated as $c_p^{rsk(\tau)}$, is the sum of the path's corresponding link travel times $t_a^{(\tau)}$, toll charges $\rho_a^{(\tau)}$, fuel costs $cf_a^{(\tau)}$, and parking fee at the destination s, $cp^{s(\tau)}$, if any:

$$c_p^{rsk(\tau)} = \sum_a \delta_{a,p}^{rs} (t_a^{(\tau)} \cdot vot^{k(\tau)} + cf_a^{(\tau)} + \rho_a^{(\tau)}) + cp^{s(\tau)}, \tag{3}$$

where $vot^{k(\tau)}$ is the value of time of residents in income group k and period τ. $\delta_{a,p}^{rs}$ is the path-link incidence indicator, equals 1 if auto link a is on the auto path p between OD pair rs; 0 otherwise.

2.1.3 Zonal attractiveness

Zonal attractiveness encompasses the amenities and/or opportunities of residential locations or workplaces. It describes residents' valuation of alternative locations, which can be measured by their associated attributes. Some attributes are exogenous, such as the natural environment; others are endogenous within the model, also known as location externalities, resulted from land-use and transport interactions, or other neighborhood interactions, such as traffic noise, segregated neighborhood in residential locations, the economies of agglomeration of firms in commercial areas, etc., which will impact residents' location choices. In this formulation, for simplicity, we focus on the impact of population allocation on zonal attractiveness, which is measured by two terms: first, the zone's intrinsic attractiveness, which is an exogenous constant; second, a measure of the effect of land use intensity or congestion on zonal attractiveness (Yang and Meng, 1998; Siu and Lo, 2009). Mathematically, the zonal attractiveness for origin ($l^{rk(\tau)}$) and destination ($l^{sk(\tau)}$) zones, respectively, are formulated as:

$$l^{rk(\tau)} = l_0^{rk(\tau)} - \theta_1^{rk(\tau)} \left(\frac{O^{r(\tau)}}{K^{r(\tau)}}\right)^{\theta_2^{rk(\tau)}}, \tag{4}$$

$$l^{sk(\tau)} = l_0^{sk(\tau)} - \theta_1^{sk(\tau)} \left(\frac{D^{s(\tau)}}{K^{s(\tau)}}\right)^{\theta_2^{sk(\tau)}}, \tag{5}$$

where,

$$O^{r(\tau)} = \sum_{j\in S} \sum_{p\in P^{rj}} \sum_k \overline{f}_p^{rjk(\tau)}, \tag{6}$$

$$D^{s(\tau)} = \sum_{i\in R} \sum_{p\in P^{is}} \sum_k \overline{f}_p^{isk(\tau)}. \tag{7}$$

$l_0^{rk(\tau)}$, $l_0^{sk(\tau)}$, respectively, are the intrinsic attractiveness for r and s as valuated by income group k in period τ, which are constants to be calibrated, reflecting the relative differences of attractiveness among locations; $K^{r(\tau)}$ and $K^{s(\tau)}$, respectively, are the holding capacities within r and s in period τ arising from land use planning; $\theta_1^{rk(\tau)}, \theta_1^{sk(\tau)}, \theta_2^{rk(\tau)}, \theta_2^{sk(\tau)}$, are coefficients to be calibrated for each origin r, destination s, and income group k; whether these parameters are positive or negative depends on model calibration. For instance, there is no obvious evidence whether the economies of agglomeration of firms have dominant positive effect on the attractiveness of a zone over other density or congestion side effects. $O^{r(\tau)}$ ($D^{s(\tau)}$) is the total productions (attractions) from origin r (to destination s) in period τ. In (6), $\overline{f}_p^{rjk(\tau)}$ is the path flow of residents in income group k between OD pair rj through path p in period τ, which can be obtained in the equilibrium formulation as described in the following section; P^{rj} refers to the generalized path set between OD pair rj. The first summation on the RHS of (6) works through each destination node j while fixing the origin at r. Likewise, in (7), $\bar{f}_p^{isk(\tau)}$ is the path flow of residents in income group k between OD pair is through p in period τ; P^{is} refers to the generalized path set between OD pair is.

2.1.4 Combined bid-rent and random utility model

A combined bid-rent and nested multinomial logit framework is developed to solve the locators' problem. The bid-rent process (Figure 2), which produces the housing rent at each residential location (labeled as $r1,...,rr$), is illustrated with the thickened arrows for residential zone $r1$ in the left diagram. The bidding process is open to all income levels (labeled as High, Low) working in different workplaces (labeled as $s1,...,ss$), after factoring in the relative location and transportation advantages of each zone. The resultant bid-rent for each zone (Rosen, 1974) then becomes the rent cost faced by locators in choosing their residence locations. As shown in the right diagram in Figure 2, after considering the overall cost/benefit, locators of each income group (illustrated with the thickened arrows for the example of high income group working at $s1$) make their location choices according to the random utility model.

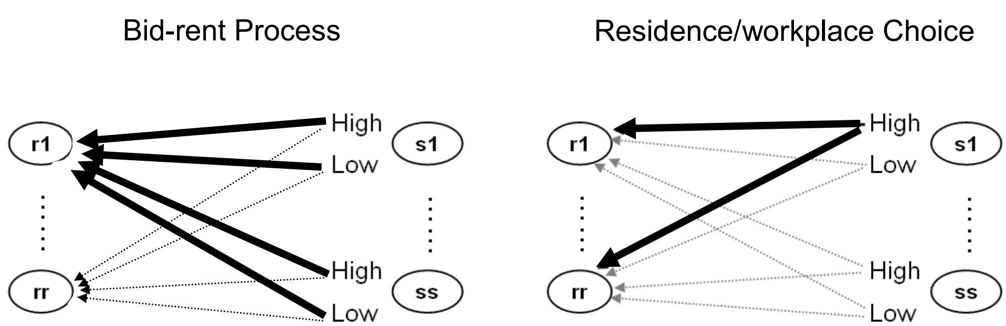

Figure 2 A schematic diagram for the bid-rent process and residence choice allocation.

