LONG-TERM PLANNING OF A CONTAINER PORT TERMINAL UNDER DEMAND UNCERTAINTY AND ECONOMIES OF SCALE

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ABSTRACT

This paper deals with a container port capacity expansion problem in which the projected demand shows a relatively high level of uncertainty. An important characteristic of most capacity expansion problems is the recognition of economies of scale when considering infrastructure investments. In many problems, backlogging of demand is not permitted and since the demand tends to increase on the long run, excess capacity will be usually observed in certain periods of time. In our analysis, the objective is to minimize the net present value of investments on ship berths along the lifespan of the project, plus ship costs and berth operating costs. Ship waiting times are estimated with an appropriate queuing model. In the literature concerning container terminals, an Erlang queue $M / E_k / c$ is usually assumed, where c is the number of berths working in parallel. In the last years container ship size has been steadily increasing, and this trend is likely to continue in the future. With increasing costs, ship waiting times must be kept within reasonable limits. Ship waiting times are introduced in the model as constraints which are based on best maritime practices among leading container terminals of the world. Adopting a continuous time representation, the optimization model assumes stochastic demand evolution and employs a Dynamic Programming formulation in order to find the best epochs to install new berths. Finally, the model is applied to the container terminal of the port of Rio Grande, south of Brazil.

Keywords: port planning, capacity expansion, dynamic programming

1 INTRODUCTION

The optimization of logistics activities within maritime container terminals has attracted the attention of researchers and practitioners for more than forty years. Novaes and Frankel

(1966) analyzed the generation of unitized cargo (container, pallet, roll-on, roll-off) in a port and its relation to liner shipping, with the aid of a bulk queue model. In 1968, the World Bank issued a seminal working paper on the optimum determination of berths at a port (De Weille and Ray, 1968). Taborga (1969), in his pioneering work on port planning, applied dynamic programming and queuing models to define the optimal epochs to add new berths to the terminal, and at same time choosing the best port technology with the objective of maximizing seaport returns. The economic appraisal of port investments was analyzed by Goss (1967). Over the past recent years, the typical operating scheme of a container terminal has changed significantly. Larger container ships that call a modern port are often "classified ships" (Daganzo, 1990; Huang *et al*., 2007), in the sense that an appropriate berth slot is allocated to the vessel prior to its arrival. A non-classified ship, on the other hand, may have to wait until a berth of appropriate length becomes free to accommodate it. This means that the classical queuing models, that assume a first-in, first-out servicing discipline, are no long adequate to represent this kind of port operation. New empirical approximate queuing approaches have been proposed to partially bypass this kind of limitation (Jagerman *et al*., 2004; Huang *et al*., 2007; Morrison, 2007). In parallel, a number of researchers have relied on simulation techniques in order to plan resource utilization at a container terminal (Legato and Mazza, 2001; Kia *et al*., 2002; Shabayek and Yeung, 2002), thus bypassing the queuing formulation drawback. But one serious limitation of the simulation approach in the context of developing countries is the lack of reliable data to be fed into the model. For instance, at the container terminal (Tecon) of the Port of Rio Grande, in southern Brazil, although classified ships do benefit from the *time slot* practice (reserved quay space), there is no reliable data that would permit a robust analysis of the effects of ship classification on waiting time and quay usage.

The use of containers for intercontinental maritime transport has dramatically increased around the world in the last decade. From about 150 million twenty feet equivalent units (TEU) in 1996, the world container turnover has reached 496 million TEU in 2008. A further increase is expected in the upcoming years. Although the flow of container in Brazil is approximately 1% of the world movement only, the participation of containers in the country's foreign trade has increased more than 11% a year in the period 1996-2008 (Rebelo *et al*., 2008).

Brazil is following a trajectory of economic growth. Part of this growth is fueled by international trade, which is expected to continue to increase in line with Brazilian economic expansion. In such a scenario, port container terminals play an important role and represent a key asset of Brazil's logistics system. In fact, the country has one of the longest coastlines in the world and the existence of container terminals in points of the Atlantic coast is an important advantage in international trade. However, in spite of this location advantage, the Brazilian port system suffers from some critical problems that contribute to high logistics costs throughout the economy. Among other limitations, the lack of harbor capacity, excessive ship waiting times, and inefficiencies in the port administration model are worth mentioning. Thus, improvements in port efficiency, particularly in container operations, would have a major impact on reducing logistic costs in the country.

Container transshipment to and from hub ports is the fastest growing segment of the container port market today (Baird, 2006). Reflecting the international trend, some port authorities in developing countries like Brazil are already considering this operating alternative with regard to future development of leading container terminals. But apparently, scale economies in South American liner shipping are not significant enough to justify the move, as compared to regions such as northern Europe and Asia.

These factors, among others, indicate that the use of robust operations research models in container port development analysis, and in the context of developing countries, is not an easy task. Forecasting future scenarios depends on a number of intervening factors, such as liner shipping strategies and operating schemes, regional port competition, government policies, and technology evolution (Daschkovska and Scholz-Reiter, 2009; Günther and Kim, 2005; Kim and Günther, 2007). As a consequence, the forecasted demand levels tend to show a relatively high degree of uncertainty. The inexistence of reliable operational data, on the other hand, precludes the use of more detailed methods of analysis such as simulation techniques, for example. And third, the inefficiencies in port administration are reflected in historical short-term solutions, without a long-term planning of port development.

The objective of this paper is to present a robust, yet simple approach to develop a long-term container terminal planning model that will only require the basic data available in most developing country ports. The first part, analyzed in Section 2, is a mathematical formulation which allows for the introduction of demand volatility in the capacity expansion model. Section 3 presents the queuing model developments that allow estimation of ship waiting times. Section 4 is dedicated to describe the container terminal of the Port of Rio Grande, Brazil, to analyze its operational data, and define the planning decision structure to be implemented into the optimization model. The structure of the model is analyzed in Section 5, including the description of a dynamic programming model used to minimize the net present value of berth and ship costs associated to the container terminal. The dynamic approach to solve transport and logistics problems is receiving increasing attention in the literature (Scholz-Reiter *et al*, 2009). Finally, in Section 6, the application of the model to the container terminal in question is presented and the results are discussed.

2 CAPACITY EXPANSION WITH DEMAND VOLATILITY

2.1 The Cost Function

Capacity expansion is the process of adding facilities over time in order to satisfy rising demand (Manne, 1961). Capacity expansion decisions generally add up to a massive commitment of capital. The efficient investment of capital depends on making appropriate decisions in expansion undertakings, in such a way the demand remains satisfied over an extended time period, with a minimum discounted lifespan cost. The literature shows, along the last forty years, an evolving sequence of capacity expansion models (Manne, 1961, 1967; Freidenfelds, 1980, 1981; Higle and Corrado, 1992; Bean *et al*., 1992; Novaes and Souza, 2005). The basic way to economically minimize a project lifespan cost is to compute its Net Present Value (*NPV*), where investments and costs are discounted using a continuous interest rate r. The project will increase its own value by adopting the investment plan that minimizes *NPV*. The classical *NPV* calculation presupposes that unknown-risk future monetary values are summarized by their expected values. Assuming no backlogs in demand are admitted, the net present value is given by

$$NPV = I_0 + \sum_{i=1}^{n} I_i \exp(-r\tau_i) - SV_T \exp(-rT) + \int_{t=0}^{T} C(t)\exp(-rt)\,dt \tag{1}$$

where I_0 is the investment at time $t=0$, I_i is the value of the money inversion of order i $(1 \le i \le n)$, r is the annual continuous interest rate, τ_i is the time instant when the investment I_i is made, $C(t)$ is the system operating cost at time t, T is the duration of the project, and SV_T is the salvage value of the project at time T.

