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THE PROBLEM OF THE LOCATION OF BUS-STOPS IN URBAN PUBLIC TRANSPORT

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Abstract

Generally it would seem that enough scientific criteria have not been used to tackle the problem of bus stops. Their location is based on the criterion (generally a qualitative one) of the urban transport system techniques, aiming only at the demand's satisfaction , not considering other fundamental aspects such as access time (home – bus-stop), waiting time, level of service cost, and other costs that affect the companies working in the transport service. These costs depend, among other factors, on the spatial and numerical situation of bus stops.

According to this point of view, the bus stops of an urban transport system line acquire particular importance.

The aims of this work are to proportionate scientific criteria concerning the situation of the bus stops and propose mathematical models that can help the organizer in his/her work. The goodness of these models is based on the following argument: the location of the bus stops is obtained with the optimization of all the costs relative to users and to transport companies (operating costs) and the level of service offered by the system.

In order to give a good service to the public transport users, it is considered essential to realize an adequate transport plan; an appropriate planning, in fact, contributes in a relevant way, not only to the full satisfaction of the demand and to the optimization of the services costs, but also to decrease other social costs such as congestion and pollution.

Keywords: Location; Optimization; Cluster
Topic Area: E1 Assessment and Appraisal Methods

1. Introduction

In the developing countries, city buses are the most popular public transportation means. In industrialised countries, too, public transportation is gaining popularity in an effort to reduce pollution and traffic problems.

Those are the reasons why, on one hand, urban communities require an ever more efficient public transportation system, while on the other hand city transportation companies are focusing their attention towards issues like increasing costs and the responsibility to offer client-oriented services.

When planning urban public transportation, a quite critical issue is the one related to the allocation of bus stops.

As of now, there are no scientific criteria to distribute bus stops, and as a consequence their allocation is based on plans devised by city transportation managers, whose primary concern is to meet the highest public demand, without taking into account other crucial factors, which play a considerable role in optimising public transportation costs.

Whereas by *costs* we mean expenses associated to public transportation client services, along with operating costs and possibly additional costs related to service quality, as perceived by customers (the comfort factor).

In the last years, an impressive number of publications have appeared on the issues related to bus stops' allocation, even though no one had ever thought about the opportunity to allocate stops, and at the same time optimising the total costs of a public transportation system.

The following are some of the papers that appeared on these issues: R. Fernandez (1993) describes a methodology aimed at planning and allocating high capacity bus stops. H. Pietrantonio (1997a) calculates the optimal distance between two consecutive stops on a public transportation urban route, taking into account geometric restrictions, visibility and concentration points as required by the public demand.

Besides the aforementioned studies, particular consideration has been devoted to more general allocation studies, such as the one written by G. Gatti e E. Cavuoti (1988): they use numerical taxonomic models, applied to problems related to interchange ramps between a motorway and secondary roads.

The present paper is aimed at devising a model of urban transportation services' bus stops, thus optimising public transportation system costs.

The first section of this paper shows a model for allocating public transportation bus stops in an urban environment, taking into account the access costs paid by customers to reach the stop, given a total number of stops, and the relevant solution algorithm. This section also includes a proposal for a variation model, which allows to determine the optimal number of stops in a single public transportation bus route, and their related locations.

In the second section, the initial formula is made more complex by adding to access costs, additional costs related to time lost while waiting for the bus (waiting time), plus in bus travel costs, and also costs pertaining access to final destination stop and operating costs. In this second section, it will become clear that bus stop allocation plays a crucial role in cost variations.

2. A model of bus stops allocation, related to customers' access costs

The first section of this paper is aimed at devising a model for the allocation of urban public transportation bus stops, minimising customers' access costs to the bus stop. Gatti, G. and Cavuoti, E., (1988) have proposed a similar model applied to motorway ramps; the present model has been duly modified and adapted to bus stops allocation.

This model's starting hypotheses are:

a) There are given routes for all urban public transportation buses.

b) There is a given maximum number of bus stops (m) to allocate for each route (supposing the local transportation authority has fixed costs quota).

c) There is a given number of travellers $V_{i \in C_{k,l}}$ that move from the centroid i to reach route l.