The bid rent theory considers that travelers bid for residential locations based on utility maximization subject to budget constraints (Rosen, 1974). And the location will be rent out to the highest bidder. While deciding on the bid, each bidder maximizes his utility by considering both the benefit associated with the residential location and other expenses, given the bidder's income $I^{k(\tau)}$ (Martínez and Araya, 2000). The term $WP^{rsk(\tau)}$, or Willingness-to-Pay (WP), represents the maximum value that a bidder is willing to pay for a residential location r while working in workplace s in period τ, in order to maximize his/her overall utility $\bar{U}^{rsk(\tau)*}$ (Rosen, 1974; Martínez, 1992). An indirect utility function conditional on the location choice can be derived by fixing the utility level at the desired level $\bar{U}^{rsk(\tau)*}$. The WP function is then expressed as:

$$WP^{rsk(\tau)} = I^{k(\tau)} - f(\bar{U}^{rsk(\tau)*}) - \mu^{rsk(\tau)} + l^{sk(\tau)} + l^{rk(\tau)} + wp \,, \tag{8}$$

where the second term of the RHS, $f(\bar{U}^{rsk(\tau)*})$, represents the expenses set aside to achieve the bidder's desired level of utility in aspects other than transport and locational choices. $\mu^{rsk(\tau)}$ is the corresponding travel cost between r and s as perceived by locators in income group k. $l^{rk(\tau)}$, $l^{sk(\tau)}$ are the zonal attractiveness as defined in (4)-(5). wp is a constant, independent of locators' choices, which adjusts bids to match the actual housing rents and can be considered as a calibration parameter. The term wp does not affect the resultant location probabilities of bidders as only the relative bids matter in the multinomial logit framework. According to Martínez and Henríquez (2007), by defining a utility index $\bar{b}^{k(\tau)}$ expressed as:

$$\bar{b}^{k(\tau)} = I^{k(\tau)} - f(\bar{U}^{rsk(\tau)*}) + wp \,, \tag{9}$$

and adjusting this utility index, the equilibrium can be attained with all locators being able to find a residence; thus WP can be rewritten as:

$$WP^{rsk(\tau)} = \bar{b}^{k(\tau)} - \mu^{rsk(\tau)} + l^{sk(\tau)} + l^{rk(\tau)} \tag{10}$$

In the bid-rent process, each residential zone r is open to bidding by all locators with different income levels and different workplaces. It is expected that the locator group who offers the highest bid or the highest WP for a particular residential location r will have the highest probability of living there. According to the logit model, the probability of a housing unit being occupied by a locator of income group k working in workplace s in period τ is expressed as:

$$\bar{\Pr}^{sk/r(\tau)} = \frac{\bar{D}^{sk(\tau)} \cdot \exp(\bar{\beta}^{(\tau)} \cdot WP^{rsk(\tau)})}{\sum_{j\in S, k'\in K} \bar{D}^{jk'(\tau)} \cdot \exp(\bar{\beta}^{(\tau)} \cdot WP^{rjk'(\tau)})} \,, \tag{11}$$

where $\bar{\beta}^{(\tau)}$ is a scale parameter of the logit model for income group k in period τ; $\bar{D}^{sk(\tau)}$ is the number of locators in income group k working in workplace s in period τ. The probability defined in (11) is the aggregate version of the multinomial model proposed by McFadden (1978). Based on that, the OD demand of locators in income group k between residential location r and workplace s in period τ can be obtained by:

$$\bar{q}^{rsk(\tau)} = S^{r(\tau)} \cdot \bar{\Pr}^{sk/r(\tau)} \,, \tag{12}$$

where $S^{r(\tau)}$ is the number of housing supply in residential location r in period τ, which is endogenously determined in the following section.

According to the discrete choice theory for the logit model (McFadden, 1978; Small and Rosen, 1981; Train, 2003), the expected maximum bid or expected maximum willingness-to-pay, which constitutes the eventual housing rent, can be defined by the well-known log-sum function, expressed as:

$$\bar{\varphi}^{r(\tau)} = E[\max_{j\in S, k'\in K}\left\{WP^{rjk'(\tau)}\right\}] = \frac{1}{\bar{\beta}^{(\tau)}} \ln\left(\sum_{j\in S}\sum_{k'\in K} \bar{D}^{sk(\tau)} \cdot \exp\left(\bar{\beta}^{(\tau)} \cdot WP^{rjk'(\tau)}\right)\right) + \frac{\gamma}{\bar{\beta}^{(\tau)}} \,, \tag{13}$$

In (13), the first summation on the RHS works through each workplace zone j; the second summation works through each income group k'; γ is the Eulers's constant. Note that (11) -(13) are derived purely according to the bidding process and that the landowner of each residential zone will charge the rent according to the expected maximum bid. In a way, the analysis is developed from the perspective of the landowner.

It is equally important to analyze the choice process from the perspective of locators, and see if consistent results are achieved as far as residential location choices are concerned. To this end, the principle of utility maximization as represented by the logit model is adopted to model locators' location choices, and locators are considered to choose their residence and work locations according to their consumer surplus, defined as the difference between their willingness-to-pay for residence at r and the rent actually charged there, expressed as:

$$\bar{C}S^{rsk(\tau)} = WP^{rsk(\tau)} - \bar{\varphi}^{r(\tau)}, \tag{14}$$

The consumer surplus can be regarded as an incentive or utility in residents' choice process. As a result, the probability of locators in income group k choosing to reside in location r and work in location s is expressed as:

$$\bar{\Pr}^{rsk(\tau)} = \frac{S^{r(\tau)} \cdot \exp(\bar{\beta}^{(\tau)} \cdot \bar{C}S^{rsk(\tau)})}{\sum_{i \in R} \sum_{j \in S} S^{i(\tau)} \cdot \exp(\bar{\beta}^{(\tau)} \cdot \bar{C}S^{ijk(\tau)})}, \tag{15}$$

where the first summation in the denominator works through each origin node $i \in R$; the second summation works through each destination node $j \in S$. Following this specification of the choice process, Martínez and collaborators (e.g., Martínez, 1992; Martínez and Henríquez, 2007) established the important result that the two perspectives, the bid and choice processes (Figure 2), i.e. (11)-(15), are entirely consistent and complementary to each other. Either (11) or (15) will produce the same set of residence allocation, and the same rent for each zone.

Accordingly, the final nested choice probability of locators in income group k choosing to reside in location r, work in location s, through path p in period τ is then defined by:

$$\bar{\Pr}_p^{rsk(\tau)} = \frac{S^{r(\tau)} \cdot \exp(\bar{\beta}^{(\tau)} \cdot \bar{C}S^{rsk(\tau)})}{\sum_{i \in R} \sum_{j \in S} S^{i(\tau)} \cdot \exp(\bar{\beta}^{(\tau)} \cdot \bar{C}S^{ijk(\tau)})} \cdot \frac{\exp(-\beta^{k(\tau)} \cdot c_p^{rsk(\tau)})}{\sum_{p' \in P^{rs}} \exp(-\beta^{k(\tau)} \cdot c_{p'}^{rsk(\tau)})}, \tag{16}$$

where,

$$\mu^{rsk(\tau)} = -\frac{1}{\beta^{k(\tau)}} \ln\left(\sum_{p' \in P^{rs}} \exp\left(-\beta^{k(\tau)} \cdot c_p^{rsk(\tau)}\right) \right), \tag{17}$$

$\beta^{k(\tau)}$ is the scale parameter of the logit model for income group k in period τ. $\mu^{rsk(\tau)}$ in (17) represents the expected minimum travel cost for income group k between OD pairs rs in period τ.