The cost functions associated with port operations reported in the literature vary slightly among authors (Huang *et al*., 2007). Usually these equations incorporate ship cost and berth operating cost in different forms. One widely used cost function in container terminal analysis is (Huang and Wu, 2005; Huang *et al*., 2007)

$$C(t) = U_s(t)\lambda(t)\left[W_q(t) + 1/\mu(t)\right] + U_b(t) \tag{2}$$

where $C(t)$ is the total operating cost at time t, $U_s(t)$ is the ship cost per unit time, $U_b(t)$ is the berth operating cost per unit time, $\lambda(t)$ is the mean ship arrival rate, $W_q(t)$ is the average ship waiting time, and $1/\mu(t)$ is the ship mean berth occupancy time (see Section 3.1 for more detailed explanation of variables $\lambda(t)$, $W_q(t)$ and $\mu(t)$). The term within brackets in the right-hand side of equation (2) represents the ship turn-around time expressed as the sum of the ship waiting time and the ship service time.

2.2 Economies of Scale

An important characteristic of most capacity problems is the recognition of economies of scale, i.e. large installations usually cost less per produced unit than small ones. But, if the demand level is continuously rising on the long run and demand backlogging is not permitted, excess capacity will occur (Figure 1). Thus, there is a trade-off between scale economies and excess capacity cost, leading to a compromised optimal solution.

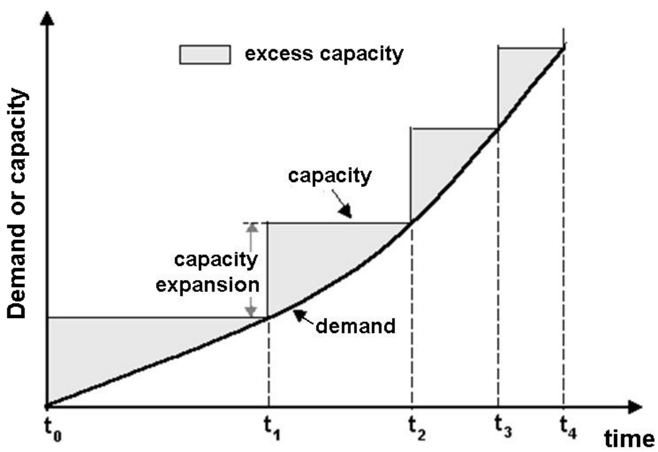

Figure 1 - The classical capacity expansion process

The term “learning curve” effect (Couto and Teixeira, 2005) states that the more time a task is performed, the less time it will be necessary to workers do each subsequent iteration. Learning curve theory states that as the quantity of units produced doubles, labor costs will decrease at a predictable rate. The experience curve, on the other hand, is broader in scope, since it encompasses far more than just labor cost. Now, each time cumulative volume doubles, value added costs (including construction, administration, logistics, etc.) fall by a constant and predictable percentage. Mathematically the experience curve is described by a power law function

$$I^{(m)} = I^{(1)} m^{-\theta} \quad (3)$$

where $I^{(1)}$ is the value of the building cost of the first unit, $I^{(m)}$ is the value of the investment on the m^{th} unit, and θ is the elasticity of building cost with regard to the number of units built in sequence. A experience curve that depicts a 20% cost reduction for every doubling of the number of built units is called a “80% experience curve”, indicating that unit cost drops to 80% of its original level when installing the second item of the series. Let β be the experience factor, obtained via (3) as

$$\beta = I^{(2)} \big/ I^{(1)} = 2^{-\theta} . \quad (4)$$

Applying logarithms to (4) one has

$$\theta = -\ln \beta \big/ \ln 2 \quad (5)$$

and the total investment related to the installation of m units in series is

$$I^{(m)} = I^{(1)} \sum_{j=1}^{m} j^{-\theta} . \quad (6)$$

Let m_i be the number of berths to be installed simultaneously at time τ_i, and $I_i^{(m_i)}$ the corresponding investment. To incorporate returns to scale in the analysis, we substitute variable $I_i^{(m_i)}$ for I_i in equation (1), thus adding the decision variable m_i into the model.

2.3 Capacity Expansion Model

2.3.1 The linear expansion model

Manne, in his seminal approach to capacity expansion problems (Manne, 1961) departed from a deterministic demand formulation growing at a constant linear rate. Under this approach it is assumed that the facility has an infinite economic life and, whenever demand catches up with the existing capacity, x additional unities of same capacity are installed. No backlog in demand is admitted. The objective is to determine the optimal capacity addition x, and the consequent time interval ΔT between increments. The time origin is the point t_0, followed by regeneration points at $t_0 + \Delta T$, $t_0 + 2\,\Delta T$, $t_0 + 3\,\Delta T$, that represent instants at which the previously existing excess capacity has just been wiped out. Note that when the system has reached $t_0 + \Delta T$, the future looks identical with the way it appeared at time t_0. The investment necessary to add a capacity increment of size x, is assumed to be given by a relationship in the form of a power function, with economies of scale

$$I(x) = kx^a, \qquad (k > 0;\ 0 < a < 1). \tag{7}$$

The demand, in this case, varies linearly over time and is expressed $d_t = \mu t$, with $\mu > 0$. The objective function to be minimized is of type (1). Let $NPV(x)$ be a continuous function of x that represents the sum of all discounted future investments looking forward from a point of regeneration t. We may write down the following recursive equation (Manne, 1961; Novaes and Souza, 2005)

$$NPV(x) = kx^a + e^{-rt} NPV(x). \tag{8}$$

The first term on the right hand side of (8) represents the investment cost incurred at the beginning of the current cycle. Since it is assumed that the facilities have an infinite economic life, the salvage value of the project in this case is nil. The second term represents the sum of all installation costs incurred in subsequent cycles, discounted from the next point of regeneration back to instant t_0, with a continuous interest rate r. It follows directly that

$$NPV(x) = \frac{kx^a}{1 - \exp(-rt)}. \tag{9}$$

But, since $t = x/\mu$, expression (9) becomes

$$NPV(x) = \frac{kx^a}{1 - \exp(-rx/\mu)}. \tag{10}$$

The optimal expansion size $\hat{x}$ is obtained by minimizing $NPV(x)$ with respect to x, and the optimal expansion time interval Δt is given by $\Delta t = \hat{x}/\mu$ (Manne, 1961,1967). Manne (1961) extended his deterministic model to a stochastic demand formulation, with an underlying linear growing rate μ and standard variation σ (Hull, 1997, Novaes and Souza, 2005). Such a model is based on Brownian motion theory, specifically the Bachelier-Wiener diffusion process in continuous time. One is interested in determining the instant t where demand first exceeds the installed capacity. At that time additional facilities will be supplied. In stochastic process terminology, one seeks the first passage time at point x, with probability $u(t,x)$. The following difference equation applies to the problem (Manne, 1961; Novaes and Souza, 2005)

$$u(t + \Delta t, x) = p\,u(t, x - \Delta x) + q\,u(t, x + \Delta x) \tag{11}$$

where p is the probability that one incremental step Δx is directed to increasing x, and $q = 1 - p$ is the probability that the step Δx is directed to the opposing direction. Expanding expression (11) according to Taylor's theorem up to terms of second order, and dividing both sides by Δt, one gets

$$\frac{\partial u(t,x)}{\partial t} = (q - p)\frac{\Delta x}{\Delta t}\frac{\partial u(t,x)}{\partial x} + \frac{(\Delta x)^2}{2\Delta t}\frac{\partial^2 u(t,x)}{\partial x^2}. \tag{12}$$