Even though it is assumed that there are fixed bus stops' numbers, in the following section of this paper there is a proposal for a methodology aimed at determining an optimal number ($\bar{m}$) of stops to allocate, and therefore to plan for each route l.

Starting data are the number of travels generated from different centroids towards the respective urban public transportation routes. The target function of the present model is the following:

$$C_a = \sum_l \sum_k \sum_{i \in C_{k,l}} \gamma_a * ta_{i,p_{k,l}} * V_{i \in C_{k,l}} = \min \quad (1)$$

where:

C_a = Access cost ($).

γ_a = unit value of access time ($/t) for each traveller.

$ta_{i,p_{k,l}}$ = Access time from centroid (i) to bus stop (p_k) of route l (t).

$V_{i \in C_{k,l}}$ = Total number of travellers moving from centroid i to reach p_k bus stop on bus route l.

$C_{k,l}$ = Set of centroids that create travels towards the p_k bus stop on bus route l.

Where:

$$ta_{i,p_{k,l}} = \frac{d_{i,p_{k,l}}}{v_m} \quad (2)$$

being:

$d_{i,p_{k,l}}$ = Access distance from centroid i to bus stop p_k of route l.

v_m = Pedestrians' average mean speed.

It is easy to notice that, as far as our optimisation problem is concerned, the only unknown variable is the access distance ($d_{i,p_{kl}}$) from the centroid i to bus stop p_k on route l, so that the target function may be simplified, giving as a result the following:

$$\sum_l \sum_k \sum_{i \in C_{k,l}} d_{i,p_{k,l}} * V_{i \in C_{k,l}} = \min \quad (3)$$

$$\sum_{i \in C_{k,l}} V_i \le K_{p_{k,l}} \qquad \forall l = 1,2,....,n \quad \forall k = 1,2,....,m_l \quad (4)$$

where:

$K_{p_{k,l}}$ = is the maximum quantity of people who can access to a given bus stop p_k pertaining to a given route l.

In addition to the preceding target function, further capacity constraints must be added, related to the maximum number of travellers who can access to a given bus stop, or other kinds of constraints. Clearly, one of the fundamental constraints is the one related to the bus stops pertaining to the mentioned bus route.

This model allows to cluster centroids for each bus route, in *m* groups C_1, C_2,…, C_k, …, C_m, where *m* is the number of bus stops for a given route (fig. 1). It is thus possible to determine the hinterland of bus stops.