Finally, the corresponding path flow of locators, $\overline{f}_p^{rsk(\tau)}$, OD demand, $\overline{q}^{rsk(\tau)}$, and link flow $x_a^{(\tau)}$ for the entire network as a whole for each time period τ are expressed as:

$$\overline{f}_p^{rsk(\tau)} = \overline{H}^{k(\tau)} \cdot \overline{\Pr}_p^{rsk(\tau)}, \tag{18}$$

$$\overline{q}^{rsk(\tau)} = \sum_{p' \in P^{rs}} \overline{f}_{p'}^{rsk(\tau)}, \tag{19}$$

$$x_a^{(\tau)} = \sum_r \sum_s \sum_k \sum_p f_p^{rsk(\tau)} \cdot \delta_{a,p}^{rs(\tau)}. \tag{20}$$

$\overline{H}^{k(\tau)}$ is the population of locators in income group k in period τ, which is exogenously given in this formulation. The path flows are used to determine the total attractions and productions for each zone in (6)-(7). The link flows in (1) are used for calculating link congestion.

According to the above formulation, any changes in the transport network and the resultant changes in travel cost will alter not only travelers' route choices, but also their willingness-to-pay, or the housing rents and their residential location choices. On the other hand, changes in the zonal attractiveness or land use planning, such as increases in the supply of residential units, will have consequential effects on locators' residence choices, which in turn will affect the transportation system performance. In this way, by capturing the location costs (i.e., housing rents) endogenously via the bid-rent process, the interaction between land use and transport can be established.

2.2 The Supplier problem

The actual bid-rent process involves both locators and suppliers, e.g. real estate developers. In this formulation, it is assumed that suppliers make housing investment decisions, i.e. the number of future housing unit to be developed in each residential location, based on the principle of profit maximization. The profit of renting out a housing unit, $\pi^{r(\tau)}$, is the difference between the housing rent and investment cost discounted over time, $b_H^{r(\tau)}$. Moreover, suppliers would charge the rent according to the expected maximum bid $\overline{\varphi}^{r(\tau)}$, which is applied to all locators regardless of their income levels. Furthermore, the location choices of developers in housing supply are estimated based on the current housing demand and profit, i.e. at time period τ; due to the lag interaction between housing investment decision and housing availability, the probability that a housing unit is invested in residential location r in period $\tau + n_2$ is expressed:

$$\Pr^{r(\tau+n_2)} = \frac{S^{r(\tau)} \cdot \exp(\lambda^{(\tau)} \cdot \pi^{r(\tau)})}{\sum_{i \in R} S^{r(\tau)} \cdot \exp(\lambda^{(\tau)} \cdot \pi^{i(\tau)})}, \ \forall \tau \geq 0, n_2 \geq 1, \tag{21}$$

where,

$$\pi^{r(\tau)} = \overline{\varphi}^{r(\tau)} - b_H^{r(\tau)}, \tag{22}$$

$\lambda^{(\tau)}$ is a logit model parameter to calibrate the location choice of developers for housing supply. $\bar{\varphi}^{r(\tau)}$ is the unit housing rent determined through the bid-rent process in (13); $b_H^{r(\tau)}$ is the unit housing investment cost in residential location r in period τ, which is exogenously given. Thereby the number of future housing supply $\bar{S}^{r(\tau+n_2)}$, is obtained by:

$$S^{r(\tau+n_2)} = \begin{cases} S^{r(0)}, \ 0 \le \tau < n_2 \\ S^{(\tau+n_2)} \cdot \Pr^{r(\tau+n_2)}, \ \tau \ge n_2 \end{cases}, \tag{23}$$

$S^{(\tau+n_2)}$ is the total number of housing supply in period $\tau + n_2$. The housing supply, $S^{r(0)}$, in the initial time period is assumed to be exogenously given. At each time period, the total housing supply is equal to the total number of locators, expressed as:

$$S^{(\tau)} = \sum_k \bar{H}^{k(\tau)}, \forall \tau \tag{24}$$

It is worth noting that the actual housing supply market may involve multiple suppliers, with different technological advantages, information, resources, and resulting profit functions. The heterogeneity of suppliers may induce a competitive housing supply sub-market as compared with the currently modeled monopoly supply market. The end result is that the suppliers may or may not be able to charge rent at the same level as the expected maximum bid, depending on the competition by other suppliers in the same as well as other locations. Besides, suppliers may develop different housing types h simultaneously. Normally, the housing types to be produced by suppliers is an important locational attribute which would affect the zonal attractiveness as defined in (4). In this formulation, for simplicity, it is assumed that there is only one supplier who develops homogenous housing units. To more completely reflect the housing supply problem, it is necessary to elevate it as a competitive equilibrium formulation, which is part of our ongoing research.

2.3 Combined Equilibrium Formulation

The combined bid-rent and nested multinomial choice framework formulated above constitutes an equilibrium model, with the time dimension of the planning horizon incorporated explicitly. The entire equilibrium problem can be cast into an equivalent Nonlinear Complementarity Problem (NCP) and solved accordingly (Lo and Chen, 2000a, 2000b). The equivalent NCP is to:

Find $\mathbf{Z}^* \ge 0$, such that

$$\mathbf{F}(\mathbf{Z}^*) \ge \mathbf{0}, \tag{25}$$

$$\mathbf{Z}^{*\mathrm{T}} \cdot \mathbf{F}(\mathbf{Z}^*) = \mathbf{0}, \tag{26}$$

where $\mathbf{Z}=\begin{pmatrix} \bar{f}_p^{rsk(\tau)}, \forall r,s,p,k,\tau \\ S^{r(\tau)}, \forall r, \tau \ge n_2 \\ \bar{b}^{k(\tau)}, \forall k, \tau \end{pmatrix}$ is a column vector of path flows, housing supply, and utility index defined in (16)-(18), (21)-(23), (9), respectively. Correspondingly,

$\mathbf{F}(\mathbf{Z})=\begin{pmatrix} \bar{f}_p^{rsk(\tau)} - \bar{H}^{k(\tau)} \cdot \overline{\Pr}_p^{rsk(\tau)}, \forall r,s,p,k,\tau \\ S^{r(\tau)} - S^{(\tau)} \cdot \Pr^{r(\tau)}, \forall r, \tau \ge n_2 \\ \sum_{r,s} S^{r(\tau)} \cdot \overline{\Pr}^{sk/r(\tau)} - \bar{H}^{k(\tau)}, \forall k, \tau \end{pmatrix}$ is a column vector.