The Laplace transform of $u(t,x)$ is

$$\bar{U}(r,x) = \int_{t=0}^{\infty} u(t,x)e^{-rt}dt \,, \tag{13}$$

where r, in this formulation, is both the transformer parameter and the discount interest rate (Manne, 1961). Applying the Laplace transform to each side of (12), and recalling the boundary condition $u(0,x)=0$, a second-order linear differential equation with respect to x is obtained (Manne, 1961)

$$\frac{\partial^2 \bar{U}(r,x)}{\partial x^2} - \frac{2\mu}{\sigma^2}\frac{\partial \bar{U}(r,x)}{\partial x} - \frac{2r}{\sigma^2}\bar{U}(r,x) = 0 \,. \tag{14}$$

The characteristic equation of $\bar{U}(r,x)$ has two real roots

$$\lambda_1 = \frac{\mu}{\sigma^2}\left[1+\sqrt{1+\frac{2r\sigma^2}{\mu^2}}\right] \text{ and } \lambda_2 = \frac{\mu}{\sigma^2}\left[1-\sqrt{1+\frac{2r\sigma^2}{\mu^2}}\right] \tag{15}$$

The general solution of the differential equation (14) is of the form

$$\bar{U}(r,x) = A(r)e^{\lambda_1 x} + B(r)e^{\lambda_2 x} \tag{16}$$

where $A(r)$ and $B(r)$ are constants whose values depend upon boundary conditions of the problem (Manne, 1961). After some transformations and simplifications, one gets

$$\bar{U}(r,x) = e^{\lambda_2 x} \,. \tag{17}$$

Note that λ_2 is a function of only μ, σ, and r being independent of the quantity x. Since $u(t,x)dt$ represents the probability with which exactly t years have elapsed between two successive points of regeneration (points between which the total demand grows by an amount x), the probability analogue of (8) is (Manne, 1961)

$$NPV(x) = kx^a + \int_{t=0}^{\infty} u(t,x)e^{-rt} NPV(x)\,dt \,. \tag{18}$$

Just as in the deterministic case, the first term on the right hand side of (18) is the present cost of installing a facility of capacity x. The second term of (18) is the probability that the next point of regeneration will occur in t units of time, discounting the corresponding cost back to the present, and integrating over all t. Departing from (18), and recalling the basic definition of Laplace transform (13), one has (Manne, 1961)

$$\frac{NPV(x)}{k} = \frac{x^a}{1-\bar{U}(r,x)} \,. \tag{19}$$

Substituting (17) for $\bar{U}(r,x)$ into (19), one has

$$NPV(x) = \frac{kx^a}{1-\exp(\lambda_2 x)} \,. \tag{20}$$

From (15) it follows that $\lambda_2 < 0$ for $\mu > 0$ and $r > 0$. Comparing (10) and (20), it is seen that the following equivalence holds

$$r^* = -\mu\lambda_2 = \left(\frac{\mu}{\sigma}\right)^2 \left[\sqrt{1+2r\left(\frac{\sigma}{\mu}\right)} - 1\right], \tag{21}$$

meaning the linear stochastic model can be solved through an equivalent deterministic model in which the original discounting rate r is substituted by the modified rate r^* (Srinivasan, 1967; Freidenfelds, 1980; Bean *et al*., 1992; Higle and Corrado,1992). Manne (1961) indicated that, in the deterministic model, the optimal expansion size $\hat{x}$ increases as the discount rate r is lowered. For the probabilistic model, Manne (1961) showed that the greater the standard variation σ, the lower will be the value of r^*, and the higher will be the optimal value $\hat{x}$.

2.3.2 Generalization to non-linear demand case

Assume now that $d(t)$, the demand for product or service, is a semi-Markov process $\{d(t),\ t \geq 0\}$ (Bean *et al*., 1992). One is interested in selecting a sequence of capacity expansions to satisfy the demand $d(t)$ over an infinite horizon at minimum expected discounted cost using a continuous interest rate r. A continuous semi-Markov process $\{d(t),\ t \geq 0\}$ is said to be *transformed Brownian motion* with underlying rate μ and standard variation σ if there exists a non-negative increasing deterministic transformation h such that $d(t) = h(d(t))$, where $\{d(t),\ t \geq 0\}$ is (linear) Brownian motion with drift $\mu > 0$ and volatility $\sigma > 0$. The function h is referred to as the transforming function. Bean *et al*. (1992) demonstrated that if $\{h(d(t)),\ t \geq 0\}$ is a transformed Brownian motion as defined above, the interest rate r^* to be used in the equivalent deterministic problem is given by the same expression (21) obtained by Manne for demand following an ordinary (linear) Brownian motion. In other words, r^* is independent of the transformation h. In more general terms, Bean *et al*. (1992) demonstrated that a stochastic capacity expansion problem may be solved via a deterministic problem formulation in which (a) the random expansion epochs $\tau(x)$ are replaced by their expected values; and (b) the original continuous discount factor r is replaced by its equivalent value r^* given by (21). In general terms, capacity expansion models with general cost structures and with demand based on a transformed Brownian motion can be solved with an equivalent deterministic problem and an equivalent discount factor (Bean *et al*., 1992).

2.3.3 First passage time

The Laplace transform of the probability distribution $u(t,x)$, given by equation (17), allows to compute the expected value and the variance of the first passage time to the level x. One has (Novaes and Souza, 2005)

$$E[\tau] = -\left.\frac{\partial \bar{U}(r,x)}{\partial r}\right|_{r=0} = \frac{x}{\mu} \quad \text{and} \tag{22}$$

$$\mathrm{var}\left[\tau\right] = -\left.\frac{\partial^2 \bar{U}\left(r,x\right)}{\partial r^2}\right|_{r=0} - \left\{E\left[\tau\right]\right\}^2 = \frac{x\sigma^2}{\mu^3} = \left(\frac{\sigma}{\mu}\right)^2 E\left[\tau\right]. \tag{23}$$

For the general case with $d\left(t\right) = h\left(d\left(t\right)\right)$ one has (Novaes and Souza, 2005)

$$E\left[\tau\right] = \frac{h\left(x\right)}{\mu} \quad \text{and} \quad \mathrm{var}\left[\tau\right] = \left(\frac{\sigma}{\mu}\right)^2 E\left[\tau\right]. \tag{24}$$

3 QUEUING MODELS APPLIES TO CONTAINER PORT PLANNING

3.1 The Mathematical Approach

Researchers and practitioners have applied queuing models to port planning for more than forty years (Novaes and Frankel, 1966; De Weille and Ray, 1968; Taborga, 1969; Noritake, 1978; Noritake and Kimura, 1983; Huang *et al*., 2007). The first papers dealt with general cargo operations, with no product specialization, for which the queue $M/M/c$, in Kendall's notation (Page, 1972), was usually employed. For this type of queue, ship arrivals are Poisson distributed, the service time is represented by an exponential distribution, there are c service units (berths) working in parallel, and the queue discipline is *first-come, first-served*. Novaes and Frankel (1996) utilized bulk queues in their research work, but the objective of the analysis was not to compute ship waiting times, but the cargo generating process at the port. Along the years, the handling of cargo at ports became more and more specialized. As a consequence, the queue models that best fit the ship waiting time process have changed, with the Erlang $E_{k_1}/E_{k_2}/c$ and the $M/E_k/c$ types being the most commonly used in port planning (Huang *et al*., 1995; Kozan, 1997; Huang *et al*., 2007).