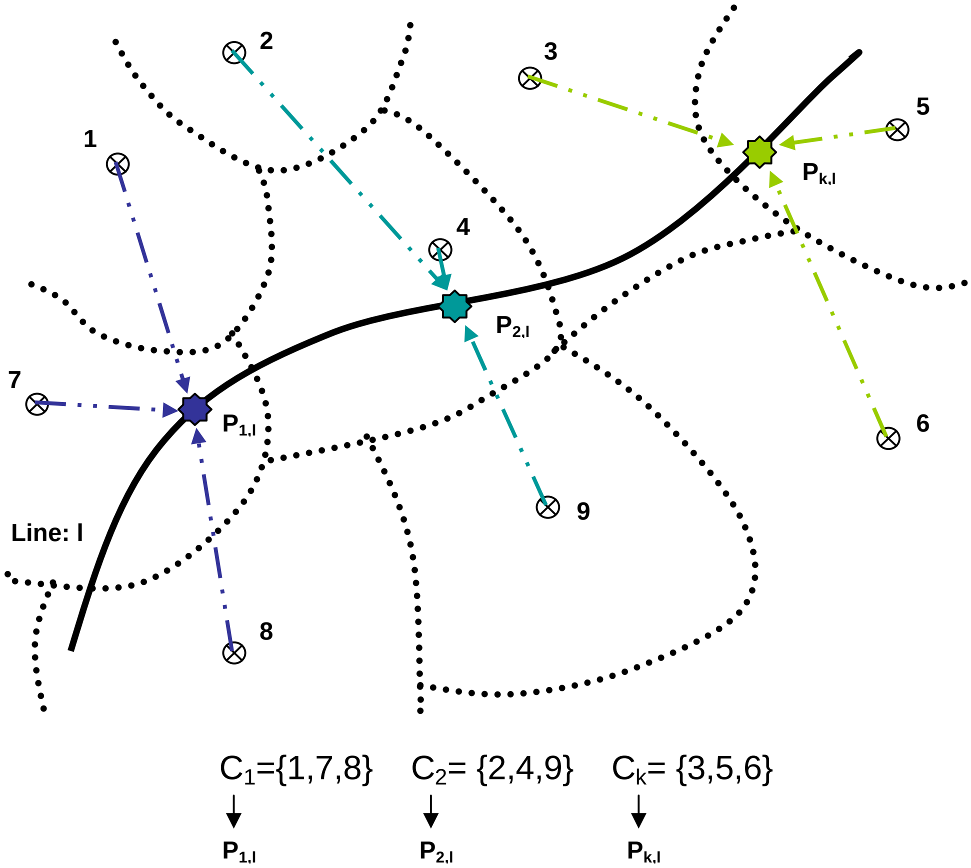

Figure 1. Diagram to show a part of the area under study in the model.

2.1 Solution algorithm

We now propose an algorithm to solve this problem. It is a recurring algorithm that starts from an arbitrary division of centroids in *m* groups (a number of groups that reflects the bus stops to allocate).

This algorithm is calculated through 7 steps:

Step 1: It is hereby defined the maximum number of bus stops m_l for each route.

Step 2: Centroids are divided in *m* groups (the initial division is arbitrary, except when the capacity constraint is satisfied); this operation is then repeated for *n* routes, accounting for a complete public transportation system. To word it differently, groups are selected so that the sum of V_i belonging to C_k is inferior to the bus stop capacity for travellers coming from each centroid, taken from the given group.

Step 3: For each group, we search for the most accessible route point, minimising the following function:

$$\sum_{l}\sum_{k}\sum_{i\in C_{k,l}} d_{i,p_{k,l}} * V_{i\in C_{k,l}} = \min \qquad (5)$$

Step 4: For each route, it is calculated the distance needed to move from each centroid to all bus stops ($d_{i,p_{k,l}}$).

Step 5: For each centroid *i*, it is calculated the minimum $d_{i,p_{k,l}}$ distance;

Step 6: If the minimum $d_{i,p_{k,l}}$ distance corresponds to the bus stop related to the group to which that centroid belongs, this centroid remains in said belonging group; if the bus stop corresponds to a different group from the one the centroid belongs, this centroid is thus moved to the group related to the stop that matches the minimum distance from the group.

Step 7: Based on what has been explained in Step 6, if one or more centroids are moved from a group C_k to a different group, we must go back to Step 3; if no node is moved, the algorithm is stopped.

There may exist an alternative way, which is applicable when capacity restrictions occur: this solution presupposes a limited minimum, which may be solved with the method devised by Kuhn Tucker.

However, this method cannot be applied in this particular case, because the value of the target function varies following discreet quantities, that is random quantities and not infinitesimal ones, nor it follows a law that may be analytically expressed.

Therefore, the aforementioned procedure has been modified, in order to proceed with an heuristic solution of the problem.

2.2 Establishing an optimal bus stops number

One of the fundamental hypothesis of the present model, is that the maximum bus stops number, rather than the optimal number, is identified (as can be seen in step 1 of the model's solution algorithm).

In order to obtain this result, we drawn from a paper by G. Gatti and E. Cavuoti (1988), who propose two criteria.

In order to apply the first methodology, it is necessary to slightly modify the target function as follows:

$$\sum_{l} \sum_{k} \sum_{i \in C_{k,l}} \gamma_d d_{i,p_{k,l}} * V_{i \in C_{k,l}} + \sum_{l} m_l c_m = \min \qquad (6)$$

where:

γ_d = unit value of the travel distance $d_{i,p_{k,l}}$ for each traveller.

m_l = number of bus stops on route l.

c_m = construction costs associated to single bus stop.

This way, by applying the aforementioned algorithm and changing the number *m* of bus stops, it is possible to show on a chart the two addenda of the target function, which stand for "Access cost" and "Bus stops construction costs" (fig.2).

In figure 1, travel cost and bus stop construction cost vary following the chart curves. In this chart, $\bar{m}_l$ stands for the optimal number of bus stops to be built and to allocate for each given route. The value of $\bar{m}_l$ is obtained by minimising the total cost function (access cost plus bus stop's construction cost).