The nonlinear complementarity conditions (25)-(26) can be written as:

$$\bar{f}_p^{rsk(\tau)}(\bar{f}_p^{rsk(\tau)} - \bar{H}^{k(\tau)} \cdot \overline{\Pr}_p^{rsk(\tau)}) = 0, \quad \forall r,s,p,k,\tau \,, \tag{27}$$

$$\bar{f}_p^{rsk(\tau)} - \bar{H}^{k(\tau)} \cdot \overline{\Pr}_p^{rsk(\tau)} \ge 0, \quad \forall r,s,p,k,\tau \,, \tag{28}$$

$$S^{r(\tau)}(S^{r(\tau)} - S^{(\tau)} \cdot \Pr^{r(\tau)}) = 0, \forall r, \tau \ge n_2 \,, \tag{29}$$

$$S^{r(\tau)} - S^{(\tau)} \cdot \Pr^{r(\tau)} \ge 0, \forall r, \tau \ge n_2 \,, \tag{30}$$

$$\bar{b}^{k(\tau)}(\sum_{r,s} S^{r(\tau)} \cdot \overline{\Pr}^{sk/r(\tau)} - \bar{H}^{k(\tau)}) = 0, \quad \forall k, \tau \,, \tag{31}$$

$$\sum_{r,s} S^{r(\tau)} \cdot \overline{\Pr}^{sk/r(\tau)} - \bar{H}^{k(\tau)} \ge 0, \quad \forall k, \tau \,, \tag{32}$$

$$\bar{f}_p^{rsk(\tau)} \ge 0, \ \forall r,s,p,k,\tau \,, \tag{33}$$

$$S^{r(\tau)} \ge 0, \ \forall r, \tau \ge n_2 \,, \tag{34}$$

$$\bar{b}^{k(\tau)} \ge 0, \ \forall k, \tau \,. \tag{35}$$

According to (27), since $\bar{f}_p^{rsk(\tau)} > 0$, given that the logit model is adopted in the equilibrium formulation, then $\bar{f}_p^{rsk(\tau)} - \bar{H}^{k(\tau)} \cdot \overline{\Pr}_p^{rsk(\tau)} = 0$ or $\bar{f}_p^{rsk(\tau)} = \bar{H}^{k(\tau)} \cdot \overline{\Pr}_p^{rsk(\tau)}$. That is, the path flows for locators are assigned exactly according to the probability as defined in (15). Similarly, (29)-(30) model the housing supply equilibrium, where suppliers maximizing their profits. The conditions defined in (31)-(32) assure that every locator can eventually be located, i.e. the total housing supply equals the total housing demand in each time period.

The above reformulation can be reformulated as an unconstrained optimization problem (Lo and Chen, 2000a), by minimizing the following gap function to zero (Fischer, 1992; Facchinei and Soares, 1995).

$$\begin{aligned}\min G(\mathbf{Z}) = &\sum_{\tau}\sum_{rskp}\vartheta\left(\bar{f}_p^{rsk(\tau)},\ \bar{f}_p^{rsk(\tau)} - \bar{H}^{k(\tau)}\cdot\overline{\Pr}_p^{rsk(\tau)}\right)\\ &+\sum_{\tau}\sum_{r}\vartheta\left(S^{r(\tau)},\ S^{r(\tau)} - S^{(\tau)}\cdot\Pr^{r(\tau)}\right) \qquad , \\ &+\sum_{\tau}\sum_{k}\vartheta\left(\bar{b}^{k(\tau)},\ \sum_{r,s}S^{r(\tau)}\cdot\overline{\Pr}^{sk/r(\tau)} - \bar{H}^{k(\tau)}\right)\end{aligned} \tag{36}$$

where $\vartheta(\cdot)$ is defined as:

$$\vartheta(c,d) = \frac{1}{2}\phi^2(c,d), \tag{37}$$

$$\phi(c,d) = \sqrt{c^2+d^2} - (c+d). \tag{38}$$

c and d in (37)-(38) are any real numbers. By minimizing (36), the path flows $\bar{f}_p^{rsk(\tau)}$ and housing supply $S^{r(\tau)}$ for each time period can be obtained. Equivalently, the gap function is simply posed to be zero, i.e. $G(\mathbf{Z}) = 0$ and consider it as a set of equilibrium constraints to be satisfied. It can be verified readily that when $G(\mathbf{Z})$ attains the value of zero, the entire NCP (27)-(35) is satisfied (Lo and Chen, 2000a).

2.4 System Optimization with Recourse Considerations

The above combined equilibrium formulation describes the resultant residents' location and travel choices and the housing supply choices of real estate developers, given a fixed TS-DM strategy and fixed demand, i.e. the total number of locators, in one time period. However, the ultimate goal is to devise adaptive optimal transport management policies under demand uncertainty over time, so as to achieve the desired planning perspectives. Two issues have to be dealt with in the formulation, i.e. the adaptive optimal policy structure and demand uncertainty.

2.4.1 Adaptive optimal policy

In our previous research (Ma and Lo, 2009), different planning objectives were developed to investigate the resultant urban development patterns and system performances. For example, the welfare-oriented objective is to maximize the total social welfare for the whole planning horizon, defined as the combined producer surplus, i.e. real estate developers and transport infrastructure investors, and consumer surplus, i.e. locators, discounted to present-value terms. The problem is formulated as a mathematical program with equilibrium constraints (MPEC), by expressing the combined equilibrium model as constraints to be satisfied, i.e. $G(\mathbf{Z}) = 0$. The optimal solution takes the form of time-dependent TS-DM strategies based on the condition that the future demands will be realized as anticipated. It prescribes the level of investment and tolling scheme in each time period, given that the population growth is fixed (or the demand forecast is 100% accurate). However, if the realized demand in any period is different from the prediction, the TS-DM strategy determined in the earlier planning stage may not be optimal.

In this study, the system optimization is extended to allow for the development of adaptive TS-DM strategies over time as the population growth uncertainty is gradually revealed. The resultant formulation takes the form of a multi-stage stochastic program with equilibrium constraints, with the planning horizon being divided into several stages, say each 5-10 years. Compared with the previous one time optimization solution, the new solution takes the form of an optimal policy, known as a closed-loop control, which not only provides an expected overall system performance over the planning horizon, but also adapts the decisions as the uncertain population growth reveals gradually as the plan proceeds forward. As shown in Figure 2, the discrete time periods are treated as *stages*, while potential demand level, $\bar{H}^{k(\tau)}$, and realized transport network provision, $C_a^{(\tau)}$, representing system *states*. At each *state* and *stage*, there exists an optimal TS-DM strategy, $(\mathbf{y}, \boldsymbol{\rho})$, to be determined and implemented. The details of this formulation will be further elaborated in the next section.

Note that the system is modelled as a quasi-dynamic structure (Wegener, 1994, 1998; Lo and Szeto, 2004; Szeto and Lo, 2008; Ma and Lo, 2009). For instance, as defined in the previous sections, the new investment plan determined in the current period can only be operational in the next time period, while the decisions on toll charge can be implemented at once if needed.

