AN HEDONIC APPROACH OF THE IMPERFECT MARKET OFFREIGHT TRANSPORT BY FRONTIER ANALYSIS *

Michel Mouchart and Marie Vandresse
Institut de statistique, Universit´e catholique de Louvain, Louvain-la-Neuve,Belgium.
Institut de statistique, 20 Voie du Roman Pays, 1348 Louvain-la-Neuve, Belgium;
vandresse@stat.ucl.ac.be

Abstract

As the market for freight transport is all but prefect, notably due to the heterogeneity of the goods and the firms, it should exist space for negotiation between the supply and the demand sides. The negotiation are based on the price and the attributes of the mode, but not on the mode itself (hedonic approach). We consider also that those attributes are endogenous, i.e. dependent of other variables as the characteristics of the goods. In order to take those characteristics into account, we propose to analyze the joint distribution of the price and the attributes rather that the conditional expectation, as done notably in the logistic or hedonic regressions. This model allows one to measure the market imperfection and then the bargaining power of the agents, by extending the methods of frontier analysis. The illustration is based on a study conducted in the Belgian sector of freight transport. The data were obtained by face-to-face interviews.

Keywords: Frontier analysis, supply and demand bid functions, imperfect competition, bargaining power, freight transport.
Topic Area: D2 Freight Transport Demand Modelling

1. An hedonic approach to discrete choice

Modal choice models classically used in transport economics are based on a random utility approach to discrete choice. The well-known models are the logit, probit, nested logit or more recent ones as mixed logit. These models call for three remarks. Firstly, the choice of the decision-makers is made dependent on the price and on the characteristics not only of the chosen mode but also of the competing modes. So, from an econometric point of view, not only the characteristics and the price of the chosen mode have to be known, but also the characteristics of the unchosen modes. Obtaining this information in the freight transport sector may be quite difficult, if not impossible. Secondly, experience based on interviews, conducted in the framework of the present study, suggests that the preferences of the decision-makers are based on the characteristics of the mode without preference for the mode itself. Even if a preference for the mode is observed, this might be interpreted as preference for unobservable characteristics of the mode. Thirdly, the supply side of the market is generally considered as given, i.e. the endogenous choice of the demand is modelled by assuming exogenous price. This last assumption is questionable for the market of freight transport. In view of those three arguments the hedonic models may provide an interesting alternative for modelling freight transport.

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In the hedonic approach, as pioneered in Griliches (1971) and Rosen (1974), the decision-maker maximizes a utility function defined on a vector of characteristics, say $U(z)$, subject to a constraint on the price specified as a function of the same characteristics, say $y(z)$ (the hedonic price function). When R represents the budget of the decision-maker, the demand is determined by the following maximization:

$$Max_z\, U(z) \text{ s.t } y(z) \leq R$$

However, as $y(z)$ is the result of a market clearing condition, the interpretation of the estimated parameters of the hedonic price function is not obvious as we face a problem of identification. Indeed, even under perfect competition, a relatively high value for one parameter can be due to a great importance of the associated characteristics for the demand side, or by a high production cost of this characteristics for the supply side. Furthermore, taking into account characteristics of the freight market (such as heterogeneity of the flows, of the firms or the presence of captive market), one may suspect that it exist a space for negotiation with bargaining power (partially or exclusively) shared between the demand and the supply side, so that perfect competition should often be excluded.

During the interviews, it has also been noticed that decision-makers have a minimal requirement on each characteristics z_k , for instance a minimal speed, or a minimal level of reliability. In order to represent this element in the maximization problem, one hedonic constraint could be added. The maximization problem becomes:

$$Max_z\, U(z) \text{ s.t } y(z) \leq R \text{ and } z \geq \underline{z}$$

where $\underline{z}$ represents the vector of required minimal level of the characteristics.

Note that in freight transport, it seems more appropriated to think in terms of minimization of the expenses rather than maximization of the utility. The minimization problem can be written as follows:

$$Min_z\, y(z) \text{ s.t } z \geq \underline{z} \text{ and } U(z) \geq \underline{U}$$

where $\underline{U}$ represents the required minimal level of utility.