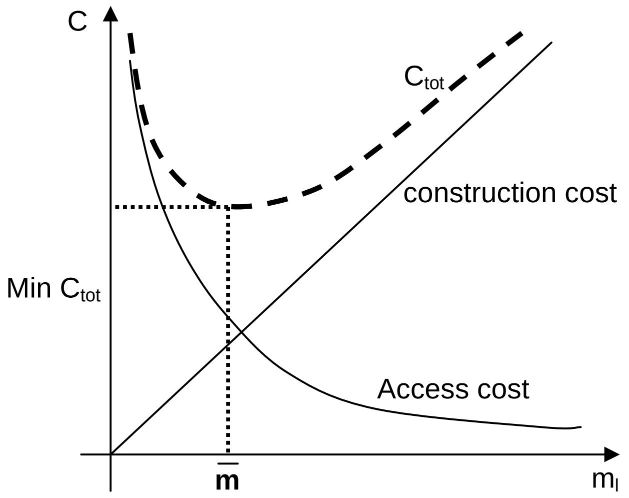

Figure 2. Optimal number of bus stop minimising the total cost function.

The second criteria refers to the bus stops' capacity limit, giving to this limit a standard value for each stop.

As a matter of fact, the value $\left(\sum_{i\in C_{k,l}} V_i\right)_{Max}$ is decreasing when related to the number *m* of the examined bus stops, as can be seen in figure 3. If V_K is the capacity limit for each single stop, in order to meet passengers' needs and at the same time minimise construction costs, the optimal number of stops to be allocated and therefore to be built will be $\bar{m}$.

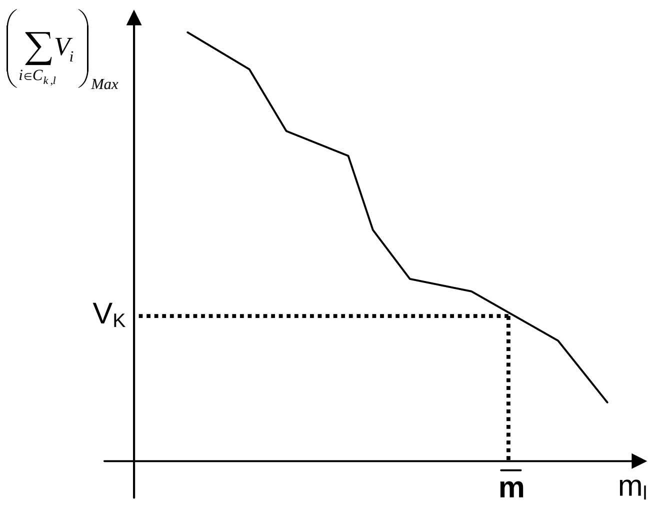

Figure 3. Optimal number of bus stop considering capacity constraint.

3. Incorporation of other costs inside the model

As we have already described, access cost is not the only one to influence bus stops allocation. This cost can be classified as a customer-related cost, however this is not the only one: there are costs related to the waiting time, costs related to the in bus travel time, and costs related to access the final destination.

In other words, costs related to a customer C_u may be summarised in the following expression:

$$C_u = C_a + C_w + C_r + C_{af} \tag{7}$$

Where:

C_a = bus stops access cost ($).

C_w = bus stops waiting cost ($).

C_r = in bus travel cost ($).

C_{af} = final destination access cost ($).

As regards costs related to bus stops, we have already seen how they have been formulated; as regards other costs, we mainly referred to papers by Vega, A. (2002) and Byrne, B. F., (1.976).

Cost functions, as proposed by the two authors, have been duly modified so as to fit the problem targeted in the present study.

The cost associated with time lost while waiting is obtained from the following expression:

$$C_w = \sum_l \sum_k \sum_{i \in C_{k,l}} \gamma_w * tw_{i,p_{k,l}} * V_{i \in C_{k,l}} \tag{8}$$

Where:

γ_w = unit value for waiting time $tw_{i,p_{k,l}}$ ($/t) for each traveller.

$tw_{i,p_{k,l}}$ = customer's waiting time, for a customer moving from centroid i to bus stop p_k on route l.