Let λ be the mean ship arrival rate at the terminal and μ the mean service rate. The average berth utilization factor ρ is given by $\rho = \lambda/\mu c$ (Saaty, 1961; Page, 1972) and it must be less than the unit to let the queue attain a steady state solution. For the queue $M/M/c$, the mean waiting time can be expressed mathematically in an explicit form. It is customary to represent the mean waiting time as a dimensionless figure, expressing it as a fraction of the mean service time. For the queue $M/M/c$ one has (Saaty, 1961; Page, 1972)

$$q = \rho/\left(1-\rho\right) \quad \text{for} \quad c = 1 \text{ and} \tag{25}$$

$$q = \pi_0 \left(\rho c\right)^c / \left(1-\rho\right)^2 c!c \quad \text{for} \quad c > 1 \tag{26}$$

where q is the mean waiting time expressed as a fraction of the mean service time W_q, and π_0 is the probability of having zero elements in the waiting line, given by

$$\pi_0 = \frac{1}{\sum_{j=0}^{c-1} \frac{\left(c\rho\right)^j}{j!} + \frac{\left(c\rho\right)^c}{c!\left(1-\rho\right)}}. \tag{27}$$

Practical results for the queuing system with general independent interarrival times, a general service-time distribution, and c servicing units in parallel, denoted $GI/G/c$ in Kendall's notation, cannot be expressed mathematically in explicit form (Cosmetatos, 1976). However, some particular configurations of the queue $GI/G/c$, have been solved mathematically. For instance, the queue $M/G/1$ can be solved with the Pollaczek-Khintchine equation (Saaty, 1961)

$$W_q = \frac{E[t_s]}{2}\frac{\rho}{1-\rho}\left[1-v_s^2\right] \tag{28}$$

where W_q is the average waiting time, $E[t_s]$ is the average service time, and v_s is the coefficient of variation of the service time distribution function, given by $v_s = \sigma_{ts}/E[t_s]$.

The mathematical solution for the queue $M/D/c$, with Poisson input and constant service times is analyzed by Saaty (1961) and Page (1972), but the process, based on an iterative numerical computation, generates rounding errors that will build up in the calculation of the steady state probabilities $P(j),\ j=0,1,2,\cdots$ of the number of units in the queue (Page, 1972). Thus, the values obtained with the direct mathematical approach are used as initial values for an iterative procedure intended to improve the quality of the results. By the same token, the queue $D/M/c$, with constant interarrival times and negative exponential service time distribution, can be also solved mathematically in a similar way, yielding the steady state probabilities $P(j),\ j=0,1,2,\cdots$ which are computed at points in time where customers arrive (Page, 1972). On the other hand, the queue $D/D/c$, with constant interarrival time and constant service time, is a trivial case where no queuing occurs, unless the service system is overloaded (Page, 1972).

A particular case of the series $GI/G/c$ is based on the Erlang distribution, whose probability density function is

$$f(x) = \lambda^k x^{k-1} e^{-\lambda x}/(k-1)!, \quad (t \geq 0,\ k=1,2,\cdots,\infty) \tag{29}$$

where x is the random variable under analysis, λ is the mean arrival rate, and k is the order of the distribution. The Erlang distribution can be viewed as being composed by k phases, each phase formed by a negative exponential distribution with the same average length $1/\lambda$. The value of k must be an integer, and for $k=1$ the distribution repeats the negative exponential distribution. The mean and the standard variation for the Erlang distribution (29) are $E[x]=k/\lambda$ and $\sigma_x = \sqrt{k}/\lambda$, respectively.

In general, the Erlangian queue is represented by $E_{k_1}/E_{k_2}/c$, where the interarrival times and service times are distributed according to Erlang distributions of order k_1 and k_2, respectively. Page (1972) proposed an interpolation formula to get the waiting times for the queue $E_{k_1}/E_{k_2}/c$, based on the combination of the extreme cases $M/M/c$, $D/M/c$, $M/D/c$ and $D/D/c$. This interpolation approach will be presented in Section 3.2. More complex queues can also be treated mathematically. The reader is referred to Saaty (1961) and Takács (1962).

3.2 Approximate Methods

For more complex applications, involving $GI/G/c$ queues with general independent interarrival times and a general distribution of service time, it is customary to use numerical tables or approximate formulas to estimate mean waiting times. Page (1972) proposed an interpolation formula using the numerical results of the four extreme cases represented by the queue cases $M/M/c$, $D/M/c$, $M/D/c$ and $D/D/c$. As explained, the resulting queue length for the $D/D/c$ case is nil. Expressing the mean waiting time as a fraction of the mean service time, Page's interpolation formula is

$$q^{(E_{k_1}/E_{k_2}/c)} \cong \left(1-v_a^2\right)v_s^2 w_q^{(D/M/c)} + \left(1-v_s^2\right)v_a^2 w_q^{(M/D/c)} + v_a^2 v_s^2 w_q^{(M/M/c)} \tag{30}$$

where $q^{(E_{k_1}/E_{k_2}/c)}$ is the relative waiting time for a $E_{k_1}/E_{k_2}/c$ queue, and $q^{(D/M/c)}$, $q^{(M/D/c)}$ and $q^{(M/M/c)}$ are the relative waiting times for the queues $D/M/c$, $M/D/c$ and $M/M/c$, respectively. In (30), v_a and v_s are coefficients of variation of the interarrival time and of the service time distribution functions respectively, given by

$$v_a = \sigma_{ta} \big/ E\left[t_a\right] \quad \text{and} \quad v_s = \sigma_{ts} \big/ E\left[t_s\right] \tag{31}$$

where t_a and t_s are the ship interarrival time and the service time, respectively. For the Erlang queue $E_{k_1}/E_{k_2}/c$, one has $v_a = 1\big/\sqrt{k_1}$ and $v_s = 1\big/\sqrt{k_2}$. Page (1972) presented numerical results in table format, which requires manual interpolation for intermediate values of ρ. Furthermore, Page's tables are restricted to $c \le 10$.

Another approximation, valid for the queue $M/E_k/c$, is (Cosmetatos, 1976; Huang *et al.*, 2007)

$$W_q = \frac{\pi_0 \rho \left(\lambda/\mu\right)^c}{\lambda c!\left(1-\rho\right)^2}\left\{\frac{1+k}{2k} + \left(1-\frac{1}{k}\right)\left(1-\rho\right)\left(c-1\right)\frac{\sqrt{4+5c}-2}{32\rho c}\right\}, \tag{32}$$

where π_0 is given by (27). Recalling that $\rho = \left(\lambda/\mu c\right)$, $E\left[t_s\right] = 1/t_s$ and $q = W_q \big/ E\left[t_s\right]$, substituting in (32), and making the necessary simplifications, one has

$$q = \frac{\pi_0 \left(\rho c\right)^c}{c c!\left(1-\rho\right)^2}\left\{\frac{1+k}{2k} + \left(1-\frac{1}{k}\right)\left(1-\rho\right)\left(c-1\right)\frac{\sqrt{4+5c}-2}{32\rho c}\right\}. \tag{33}$$

Since most of the main container terminals in the world show a $M/E_k/c$ queuing system, the above formula is extensively used in their planning process. Again, q in (33) expresses the ship waiting time as a fraction of the mean service time.

Another approximate formula (Sakasegawa, 1976; Morrison, 2007), which is applied to the general queue $GI/G/c$, is defined as

$$q \cong \left(\frac{v_a^2 + v_s^2}{2}\right)\left(\frac{\rho^{-1+\sqrt{2c+2}}}{c\left(1-\rho\right)}\right). \tag{34}$$

A numerical comparison was made among the approximations due to Cosmetatos (1976) and Sakasegawa (1976), for the queue $M / E_3 / c$, which is largely used in container terminal planning. For c in the range 1 to 8, the results of the two methods agreed quite well. On the other hand, Page tables are limited to $c = 10$, which, together with the table format of the results, restrict the applicability of his methodology. The literature shows that the queue $M / E_k / c$, adopted by Cosmetatos (1976), fits most container port cases. Thus, Cosmetatos formula (33) was selected to be used in the application described in Section 4.