The classical hedonic approach furthermore assumes that the characteristics are exogenous, but, again from experience, it has been observed that the contracts are based on a bargaining on the price and the characteristics simultaneously. Those characteristics can thus not be considered as exogenous.

For example, for the demand side, the requirement on the transport attributes typically depends on the specificity of the firm and/or of the freight. So, the presence of market imperfection and the endogeneity of the characteristics suggest to focus the attention on the joint distribution of prices and characteristics, rather than on the conditional distribution of the price given the characteristics. This remark suggests an approach similar to the frontier analysis often used in productivity analysis. Next section displays an economic analysis of market imperfection by means of a frontier model. Section three presents the numerical results of the analysis of data obtained through a survey by interview of 74 companies using the services of freight transport. These results include an estimation of the measure of market imperfection and of the bargaining powers of both side of each contract. A sensitivity analysis assesses the robustness of the results. Finally, a test allowing to give an indication of the importance of each attribute in the bargaining will be presented.

2. Freight transport as an imperfect market

Let us consider a data set constituted, for each contract of a representative sample, of the price y_i of the transaction i and a vector z_i of characteristics of the transaction i. These characteristics are coded in such a way that each one is a source of cost for the supply and a source of satisfaction for the demand. In the application presented below, the characteristics z are the speed, the frequency, the reliability, the safety and the flexibility.

Let us summarize the main ideas exposed more completely in Mouchart en Vandresse (2003). In order to measure the market imperfection, two bids functions are defined; the demand bid function, $P^*(z)$, representing the maximum that the demand aggrees to pay for the set of characteristics z (the upper frontier of the set of observed contracts) and the supply bid function, $P_*(z)$, representing the minimum the supply agrees to receive for the set of characteristics z (the lower frontier of the set of observed contracts). Figure 1 represents the demand and supply bid functions in the two dimensional case (i.e. one characteristic z and the price y); the observed contracts are represented by a cross. If the market were perfect, both estimated bid functions should be close to each other. On the contrary, more the market is imperfect, more the distance $(P^*(z) - P_*(z))$ between the two bid functions will be important (we will observe different prices for a same set of characteristics). Note that we are more interested in price variation for a same set of characteristics than in characteristics variation for a same price (so, in Figure 1, the market imperfection is represented by the vertical distance between the two bids functions.)

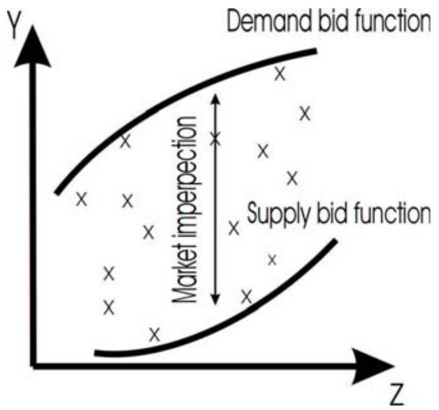

Figure 1: The market imperfection

Once a measure of market imperfection is obtained, it is also possible to obtain a measure of the bargaining power of each agent. Let us define the demand surplus as the difference $P^*(z) - P(z)$ and the supply surplus by the difference $P(z) - P_*(z)$, where P(z) represents the price of the observed contract, $x = (y,z)$, with the characteristics z (Figure 2). Then, the bargaining power of each agent involved in the contract is defined as:

$$\begin{aligned}\pi_D &= \text{demand bargaining power} \\ &= \frac{\text{surplus of the demand}}{\text{market imperfection}} \\ &= \frac{P^*(z) - P(z)}{P^*(z) - P_*(z)}\end{aligned}$$

$$\begin{aligned} \pi_S &= \text{supply bargaining power} \\ &= \frac{\text{surplus of the supply}}{\text{market imperfection}} \\ &= \frac{P(z) - P_*(z)}{P^*(z) - P_*(z)} = 1 - \pi_D \end{aligned}$$