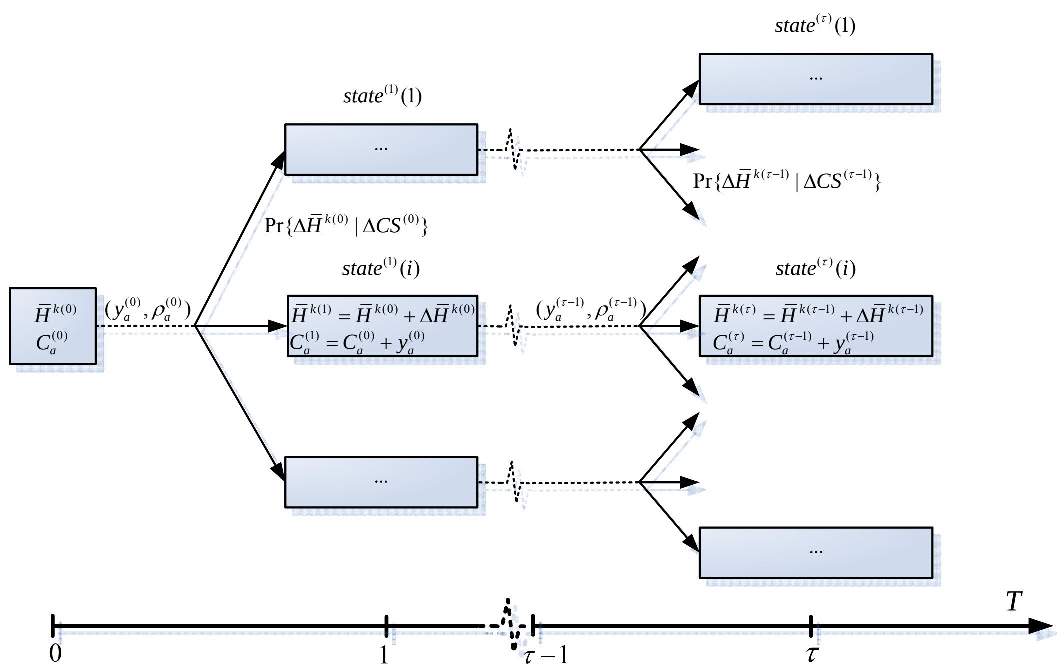

Figure 2 Structure of the multi-stage stochastic demand

2.4.2 Demand uncertainty

Ukkusuri and Patil (2009) interpreted the origination of travel demand uncertainty from two aspects, i.e. demand stochasticity and demand elasticity. The former influences the population natural growth and the level of immigration/emigration within a study region, hence the change in potential demand, i.e. the total number of locators in each time period. It can be considered as an exogenous factor outside the modeling system, e.g. economic growth, social and cultural evolution, and medical and sanitary level of development. The latter is normally due to endogenous factors, such as travel cost or locational benefit, normally modeled by a demand function. Figure 2 shows the stochastic demand tree under discrete probability distribution in this formulation. The demand change and uncertainty arise from the change in total potential demand and the planning decisions undertaken, e.g. TS-DM strategies. For instance, transport infrastructure provision will improve the system performance (as measured by consumer surplus), which will in turn induce more demand in the next period. Mathematically, the probability distribution of population change in the next planning period, $\Delta\overline{H}^{k(\tau)}$, is conditional on the change of consumer surplus in the previous period, $\Delta C\overline{S}^{k(\tau)} = C\overline{S}^{k(\tau)} - C\overline{S}^{k(\tau-1)}$, expressed as $\Pr\left\{\Delta\overline{H}^{k(\tau)} \mid \Delta C\overline{S}^{k(\tau)}\right\}$. As a result, the total number of locators in the next period is $\overline{H}^{k(\tau)} = \overline{H}^{k(\tau-1)} + \Delta\overline{H}^{k(\tau)}$, as shown in Figure 2.

2.4.3 Optimization

In this study, the TS-DM strategy is represented by the combination of highway link expansions, $y_a^{(\tau)}$, and auto toll charge, $\rho_a^{(\tau)}$. The planning decisions on link expansions include where and when, and also the expansion level among alternative highway links. To reduce the computation load during recursive iterations without losing practicability, the alternatives are predetermined among all the links in the transport network. And the potential expansion levels are limited to a few discrete values, i.e. $y_a^{(\tau)} \in \{y_a^h, y_a^l, 0\}$. For instance, “high level” refers to adding two lanes for a two-lane highway, while “low level” refers to adding one extra lane. In further research, suitable approximation approaches will be developed to identify alternative highway links for this purpose. On the other hand, the optimized auto tolls are continuous and can be charged on any auto links, bounded by a reasonable range, i.e. $\rho_a^{(\tau)} \in [0, \rho_a^u]$. At any realized system *state* at any *stage*, $(\overline{H}^{k(\tau)}, C_a^{(\tau)})$, the objective is to maximize the discounted social welfare from the current period to the end of planning horizon, expressed as:

$$SW^{(\tau)*} = \underset{\substack{\rho_a^{(\tau)} \in [0, \rho_a^u], \\ y_a^{(\tau)} \in \{y_a^h, y_a^l, 0\}}}{\text{Maximize}} \left\{ SW^{(\tau)} + \sum_{\Delta H^{k(\tau)}} \Pr(\Delta H^{k(\tau)}) \cdot SW^{(\tau+1)*} \right\} \tag{39}$$

where,

$$SW^{(\tau)} = \upsilon(dr, \tau) \cdot \left(\sum_{rs} \sum_{k} \overline{q}^{rsk(\tau)} \cdot \overline{C}S^{rsk(\tau)} + PS^{(\tau)} \right) \tag{40}$$

$$PS^{(\tau)} = (R_T^{(\tau)} + R_H^{(\tau)}) - (B_T^{(\tau)} + B_H^{(\tau)}) - (M_T^{(\tau)} + M_H^{(\tau)}) \tag{41}$$

In (39), the first term in the bracket, $SW^{(\tau)}$, is the discounted social welfare in time period τ, which is the sum of consumer surplus and producer surplus; the second term the expected maximum social welfare from period $\tau+1$ to the end of planning horizon. The producer surplus, in (41), is the difference of total revenue, being collected from housing rent, $R_H^{(\tau)}$, and toll, $R_T^{(\tau)}$, and total investment cost for housing, $B_H^{(\tau)}$, and highway expansion, $B_T^{(\tau)}$, and total maintenance cost for housing, $M_H^{(\tau)}$, and highway expansion $M_T^{(\tau)}$. $\upsilon(dr,\tau)$ is a discount factor depending on the annual discount rate dr and time period τ $(\tau\in[0,T])$. It converts the revenue collected in each time period τ to present value terms of the base year.