4 CONTAINER TERMINAL, PORT OF RIO GRANDE, BRAZIL

4.1 The *Tecon*

Brazil follows a trajectory of higher economic growth, with rates that are expected to surpass 5% per year. Part of the economic growth is fueled by international trade, which is expected to continue to grow in line with Brazilian economic expansion. Brazil's exports by value and by volume have grown significantly during the last thirty years. More importantly, the data indicates that export value has increased more rapidly than export volume. For example, the index of manufactured goods by value, increased almost 23-fold over the period examined. The volume of these exports, however, increased only about 12-fold over the same period of time. Overall, the value of total exports grew 50% more than their volume. Thus, the opportunity cost of marginal time spent in logistics activities has become higher. In this context, time, reliability, just-in-time delivery, and security are critical considerations now. Therefore, the ability to trade with other countries and to grow competitively will be increasingly intertwined with the country logistics system (Rebelo *et al*., 2008).

In this context, ports are a key asset in Brazil's logistics context, serving the entire coastline, one of the longest in the world. However, in spite of the locational advantage, the port system suffers from several critical problems that impair its development and contribute to high logistics costs throughout the economy. These include equipment obsolescence, inefficiencies in labor development and labor allocation, lack of harbor capacity, and inefficiencies in the port administration model. Waiting times for berthing are too high and yard space is insufficient in several ports (Rebelo *et al*., 2008). But it is well known that serving large container ships requires the right treatment of several terminal problems, including ensuring adequate depth berths, wider and deeper channels, suitable high-speed cargo-handling equipment, a highly productive labor force, and good road and rail connections to inland destinations (Ircha, 2001).

The container terminal of the Port of Rio Grande - *Tecon* - is located in the southern part of Brazil, and is operated on a 24-hour/day basis by a private company, Wilson and Sons, since 1997, under a 25-year concession period, renewable for an equal time, and granted by the Brazilian Federal Government. The main products exported through the *Tecon* are tobacco, thermoplastic resins, shoes, frozen poultry, furniture, leather, and auto-parts. Presently, the terminal has three berths, with a total extension of 850.0 m, and 12.5 m depth. The quay has four ship-to-shore gantry cranes Superpost Panamax, and three Gottwald HMK cranes. The yard covers a paved area of 250,000 sq.meter, with an additional area of 486,000 sq.meter for future expansions. Container ship waiting times are greater than the ones observed in other international leading ports of similar capacity. For instance, 33% of container ships that arrived in Rio Grande during 2008, waited more than 12 hours, and 14% waited more than 24 hours. These figures are similar to the results shown by the Port of Santos, the largest in the country, where 35% of ships waited more than 12 hours and more than half of these waited more than 24 hours in the same time period (Rebelo *et al*., 2008). In 2008, the *Tecon* operated 356,575 containers, equivalent to 598,196 TEUs.

4.2 Data Analysis

Table 1 shows the distribution of containers (both load and unload), and expressed in TEU), among the various ship capacity classes, at the *Tecon* in 2008. About 45% of the container flow was performed by ships in the capacity range 1500-3000 TEU. On the other hand, about 37% of the movement was done by Post-Panamax ships in the capacity range 4000-5500 TEU. In 2008, the average number of containers loaded + unloaded per ship call was approximately 695 TEU. It can observed in Table 1 that the volume of containers operated per ship call, in absolute terms, and considering both loading and unloading, tends to increase with ship size (column *d*). However, when one analyzes the ratio between the number of TEU operated per ship call, and the corresponding ship capacity, this index drops expressively with ship size. For instance, for ships in the range 5000-5500 TEU, the number of containers operated per ship call is only 21% of the ship capacity, while for vessels in the 500-1000 TEU range, the ratio is more than double that figure.

Another important index, considered when analyzing terminal productivity, is the average number of containers loaded/unloaded per ship-hour. Let δ represent the fraction of forty-foot containers on the total number of boxes. The berth productivity index, expressed as a number of containers transferred per hour, per ship (load/unload), can be estimated as

$$pr = \frac{n_{TEU}}{(1+\delta)} \times \frac{1}{E[t_s]} \quad (35)$$

where n_{TEU} is the mean number of TEUs transferred per ship call (load/unload) and $E[t_s]$ is the average berth occupancy time. The variable δ did not vary significantly in the period 2001-2009, with a trend toward $\delta \cong 0.70$, value that has been adopted in the application.

On the other hand, $E[t_s]$ can be estimated as a function of n_{TEU}, as depicted in Figure 2, and expressed approximately as

$$E[t_s] \approx 0.897 n_{TEU}^{0.417}. \quad (36)$$

It can been seen that there is a decreasing return to scale in terms of the time necessary to load/unload a ship, as a function of n_{TEU}.

Table 1 - Port of Rio Grande *Tecon*: Container movement per ship capacity class

(a) Ship capacity (*TEU*s)	(b) Number of *TEU*s (load/unload) (2008)	(c) % in each class	(d) Average nº of *TEU*s operated per ship call	(e) Ratio between # of *TEU*s per ship call, and ship capacity (%)
0-500	10,144	1.7	344	73
500-1000	4,524	0.8	342	46
1000-1500	36,070	6.0	473	34
1500-2000	107,043	17.9	572	33
2000-2500	68,996	11.5	659	28
2500-3000	92,785	15.5	630	23
3000-3500	15,298	2.6	655	20
3500-4000	31,518	5.3	705	19
4000-4500	86,853	14.5	960	22
4500-5000	25,121	4.2	797	17
5000-5500	108,821	18.2	1,093	21
5500-6000	4,700	0.8	1,156	20
6000-6500	6,323	1.1	889	14
TOTAL	598,196	100.0	695 (Average)	

As a general rule, the container ships travelling southbound in the Atlantic East Coast of South America stop in Santos and proceed directly to Buenos Aires, Argentina. In the way back, they stop in Montevideo (Uruguay) and, next, in Rio Grande. Thus, the vessels arrive at *Tecon* with a light load, generally carrying only the containers embarked in Buenos Aires and Montevideo, plus the units to be discharged at the *Tecon*. One important characteristic of the Rio Grande's *Tecon* operation observed in the last years is that the growth of demand is satisfied mostly by increasing the number of ship calls, instead of augmenting the number of containers per stop. As a result, the *Tecon* depth of 12.5 m is not a restriction today and probably not in the near future too, but it could be a severe constraint if the shipping lines substantially change their operating strategy such as, for instance, the establishment of a hub pole in Rio Grande (SCP, 2005).

Figure 3 shows the monthly evolution of the *Tecon* throughput from March 2001 to June 2009. From March 2001 to December 2003 the throughput (expressed in TEU) grew at an average rate of 22% a year. Due to the recent international crisis, the *Tecon* throughput has been increasing at a lower rate, but the trend has been changing during the first part of 2010.

The volatility of the monthly series of throughput, from March 2001 to June 2009, is now computed. To do this (Hull, 1997) we convert these absolute monthly values $D(t),\ t=0,1,2,\cdots$ into relative returns $u(t)=D(t)/D(t-1),\ t=0,1,2,\cdots$, and then taking the natural logarithms, thus forming a series

$$u(t)=\ln\left[D(t)\right]-\ln\left[D(t-1)\right], \qquad t=0,1,2,\cdots. \tag{37}$$

The standard deviation of $u(t)$ in the series is $\sigma_m = 0.176$. Converting it to annual basis, one gets the demand volatility $CV=\sigma_m\sqrt{12}=0.61$ which will be used in our analysis.