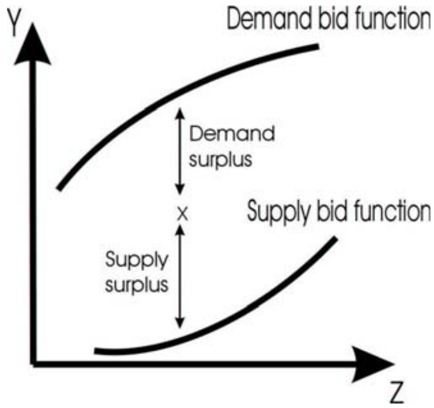

Figure 2: The demand and supply surpluses

The two bid functions, P*(z) and P*(z) are estimated through the upper and lower frontiers of the convex closure of the set of observed contracts, $\hat{C}$, with $\hat{C}$ respecting simultaneously the free disposability assumption of the demand and the supply, *i.e.* if (y_i,z_i) is observed, the demand (resp. supply) will accept $(y_i,z'_i)$with $z_i < z'_i$ (resp. $z_i > z'_i$) or (y'_i,z_i)) with $y'_i < y_i$ (resp. $y'_i > y_i$) :

$$\hat{P}_*(z) = \min\{y \mid (y,z) \in \hat{\mathcal{C}}\} \tag{1}$$

$$\hat{P}^*(z) = \max\{y \mid (y,z) \in \hat{\mathcal{C}}\} \tag{2}$$

For a given contract $(y,z) \in \hat{C}$ let us define a *demand price multiplier* (resp. *supply price multiplier*) as a factor of dilatation (resp. of contraction) that would bring y up to the frontier P*(z) (resp. P*(z)):

$$\alpha : \hat{\mathcal{C}} \to [1,\infty) \qquad \alpha(y,z) = \max\{a \mid ay \leq \hat{P}^*(z)\} \tag{3}$$

$$\beta : \hat{\mathcal{C}} \to (0,1] \qquad \beta(y,z) = \min\{b \mid by \geq \hat{P}_*(z)\} \tag{4}$$

The frontiers are characterized by the efficient contracts for which $\alpha = 1$ or $\beta = 1$, respectively. Once those efficient contracts have been empirically identified, the demand and supply bid functions are constructed by linear interpolation.

For each observed contract, x_i, the value of the functions α $(y_i,z_i)$and $\beta(y_i,z_i)$ may be estimated by linear programming, namely :

$$\hat{\alpha}(y_i,z_i) = \sup\{a \mid \exists\gamma \in S_{[n-1]} : ay_i \leq y'\gamma, Z'\gamma \leq z_i\} \tag{5}$$

$$\hat{\beta}(y_i,z_i) = \inf\{b \mid \exists\gamma \in S_{[n-1]} : by_i \geq y'\gamma, Z'\gamma \geq z_i\} \tag{6}$$

where $y = (y_1, .., y_i, .., y_n)'$: $n \times 1$, the vector of observed prices.

$z = (z_1, .., z_i, .., z_n)'$: $n \times k$, the matrix of observed characteristics.

Once α and β have been estimated for each contract x_i, the market imperfection and the bargaining powers associated to each contract x_i can be estimated as follows :

$$\hat{I}(y_i, z_i) = [\hat{\alpha}(y_i, z_i) - \hat{\beta}(y_i, z_i)] \times y_i \quad (7)$$

$$\hat{\pi}_D(y_i, z_i) = \frac{\hat{\alpha}(y_i, z_i) - 1}{\hat{\alpha}(y_i, z_i) - \hat{\beta}(y_i, z_i)} \quad (8)$$

$$\hat{\pi}_S(y_i, z_i) = \frac{1 - \hat{\beta}(y_i, z_i)}{\hat{\alpha}(y_i, z_i) - \hat{\beta}(y_i, z_i)} \quad (9)$$

3. Empirical Analysis

3.1. Descriptive analysis of the data

The database includes 74 observations from a Belgian survey realized in the sector of freight transport. The data where obtained through face-to-face interviews during which it was asked to describe the value of the criteria of quality and the cost for a typical contract, x_i:

- The cost of the transport (EUR/km× ton).
- The frequency : number of trips executed per week.
- The reliability : the percentage of time the goods are delivered within the required time.
- The time : the door to door transport time, including loading and unloading.
- The loss : the percentage of time a loss (or damage) is incurred.
- The flexibility : the percentage of time that unexpected demands are met by the carrier.