Putting everything together, the final MPEC with the welfare-oriented objective is expressed as:

$$SW^{(0)*}=\underset{\substack{\rho_a^{(0)}\in[0,\rho_a^u],\\ y_a^{(0)}\in\{y_a^h,y_a^l,0\}}}{\text{Maximize}}\left\{SW^{(0)}+\sum_{\Delta H^{k(\tau)}}\Pr(\Delta H^{k(0)})\cdot\left\{\underset{\substack{\rho_a^{(1)}\in[0,\rho_a^u],\\ y_a^{(1)}\in\{y_a^h,y_a^l,0\}}}{\text{Maximize}}\left\{SW^{(1)}+\sum_{\Delta H^{k(\tau)}}\Pr(\Delta H^{k(1)})\cdot\{...\}\right\}\right\}\right\} \tag{42}$$

subject to

$$G(\mathbf{Z})=0\,, \tag{43}$$

Constraints (1)-(24) ,

$$\bar{f}_p^{rsk(\tau)}\geq 0,\ \forall r,s,p,k,\tau\,, \tag{44}$$

$$S^{r(\tau)}\geq 0,\ \forall r,\tau\geq n_2\,, \tag{45}$$

$$\bar{b}^{k(\tau)}\geq 0,\ \forall k,\tau\,, \tag{46}$$

$$y_a^{(\tau)}\in\{y_a^h,y_a^l,0\},\ \forall a,\tau\,, \tag{47}$$

$$\rho_a^{(\tau)}\in[0,\rho_a^u],\ \forall a,\tau\,. \tag{48}$$

The above mathematical program is non-linear and non-convex, as is typical for MPEC. Nevertheless, as non-linear programs, they can be solved readily by commercial non-linear mathematical programming solvers. One caution is that global optimality of solutions is not guaranteed (Wang and Lo, 2010).

3. NUMERICAL STUDIES

3.1 Scenario Setting

Without loss of generality, the formulation can be applied to any transportation networks. For illustration purposes, the modeling framework is applied to the network as shown in Figure 3 over a planning horizon of thirty years (in three 10-year periods). The example network consists of two origin nodes (Zones 1, 2), two destination nodes (Zones 5, 6), and seven connected links.

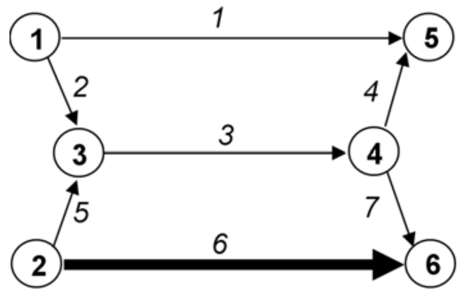

Figure 3 The example network

As shown in Table 1 and Table 2, intrinsically, Zone 2, a developed residential area, and Zone 6, a developed CBD, will attract more locators due to their higher zonal attractiveness and convenient travel options (Link 6 being a highway). However, due to the ever increasing population, the government wants to develop a new residential area (Zone 1) and a new CBD (Zone 5) to mitigate the congestion in the old area by proposing link expansion and tolling schemes on the local streets linking the new and old areas. The initial housing supply in Zones 1 and 2, are, 30 and 70 units, respectively, which just meet the total demand of 100 units of locators. The initial link capacities, free-flow travel times and coefficients of their BPR functions are shown in Table 2.

The locators are stratified into two income groups: $I^{high} = \text{HK\$ } 500/\text{day}$ and $I^{low} = \text{HK\$ } 400/\text{day}$ with an average increase of $\text{HK\$ } 40 \sim 50/\text{day}$ per time period. For simplicity, it is assumed that each household contains one person, who uses car as the only commute mode. The population will grow over time, subject to uncertainty, as will be further elaborated in the next section.

Table 1 Zonal characteristics

Zone	Intrinsic attractiveness l_0^{rk} / l_0^{sk}	Description
1	15	New residential
2	30	Old residential
5	15	New CBD
6	30	Old CBD

Table 2 Link characteristics

Link	Initial Capacity C_a^0 (veh/hr)	Free-flow time t_a^0 (min)	α_1	α_2	Link type
1	60	15	0.25	4	Local street
2	60	5	0.25	4	Local street
3	60	10	0.25	4	Local street
4	60	5	0.25	4	Local street
5	60	8	0.25	4	Local street
6	120	10	0.85	5.5	Highway
7	60	5	0.25	4	Local street

3.2 Optimal Policy

In this example, the objective is to determine the optimal policy such that the expected social welfare from any system state at any time period to the final planning horizon is maximized. Link 3 and Link 6 are pre-chosen as the alternative links to be invested. The investment decision is to determine whether to add 50% of their original capacities and if so, when, i.e. $y_a^{(\tau)} \in \{0.5C_a^0, 0\}, C_a^{(\tau)} \le 1.5C_a^0$. The tolls can be charged at any level with any combinations of link improvements.

To represent the link expansion states of the system, let $s(\tau, \varpi)$ denote the ϖ th supply state, i.e. link expansion, at time period τ, where:

- $\varpi = 1$ indicates none link is expanded
- $\varpi = 2$ indicates expansion at link 3 but no expansion at link 6
- $\varpi = 3$ indicates expansion at link 6 but no expansion at link 3
- $\varpi = 4$ indicates expansions at both link 3 and link 6

Since it is assumed that link expansions will not be demolished, therefore link expansions implemented at an earlier time (i.e. smaller τ) will remain for later times (i.e. higher τ). Hence, while it is possible to transition the system from $s(0,1)$ to $s(1,2)$ - indicating expansion at link 3 to be implemented at $\tau = 1$; it is not possible to transition the system from $s(1,2)$ to $s(2,3)$, since the expansion at link 3 shall remain at $\tau = 2$. In fact, the possible expansion states are listed in Table 3. Clearly, each pair of adjacent system states over time defines a unique investment decision. For instance, the transition of $s(1,2)$ to $s(2,4)$ refers to the expansion on Link 6 determined at $\tau = 1$, while Link 3 has been already expanded in the previous time period.

Table 3 Possible link expansion states

Link expansion status	Associated system states $s(\tau, \varpi)$
$C_3^0 = C_3^{(1)} = C_3^{(2)},\ C_6^0 = C_6^{(1)} = C_6^{(2)}$ (i.e. no expansion at all)	$s(0,1), s(1,1), s(2,1)$
$C_3^{(1)} = C_3^{(2)} = 1.5C_3^0,\ C_6^{(1)} = C_6^{(2)} = C_6^0$ (expand link 3 at $\tau = 1$ but not link 6)	$s(0,1), s(1,2), s(2,2)$
$C_3^{(1)} = C_3^{(2)} = C_3^0,\ C_6^{(1)} = C_6^{(2)} = 1.5C_6^0$ (expand link 6 at $\tau = 1$ but not link 3)	$s(0,1), s(1,3), s(2,3)$
$C_3^{(1)} = C_3^{(2)} = 1.5C_3^0,\ C_6^{(1)} = C_6^{(2)} = 1.5C_6^0$ (expand both links 3 and 6 at $\tau = 1$)	$s(0,1), s(1,4), s(2,4)$
$C_3^{(1)} = C_3^0; C_3^{(2)} = 1.5C_3^0,\ C_6^{(1)} = C_6^{(2)} = 1.5C_6^0$ (expand link 6 at $\tau = 1$ and link 3 at $\tau = 2$)	$s(0,1), s(1,3), s(2,4)$
$C_3^{(1)} = C_3^{(2)} = 1.5C_3^0,\ C_6^{(1)} = C_6^0; C_6^{(2)} = 1.5C_6^0$ (expand link 3 at $\tau = 1$ and link 6 at $\tau = 2$)	$s(0,1), s(1,2), s(2,4)$