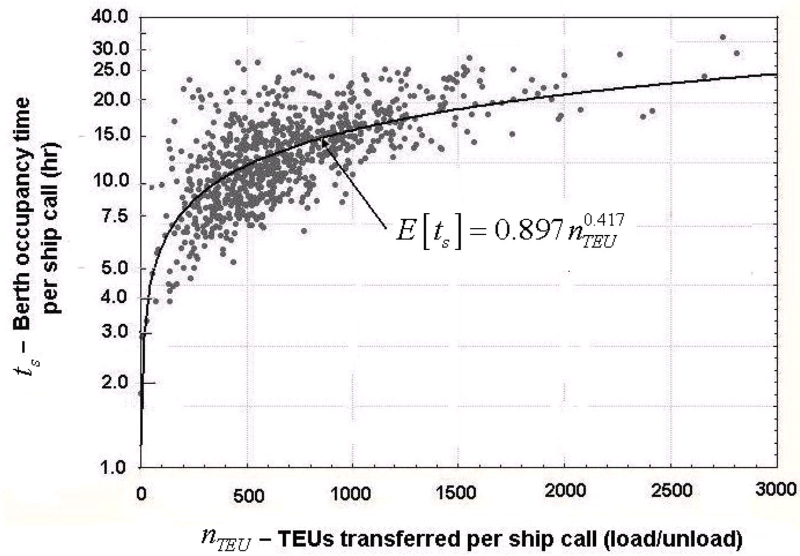

Figure 2 - Relationship between $E\left[t_s\right]$ and n_{TEU}

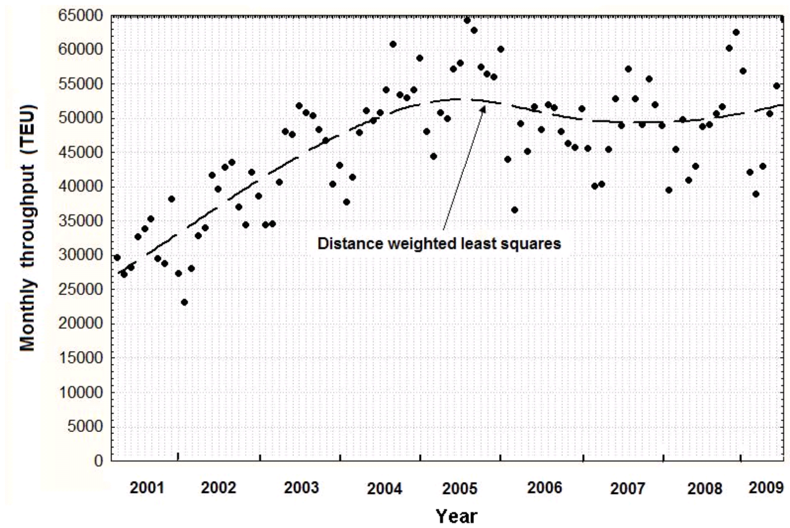

Figure 3 - Rio Grande's *Tecon*: Monthly throughput 2001-2009 (*TEU*)

Ship interarrival times at the *Tecon* are considered next. The analysis of 846 container ship arrivals in 2008 led to a mean interarrival time of 10.3 hours. Figure 4 shows the fitting result of a negative exponential distribution to the data. Ship service time, on the other hand, comprises the period of time the vessel effectively occupies a berth. Analyzing the same 2008 ship sample, the mean service time was 13.7 hours. An Erlang distribution with $k = 6$ was fitted to the data, as shown in Figure 5. Although similar studies reported in the literature have indicated Erlang distributions of order 3 (Huang *et al.*, 1995, 2007), the *Tecon* result can be justified by the shorter spread of the number of containers loaded/unloaded per ship call, as compared to other container ports around the world.

In the year 2008, the mean ship waiting time, computed for the sample of 846 container ship arrivals, was 12.3 hours. Cosmetatos (1976) equation (33) is applied next to the observed 2008 data. The number of berths was $c = 3$, the average berth utilization factor was $\rho = 0.45$, and $k = 6$. Applying (27) one gets $\pi_0 = 0.250$ and equation (33) yields $q = 0.071$. Thus, the expected ship waiting time is $T_q = q \times E\left[t_s\right] = 0.071 \times 13.7 = 0.97$ hours. This means that ship waiting time at the *Tecon* was twelve times the standard figure in 2008. This result is perhaps questionable since the third berth was put into service only in October 2008, but a report from the World Bank confirms the excessive ship waiting time at the *Tecon* (Rebelo *et al*., 2008).

4.3 Planning Decisions

The objective of the model is to minimize the net present value of the sum of investments, ship cost and berth operating cost that occur along the project lifetime, minus the net present value of the project salvage value, all represented by (1). To do this, the variables of the optimization model are: (a) the instant $\tau_i \left(i = 1, 2, \cdots, n\right)$ when the investment I_i in berths is made, and (b) the number m_i of berths to be installed simultaneously at time τ_i.

In addition to the objective function (1) to be minimized, one constraint limiting ship waiting time is added to the model. Let $\overline{W}_q\left(t\right)$ be the maximum tolerable ship waiting time at instant t. Thus, the estimated ship waiting time at time t must respect the constraint

$$W_q\left(t\right) \le \overline{W}_q . \tag{38}$$

Figure 4 - Ship interarrival time distribution

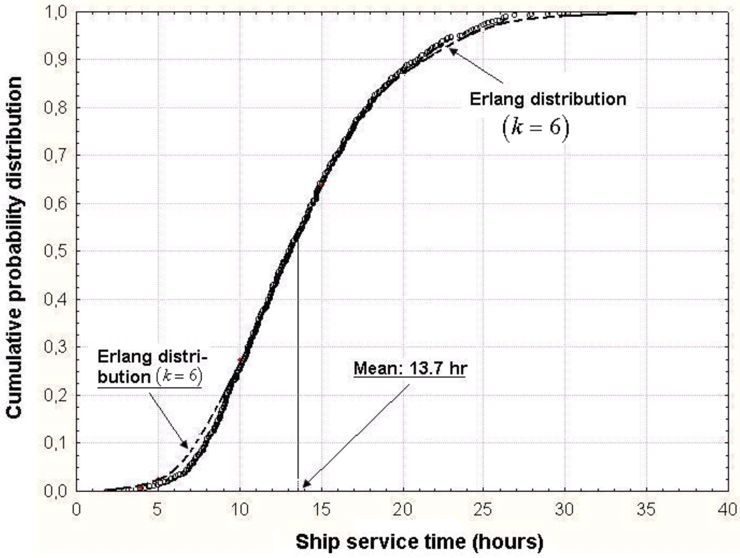

Figure 5 - Ship service time distribution

On the other hand, the berth loading/unloading productivity index is represented by the mean number of containers that are loaded/unloaded per ship, per hour. Considering a project lifetime of $T = 25$ years, we divided it into five-year periods, as indicated in Table 2, column (a). Column (b), of Table 2, shows the maximum ship expected capacity to be observed in each period, varying from 6000 TEU to 14000 TEU. Column (c) shows the required berth length to receive ships with the capacities listed in column (b). Column (d) indicates the mean ship capacity in each epoch. Column (e) shows the expected number n_{TEU} of TEUs operated per ship call in each time horizon. The mean berth occupancy time $E\left[t_s\right]$ is exhibited in column (f). Finally, the berth productivity index, represented by the mean number of containers loaded/unloaded per ship-hour, are shown in column (g). These figures are the expected coefficients to prevail during the lifetime of the project.

The growth of the variable n_{TEU} at the *Tecon* was admitted to follow the polynomial expression

$$n_{TEU}\left(t\right) = 695 + 19.11t + 1.0036t^2 \quad (39)$$

with t expressed in years. The values of $n_{TEU}\left(t\right)$ computed via (39) for the time horizons of the project are exhibited in column (e), of Table 2. For each t, the value of $n_{TEU}\left(t\right)$ is computed. Then, with (36), the variable $E\left[t_s\right]$ is estimated. Finally, the values of $\overline{pr}\left(t\right)$ are computed using expression (35).

Column (g), of Table 2, depicts satisfactory berth productivity values to be respected along the lifetime of the project. In fact, the productivity of *Tecon* in 2003 was 21 containers/ship/hour (SCP, 2005), which has improved substantially during the last years, reaching about 30 containers/ship/hour in 2008. Thus, the productivity levels indicated in Table 2 represent, in fact, feasible targets, and were adopted in the optimization model.