Considering that:

$$tw_{i,p_{k,l}} = \beta_0 + \beta_1 \left(\frac{1}{K_{i,j,l}\left(\frac{1}{h_l}\right)} \right) \tag{9}$$

Where:

β_0, β_1 = parameters to be estimated.

$K_{i,j,l}$ = routes' density for passengers travelling from centroids i and j.

h_l = headway between buses on line (l).

The cost related to the bus travel is given by the following expression:

$$C_r = \sum_l \sum_k \sum_{i \in C_{k,l}} \sum_{k'} \sum_{\substack{j \in C_{k',l} \\ k \neq k'}} \gamma_r * tr_{i,j,l} * V_{i \in C_{k,l}} \tag{10}$$

Where:

γ_r = unit value for travelling time in bus ($/t) for each traveller

$tr_{i,j,l}$ = bus travelling time for a customer moving from area $i \in C_{k,l}$ (starting trip from bus stop $p_{k,l}$) and area $j \in C_{k',l}$ (ending trip on bus stop $p_{k',l}$).

Considering that:

$$tr_{i,j,l} = \alpha_0 + \alpha_1 N_{i,j,l} + \alpha_2 \left(\frac{l_{i,j,l}}{v_{0i,j,l} - \alpha_3 i_{i,j,l}{}^{\rho_0}} \right) \tag{11}$$

$\alpha_0, \alpha_1, \alpha_2, \alpha_3, \rho_0$ = parameters to be estimated

$N_{i,j,l}$ = number of stops between stop $p_{k,l}$ and $p_{k',l}$.

$l_{i,j,l}$ = road distance between $p_{k,l}$ and $p_{k',l}$ stops.

$v_{0i,j,l}$ = average speed (no rush hour) between $p_{k,l}$ and $p_{k',l}$ stops.

$i_{i,j,l}$ = average vehicle transits between $p_{k,l}$ and $p_{k',l}$ stops.

It is worth noticing that in the formula related to travelling time $tr_{i,j,l}$, i and j, regularly indicating indexes representing centroids, in this case represent the two stops between which the bus travel takes place. As a matter of fact, centroid i belongs to a group of centroids $C_{k,l}$ associated with a bus stop $p_{k,l}$.

It may as well be noticed that this model does not take into account bus transfers, since the model itself has been devised on behalf of the municipality of Santander (Spain), whose shape and bus routes hardly need bus transfers for travellers, as a study pointed out.

Final destination access cost C_{af} has the same formula as the access cost, however in this case we considered travels directed to the area j, $V_{j \in C_{k',l}}$.

$$C_{af} = \sum_{l} \sum_{k'} \sum_{j \in C_{k',l}} \gamma_a * ta_{p_{k',l},j} * V_{j \in C_{k',l}} \tag{12}$$

$ta_{p_{k',l},j}$ = access time from bus stop p_k on route l to centroid j.

$V_{j \in C_{k',l}}$ = number of people moving from bus stop p_k on route l to centroid j.

The operating cost is obtained from the following simplified expression, since this kind of cost depends from many more variables:

$$C_o = \sum_{l} \left(\delta_0 + \delta_1 Nb_l \right) \tag{13}$$

Where:

δ_0, δ_1 = parameters to be estimated

Nb_l = number of buses travelling on route l.

It is also possible to add costs related to level of service, C_{ls}, that is a cost that calculates passengers’ comfort and the general quality of the service. As regards these costs, there are various publications that address the problem, such as Robuste, F. and Merino, E. (1988), and Northeim, B. (1988).

4. A global modelling of the system

Drawing from all we have examined so far, it is possible to devise a new target function, which, by optimising public transportation system's costs, allows to allocate bus stops, to calculate their optimal number for each route l, and also to establish important parameters, such as the optimal headway between buses h for each route and therefore the number of buses. The related target function will be the following:

$$\underbrace{C_a + C_w + C_r + C_{af}}_{user} + \underbrace{C_o}_{operating} + \underbrace{\sum_l m_l c_m}_{construction} = \min \tag{14}$$

$$\sum_{i \in C_{k,l}} V_i \le K_{p_{k,l}} \qquad \forall l = 1,2,....,n \quad \forall k = 1,2,....,m_l \tag{15}$$