In Figure 4, let $d(\tau, \nu)$ denote the ν th possible demand state, i.e. the amount of locators, at time period τ. For instance, $d(1,1)$ is the first demand state at $\tau = 1$, where there are 90 locators of high income and 130 locators of low income. The realization of the demand state could depend on both endogenously or exogenously determined probabilities. In this example, for simplicity, the probabilities are assumed to be exogenously given, as shown in Figure 4. The demand state $d(\tau, \nu)$ and supply state $s(\tau, \varpi)$ together constitute a specific system state. Eventually, the optimal policy can be determined recursively from the final stage, i.e. $\tau = 2$, as shown in Table 4. At each possible system state, an optimal policy decision is determined so as to maximize the expected social welfare onward. Note that, at $\tau = 2$, no investment decision needs to be determined as the planning process terminates.

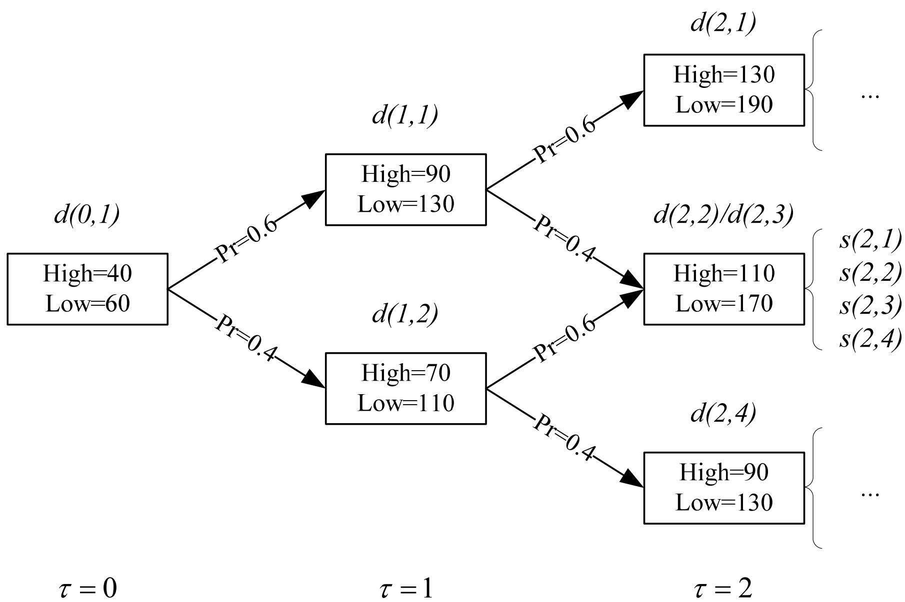

Figure 4 Population growths over time under uncertainty

In Table 4, the results show that link 6 will be expanded at $\tau = 0$ for sure. Whether link 3 will be expanded or not depends on the demand realization at $\tau = 1$. If the demand is realized as indicated by $d(1,1)$, then link 3 will be expanded; otherwise, for the realization at $d(1,2)$, link 3 will not be expanded. The optimal policy illustrates the adaptive nature of the solution. It does not pre-determine the link expansions at $\tau = 0$ but wait for the demand realization at $\tau = 1$ before deciding. Likewise, the optimal tolls also adaptively adjust themselves depending on the demand realization and link expansion status.

Table 4 Optimal policy

		$\tau=0$		$\tau=1$		$\tau=2$	
Current state		$d(0,1)$	$d(1,1)$	$d(1,2)$	$d(2,1)$	$d(2,2)/$ $d(2,3)$	$d(2,4)$
Total demand		100	220	180	320	280	220
$y_a^{(\tau)}$	Link 3	0.0	30.0	0.0	-	-	-
	Link 6	60.0	0.0	0.0	-	-	-
$\rho_a^{(\tau)}$	Link 1	1.1	1.9	2.2	0.3	3.7	0.0
	Link 2	0.0	4.2	1.1	11.9	11.6	1.8
	Link 3	0.0	7.4	4.7	9.8	4.6	6.4
	Link 4	0.0	2.0	0.3	10.8	2.8	3.4
	Link 5	0.0	11.5	5.7	13.0	4.8	7.2
	Link 6	0.2	31.0	34.4	60.8	36.6	39.7
	Link 7	0.0	4.8	7.8	0.1	1.6	2.5
$SW^{(\tau)*}$ (1)		3.09E+08	2.44E+08	2.09E+08	1.19E+08	1.17E+08	9.34E+07

(1) *The optimal social welfare is discounted to the present value at* $\tau=0$

3.3 Comparison with None Recourse Consideration

To compare performance of the optimal policy determined by this proposed stochastic multistage framework, a typical optimal time-dependent TS-DM strategy without recourse consideration is also determined to maximize the total discounted social welfare over the planning horizon (Ma and Lo, 2009). The demand at each stage/time period is fixed by taking the mean of the given population distribution. And the resultant TS-DM strategy is casted into the *actual* potential population growing trends, i.e. $d(0,1)$ - $d(1,1)$ - $d(2,1)$, $d(0,1)$-$d(1,1)$-$d(2,2)$, $d(0,1)$-$d(1,2)$-$d(2,3)$, $d(0,1)$-$d(1,2)$-$d(2,4)$. Eventually the final expected social welfare is calculated by taking the mean of all the four resultant welfare measures, which is 3.03E+08. Comparing with the optimal welfare determined in the previous section, i.e. 3.09E+08 as shown in Table 4, the optimal policy is 7% better off. This figure, obviously, is network and scenario specific. It is provided here simply to verify the benefit of this adaptive formulation. It is worth noting that, if the population growth becomes deterministic, this adaptive formulation with recourse planning will degenerate nicely into the deterministic equivalent, producing the same optimal solution.

4. CONCLUDING REMARK

This paper developed a modeling framework to study the impact of adaptive TS-DM strategies for managing the transportation system and land use pattern under demand uncertainty. A nested multinomial logit choice model combined with the bid-rent process was developed to model residents' location and travel choices. The supplier problem was also incorporated to model the housing supply market. To achieve the network equilibrium, the model is solved by formulating it as an equivalent nonlinear complementarity problem.

Subsequently, to address the issue of demand uncertainty, we integrated the model with the approach of recourse planning, extending the model to an adaptive control formulation. The final formulation takes the form of a multi-stage stochastic program, with the network equilibrium constraints captured for each stage. The resultant optimal policy with recourse consideration prescribes adaptive TS-DM strategies over time as the demand uncertainty is gradually revealed. A small network example was constructed to illustrate and evaluate this probabilistic approach under demand uncertainty. Generally, the results demonstrated the additional benefits of this more flexible approach, as compared with traditional deterministic ones.