Table 2 - Project berth productivity per time period

(a) Time horizon (years)	(b) Max. expected ship capacity (*TEU*)	(c) Berth length (m)	(d) Mean ship capacity (*TEU*)	(e) n_{TEU}	(f) $E[t_s]$ (hours)	(g) $\overline{pr}(t)$ Berth productivity $(*)$
0	5500	280	3250	695	13.7	30
5	6000	300	3500	816	16.4	30
10	8000	340	4700	987	17.7	33
15	10000	360	6000	1207	19.3	37
20	12000	380	7100	1479	21.0	42
25	14000	480	8300	1800	22.8	47

$(*)$ Containers/ship/hour

5 THE PLANNING MODEL

5.1 The Equivalent Discount Rate

First, the project basic discount rate is computed based on the *Capital Asset Pricing Model* (*CAPM*) developed by Sharpe and Lintner forty years ago (Campbell *et al*., 1997). The *CAPM* was developed, at least in part, to explain the differences in risk premium across assets. Let $r_f^{'}$ be the discrete-time, annual rate of return on the risk-free asset. Consider, next, a portfolio involving all types of assets traded in the economy. Let $r_M^{'}$ be the average rate of return of such a portfolio. The risky assets contemplated by the investor have returns that are not known with certainty at the time the investments are made. The *CAPM* asserts that the correct measure of risk of the asset is a coefficient called *beta*. The basic equation is

$$r^{'} = r_f^{'} + \left(r_M^{'} - r_f^{'}\right)\beta \tag{40}$$

where $r^{'}$ is the expected rate of return, $r_f^{'}$ is the risk-free rate of return, and β is a measure of the relative risk of the asset with reference to the whole portfolio (Campbell *et al*., 1997). The risk-free return is the interest rate offered by entities that are entirely creditworthy during the period of a loan, such as the rates paid by the US Treasury and by some European government bonds. In the *Tecon* application we have assumed $r^{'} = 0.036$ or 3.6% a year, $r_M^{'} = 0.10$ and $\beta = 1.2$, leading to $r^{'} = 0.1128$. The corresponding continuous-time discount rate is $r = \ln\left(1 + r^{'}\right)$.

The equivalent rate of return to be used in the model is given by (21). The demand at time zero is assumed to be $D_0 = 650,000$ TEU. At time $t = 0$ one has a discrete rate $\alpha^{'} = 0.12$. Then $\mu = \alpha^{'} \times D_0 = 78,000$ TEU and $\sigma = CV \times \mu = 0.61 \times 78,000 = 47,580$ TEU. Applying (21) with $r = 0.1069$, one has $r^{*} = 0.1048$, value to be used in our analysis. As seen in Section 2.3.2, the impact of the stochastic behaviour of demand on the results is totally reflected by the discount rate reduction from r to r^* (Higle and Corrado, 1992; Bean *et al*., 1992).

5.2 Investments and Operating Costs

In 2008, a 250-meter berth was put to service at *Tecon*, with a cost of US$ 50 million. Assuming an additional of US$ 10 million to cover the acquisition of container cranes, and dividing the total investment by the berth length, one has a unit cost of US$ 240,000 per meter. Further expansion investments were assumed to be proportional to berth length, given in column (c), Table 2. Ship costs, on the other hand, will depend on vessel capacity. The daily cost of a container ship, in US dollars, is estimated as

$$U_s(t) = 7486.8 + 4758x - 82.1x^2 \tag{41}$$

where $U_s(t)$ is the ship cost, in US dollars per day, and x is the mean ship capacity, expressed in TEU, and divided by 1000. It is assumed, in the model, that the mean ship waiting time constraint **Erro! Fonte de referência não encontrada.** follows an exponentially decaying function, with $\overline{W}_q = 12$ hours at time $t = 0$, reaching $\overline{W}_q = 2$ hours at $t = 10$ years, and $\overline{W}_q = 1$ at $t = 14$ years, with $\overline{W}_q = 1$ afterwards. In fact, due to budget restrictions, inefficiencies in port administration, lack of coordination between the terminal and shipping lines and cargo forwarders, among other factors, it is expected that *Tecon* performance will not improve in too short a time.

With regard to berth operating cost, we assume it depends only on the demand level (throughput). In our approach, demand is an external variable, not depending on port costs and service level. Additionally, no demand backlog is admitted to occur, i.e. the port will provide adequate facilities before full congestion may occur. Consequently, the integral $\int_{t=0}^{T} U_b(t)\exp(-rt)\,dt$ in expression (1) is constant, and therefore does not affect the minimization of NPV.

It was admitted that a berth investment is linearly depreciated in 30 years, with nil value at the end of its lifespan. Suppose a berth is installed at time t, with an investment I_t. The salvage value of the berth at time T (when the project ends) is (see Figure 6)

$$v = I_t\left(\frac{T_B - T + t}{T_B}\right) \tag{42}$$

for $T_B - T + t > 0$, and $v = 0$ otherwise, where T_B is the berth depreciation time.

5.3 The Dynamic Programming Algorithm

The dynamic programming concept has been broadly used in the literature to solve sequential decision problems of all kinds. Specifically, it can be used to optimize decision processes over time, which is the case of the problem treated in this paper. Dynamic

Programming is, in essence, a general approach to optimization, rather than a mathematical algorithm as, for instance, the Simplex method used to solve linear programming problems. One essential feature of the dynamic programming approach is the reduction of a multivariate problem to a succession of single variable cases. At each stage of the optimization process the analysis of the resulting values of these single variables, with the appropriate decision rules, defines a *policy*. The procedures of dynamic programming are based on the *Principle of Optimality* due to Richard Bellman: an optimal policy has the property that, whatever the initial state and the initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision (Bellman and Dreyfus, 1962).

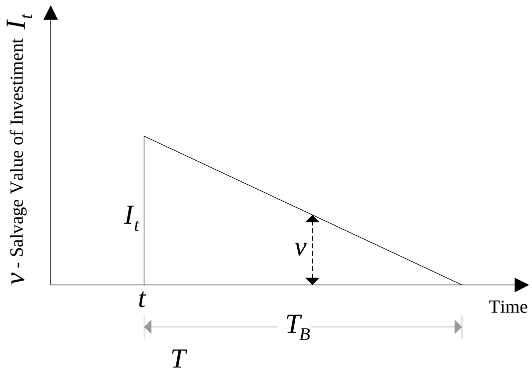

Figure 6 - Salvage value v of an investment I_t

In the dynamic programming algorithm developed for this application the *stages* represent the instants $t\ (0 \le t \le T)$ for which the model analyses the possibility of introducing additional berths. The *states* of the process, on the other hand, represent the number of berths that will be available at stage t, including the ones installed at previous stages plus the berths to be added at time t. At each stage of the process the minimum number of required berths M_{min} is externally imposed in order to guarantee $\rho(t) < 1$ and $W_q(t) \le \overline{W}_q(t)$. These restrictions help in reducing computational time, since the model investigates only feasible states. Thus, at each stage, the model first computes the minimum required number of berths. Let $c_e(t) > 0$ be the number of berths already in operation at the beginning of stage t, and let $m(t)$ e the quantity of berths to be added at that stage. The following procedure yields the value of M_{min}:

Step 0: Initialize $c_e(t)$ and make $m(t) \leftarrow 0$.

Step 1: Make $c(t) \leftarrow c_e(t) + m(t)$ and compute $\rho(t) = \lambda(t) / [\mu(t) c(t)]$ and $W_q(t) = q \cdot E[t_s]$.