This is the sum of costs related to customers (access cost, waiting cost, travel cost, final destination access cost), plus operating costs to be paid by the bus company, and bus stops' construction costs. By making the target function explicit we obtain:

$$\begin{aligned}
&\sum_l \sum_k \sum_{i \in C_{k,l}} \gamma_a * ta_{i,p_{k,l}} * V_{i \in C_{k,l}} + \sum_l \sum_k \sum_{i \in C_{k,l}} \gamma_w * tw_{i,p_{k,l}} * V_{i \in C_{k,l}} + \\
&\sum_l \sum_k \sum_{i \in C_{k,l}} \sum_{k'} \sum_{\substack{j \in C_{k',l} \\ k \neq k'}} \gamma_r * tr_{i,j,l} * V_{i \in C_{k,l}} + \quad + \sum_l \sum_{k'} \sum_{j \in C_{k',l}} \gamma_a * ta_{p_{k',l},j} * V_{j \in C_{k',l}} + \sum_l \left(\delta_0 + \delta_1 N b_l\right) \\
&+ \sum_l m_l c_m = \min
\end{aligned} \tag{16}$$

$$\sum_{i \in C_{k,l}} V_i \le K_{p_{k,l}} \qquad \forall l = 1,2,....,n \quad \forall k = 1,2,....,m_l \tag{17}$$

As we already explained, the target function may be restricted with capacity constraint (maximum number of passengers that may access to a single stop), and, for instance, with limitations to the minimum/maximum distance between two consecutive stops.

5. Summary

The problem of allocating public transportation bus stops is a rather crucial one, hardly finding adequate consideration.

As seen here, the lack of scientific criteria to allocate bus stops may severely impact costs related to travellers, C_u, and also operating costs, C_o. These costs become important because they both depend (one directly, and the other indirectly) upon the number of stops and the location where they are distributed.

As can be seen in paragraphs 2 and 2.1 of the present paper, we propose a model of city public transportation bus stops, solely taking into account the stops access costs as paid by customers, and the relevant solution algorithm.

Following this method, we can allocate the bus stops, thus minimising access cost related to the total of customers who need to access the service. In this section it is also possible to establish an optimal number of bus stops on a single public transportation route.

Moreover, in the second section the model has been completed, also considering the additional factor given by costs related to waiting time, bus travel costs, final destination access costs and operating costs. From all these calculations, it becomes clear that bus stops allocation has an impact on the variation of the above-mentioned costs.

The present model becomes important because it allows to allocate bus stops, as well as to calculate their optimal number for a given route l, and to establish important parameters, like the optimal headway between buses h for each route and consequently the number of buses to allow in each single route.

6. Conclusions

We may therefore assert that the problem of allocating bus stops is hardly an easy one, however, if appropriately addressed it may allow public transportation companies to optimise their service and to save a considerable quantity of resources. Besides, by minimising all costs associated to customers, we will obtain an even allocation of stops along the single route: this means we may allocate more stops in areas where demand is higher, and less stops where demand is decreasing, allowing for less time to access the single stops.

These results also allow to attract an increasing number of travellers, with added benefits for the bus companies. As a consequence, savings obtained following this model may be invested to increase service quality.

The model here proposed also allows to calibrate the service itself, since it estimates the optimal headway between buses for each route, so as to satisfy demand. All this is due to waiting time minimisation when waiting for the bus, a time that solely depends on said headway .

As we explained in the present model, the minimisation of bus travel time depends on the number of stops and on the existing traffic's factor between the original travel stop and the destination stop, besides the distance travelled.

Likewise, when planning a public transportation system, it is of the utmost importance to take into account construction costs, which also play a primary role when allocating an optimal number of bus stops.

This paper does not contain details related to level of service cost. These cost functions consider parameters like buses' capacity coefficients, the number of persons who travel standing or seated, etc. When all these parameters come close to overcrowding limits, in order to reduce costs it becomes necessary to decrease the headway between two consecutive buses. All this has an influence on the headway itself, and at the same time impacts waiting time and operating costs. In order to generally minimise expenses, it is essential not to neglect any kind of cost, since by adding or subtracting some costs, the variation of other costs may be influenced, costs that are strictly linked to one another.

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