The current modeling framework was constructed to model demand uncertainty with the stochastic programming method. We anticipate that the computation load will significantly increase as the network size increases. Our ongoing research pertains to how best to develop approximation schemes to make the computation tasks efficient.

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REFERENCES

Anderstig, C. and Mattsson, L.-G. (1991). An integrated model of residential and employment location in a metropolitan region. Papers in Regional Science, 70(2).

Anderstig, C. and Mattsson, L.-G. (1998). Modeling land-use and transport interaction: policy analyzes using in the IMREL Model. In: L. Lundkvist, L.-G. Mattsson, T. J. Kim, eds., Network Infrastructure and the Urban Environment, Advances in Spatial Science, 308-328. Springer Verlag Berlin Heidelberg.

Alonso, W. (1964). Location and land use. Harvard University Press.

Bertsekas, D.P. (1995). Dynamic Programming and Optimal Control, Vols. I and II. Athena Scientific, Belmont, Massachusetts.

Boyce, D.E., Mattsson, L.-G. (1999). Modeling residential location choice in relation to housing location and road tolls on congested urban highway networks. Transportation Research Part B, 33, 581-591.

Bravo, M., Briceño, L., Cominetti, R., Cortés, C.E., and Martínez, F.J. (2009). An integrated behavioral model of the land-use and transport systems with network congestion and location externalities. Transportation Research Part B.

Briceño L., Cominetti, R., Cristián E.C., and Martínez, F.J. (2008). An integrated behavioral model of land use and transport system: a hyper-network equilibrium approach. Networks and Spatial Economics, 8, 201-224.

Chang, J.S. and Mackett R.L. (2006). A bi-level model of the relationship between transport and residential location. Transportation Research Part B, 40, 123-146.

Cheshire, P. and Sheppard, S. (1995). On the price of land and the value of amenities. Economica, 62(246), 247-267.

Eliasson, J. and Mattsson, L.-G. (2001). Transport and location effects of road pricing: A simulation approach, Journal of Transport Economics and Policy, 35(3), 417-456.

Fischer, A. (1992). A special Newton-type optimization method. Optimization 24, 269-284.

Facchinei, F., Soares, J. (1995). Testing a new class of algorithms for nonlinear complementarity problems. In: Giannessi, F., Maugeri, A., eds., Variational Inequalities and Network Equilibrium Problems. Plenum Press, New York.

Jonsson, R.D. (2003). Sustainable urban development forecasting and appraisal. Licentiate Thesis, Royal Institute of Technology, Stockholm.

Jonsson, R.D. (2007). Planning Stockholm's land use and transport system using a new LUTI model: LandScapes. Paper presented at the NECTAR Conference, Porto, Portugal, May, 2007.

Lo, H. and Chen, A. (2000a). Traffic equilibrium problem with route-specific costs: formulation and algorithms. Transportation Research Part B, 34, 493-513.

Lo, H. and Chen, A. (2000b). Reformulating the traffic equilibrium problem via a smooth gap function. Mathematical and Computer Modelling, 31, 179-195.

Lo, H. and Szeto, W.Y. (2004). Planning transport network improvements over time. Urban and regional transportation modeling: Essays in honor of David Boyce, Chapter 9, 157-276, Edward Elgar.

Lo, H. and Szeto, W.Y. (2009). Time-dependent transport network design under cost-recovery. Transportation Research Part B, 43, 142-158.

Ma, X.S. and Lo, H. (2009). Impact of time-dependent transport supply and demand management on activity location costs. Transportmetrica, Submitted.

Mackett, R. (1991). A model-based analysis of transport and land-use policies for Tokyo. Transport Reviews, 11, 1-18.

Martínez, F.J. (1992). The bid-choice land-use model: an integrated economic framework. Environment and Planning A, 24, 871-885.

Martínez, F.J. (1995). Access: the transport-land use economic link. Transportation Research Part B, 39, 457-470.

Martínez, F.J. and Araya C. (2000). Transport and land-use benefits under location externalities. Environment and Planning A 2000, 32, 1611-1624.

Martínez, F.J. and Henriquez, R. (2007). A random bidding and supply land use equilibrium model. Transportation Research Part B, 41, 632-651.

Mattsson, L.-G. (1983). Equivalence between welfare and entropy approaches to residential location. Regional Science and Urban Economics, 14, 147-173. North-Holland.

Mattsson, L.-G. (2007). Road pricing: consequences for traffic, congestion and location. In: Jensen-Butler, C., Sloth, B., Larsen, M.M., Madsen, B., Nielsen O.A., eds., Road Pricing, the Economy and the Environment. Springer-Verlag.

McFadden, D. (1978). Modeling the choice of residential location. In: A. Karlqvist *et al.*, eds., Sptatial Interaction Theory and Planning Models. North-Holland, Amsterdam.

Rosen, S. (1974). Hedonic prices and implicit markets: product differentiation in pure competition. Journal of Political Economy, 62, 34-55.

Siu, B. and Lo, H. (2009). Integrated network improvement and tolling schedule: mixed strategy versus pure demand management. In: Wafaa Saleh and Gerd Sammer, eds., Travel Demand Management and Road User Pricing: Success, Failure and Feasibility. Chapter 11, 185-214, Ashgate Publishing Ltd.

Small, A. and Rosen, S. (1981). Applied welfare economics with discrete choice models. Econometrica, 49, 105-130.

Szeto, W.Y. and Lo, H. (2005). Strategies for road network design over time: robustness under uncertainty. Transportmetrica, 1, 47-64.

Szeto, W.Y. and Lo, H. (2006). Transportation network improvement and tolling strategies: the issue of intergeneration equity. Transportation Research Part A, 40, 227-243.

Szeto, W.Y. and Lo, H. (2008). Time-dependent transport network improvement and tolling strategies. Transportation Research Part A, 42(2), 376-391.

Train, K.J. (2003). Discrete choice methods with simulation. Cambridge University Press.

Wang, D.Z.W. and Lo, H. (2010). Global Optimum of the Linearized Network Design Problem with Equilibrium Flows. Transportation Research Part B, 44, 482-492.

Wegener, M. (1994). Operational urban models: State of the art. Journal of the American Planning Association, 60(1), 17-29.

Wegener, M. (1998). Models of urban land use, transport and environment. In: L. Lundqvist, L.-G. Mattsson, T.J. Kim, eds., Network Infrastructure and the Urban Environment, Advances in Spatial Science. Springer-Verlag Berlin Heidelberg.

Wilson, A.G. (1967). A statistical theory of spatial distribution models. Transportation Research, 1, 253-269.

Ukkusuri, S., and Patil G. (2009). Multi-period transportation network design under demand uncertainty. Transportation Research Part B, 43, 625-642.

Yang, H. and Meng, Q. (1998). An integrated network equilibrium model of urban location and travel choices. Journal of Regional Science, 38(4), 575-598.