Step 2: If $\rho(t) \ge 1$, or $W_q(t) > \overline{W}_q(t)$, then make $m(t) \leftarrow m(t) + 1$ and go back to Step 1.

Step 3: Return $M_{min} \leftarrow c(t)$.

Assuming the two operating restrictions are respected, the maximum number of berths to be considered in the analysis is imposed by external considerations, mainly budget constraints and physical expansion limitations. In the former procedure the average ship arrival rate is defined as $\lambda(t) = D(t)/(360 \cdot 24 \cdot n_{TEU})$ and the berth average servicing rate is $\mu(t) = 1/E[t_s]$, where n_{TEU} is given by (39) and $E[t_s]$ by (36). All states that satisfy $\rho(t) < 1$ and $W_q(t) \le \bar{W}_q(t)$ are feasible and therefore are candidates to be evaluated according to the objective function (1).

The dynamic programming transitions from the generic stage t_1 to the generic stage t_2 are depicted in Figure 7. According to this evolving structure, the problem is solved in backward steps adopting the following algorithm (Hastings, 1973):

Step 0: Initialize the values of $c_e(0)$, M_{max}, and other parameters of the problem.

Step 1: Assign value zero to the terminal states.

Step 2: Repeat Steps 3, 4, 5 e 6 for each instant t.

Step 3: Compute M_{min} and repeat Steps 4, 5 e 6 for each feasible state.

Step 4: Repeat Steps 5 e 6 for each capacity expansion decision $m(t)$, until the maximum level M_{max} is reached.

Step 5: Compute the objective function value for decision $m(t)$, at the present stage t.

Step 6: If the present decision is better than the previous one, label it as temporary optimal.

Step 7: Stop. Return the optimal capacity expansion sequence.

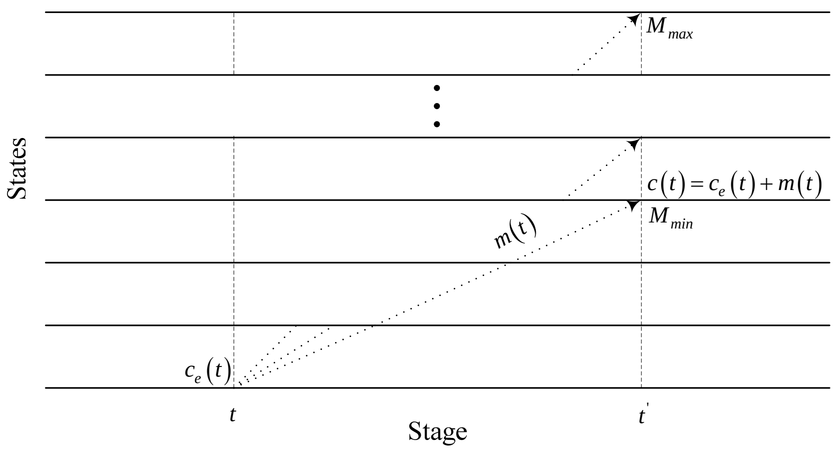

Figure 7 - Schematic evolution of the dynamic programming model

6 RESULTS AND CONCLUSIONS

It is assumed that demand, expressed in TEU, will grow at an annual discrete rate of 12.0% a year during the first five years, 9.5 % a year during the next ten years, and 7.5% a year thereafter. The terminal is already in operation at time $t = 0$, with three berths and a throughput of $D_0 = 650,000$ TEU. The project lifespan is $T = 25$ years, and berths and transferring equipments (gantry cranes) are assumed to depreciate in 30 years. The expected investment required to install a new berth, with the necessary cranes, is approximately US$ 240,000 per meter. Depending on the epoch to build a new berth, the expected ship size increases in accordance to column (b), Table 2, and the required berth length will vary accordingly as shown in column (c) of the same table. It was assumed a 80% experience curve (see Section 2.2), with $\theta = 0.3219$. In order to estimate average ship waiting times, the Cosmetatos formula (33) was employed, assuming a $M / E_6 / c$ queue type, with a threshold value of one hour to take into account ship maneuvering, customs, and other formal activities when arriving and departing.

With the above indicated assumptions the resulting optimal expansion plan is exhibited in Table 3. Apart from the three berths already in operation at time $t = 0$, it would be necessary to add five new berths, at different epochs during the project lifespan, as shown in Table 3. The capacity expansion at year 8, month 6, has benefitted from scale effect: instead of one additional berth, the model pointed out an expansion of two berths, with the second costing 80% of the first one. Figure 8 shows the evolution of the number of berths required along the project lifespan.

In our application the maximum number of berths to be considered in the analysis is ten, meaning a total number of decision variables equal to twenty, which correspond to the time t of capacity expansion and the number of berths $m(t)$ to be added simultaneously at time t. Of course, in terms of dynamic programming this is a small-size problem. When dealing with large problems it is possible that the classical dynamic programming approach may be limited by dimensional restrictions. In such a case, approximate dynamic programming algorithms (Powell, 2007) can be used.

Table 3 - Optimal capacity expansion results

Expansion Date		Number of berth to be installed	Berth extension to be added (m)	Investment value (US$ million)
Year	Month			
4	3	1	280	67.2
8	6	2	2×300 = 600	129.6
13	6	1	340	81.6
22	3	1	380	91.2
TOTAL		5	1,600	369.6

Although the model described here is theoretically sound, the container market in the globalized world has been changing dramatically in the past years. Container ship sizes are increasing steadily, and the requirements to build berths, cranes, and incorporate other technologies tend to vary substantially in the next decades. On the operational side, a number of container terminals in the world are being transformed into hub ports, and at least one East Coast South American port will probably follow this trend in the future, dramatically changing its technical and operating characteristics. With all that in mind, the use of economic analysis as the one described here, in which one assumes operating characteristics for a long period of time, seems to be not completely satisfactory for investment purposes. But the investor could follow a piece-wise dynamic sequence to do the analysis, periodically reviewing the assumptions and introducing new factors as they appear along time.

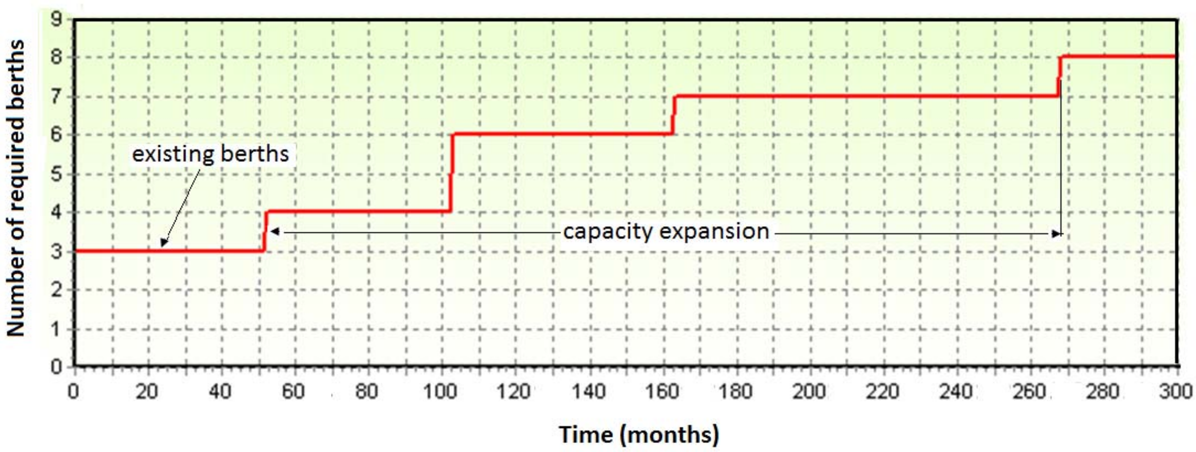

Figure 8 - Number of berths required along the project lifespan

Acknowledgments

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