The data are assembled in matrix form:

$$X = [x_i'] = [y_i \quad z_i'] = [y \quad Z]$$

where $X : n \times (1 + k), \quad y : n \times 1, \quad Z : n \times k$

Seeing that the variables have to be such that the partial derivatives of the demand utility function and the supply cost function are striclty positive, we transformed some data, i.e. the time into the speed (distance/time) and the loss into a safety index (100 -percentage of loss). The frequency is replaced by its inverse. Indeed, the cost/ton.km is negatively related to the frequency.

In Figure 3 we present the box-plots of the 6 variables. The box-plot is a graphical tool which provides 5 points summary of the data .The lower line of the box represents the first quantile, i.e. a value under where are 25% of the data. The middle line of the box represents the median. The upper line represents the third quantile, i.e. a value under where are 75% of the data. The whiskers (⊥ and T) enclose an interval defined by 1.5 times the lenght of the inner quantiles. The average is represented by a small circle between the two wiskers (and most often inside the box). The small circles outside the whiskers represent outlying data.

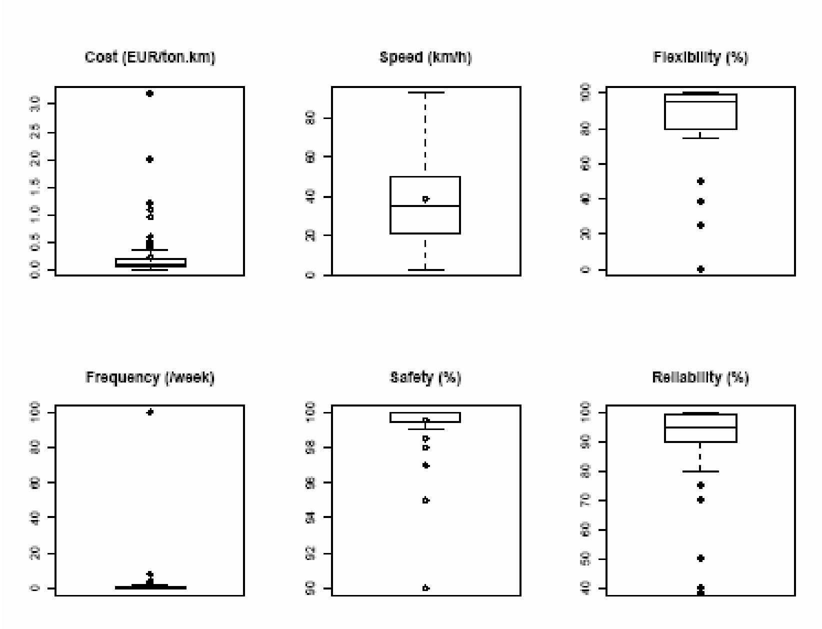

Figure 3: The box-plots of the variables

From the box-plots in Figure 3, we can observe a seemingly high concentration of the data, with the presence of some outliers.

3.2. First results and interpretation

Figure 4 provides a graph of the values of the two bid functions; on the abcissa, the labels of the contracts (ranging from 1 to n) and on the ordinates, the value of β, ranging from 0 to 1 and the value of 1/α, ranging from 0.003 to 1. The very same empirical finding may also be obtained from Figure 5 providing, in the form of box-plots, the distribution of the market imperfection and the bargaining power of each agent (remember that the sum of the two bargaining powers is equal to 1).

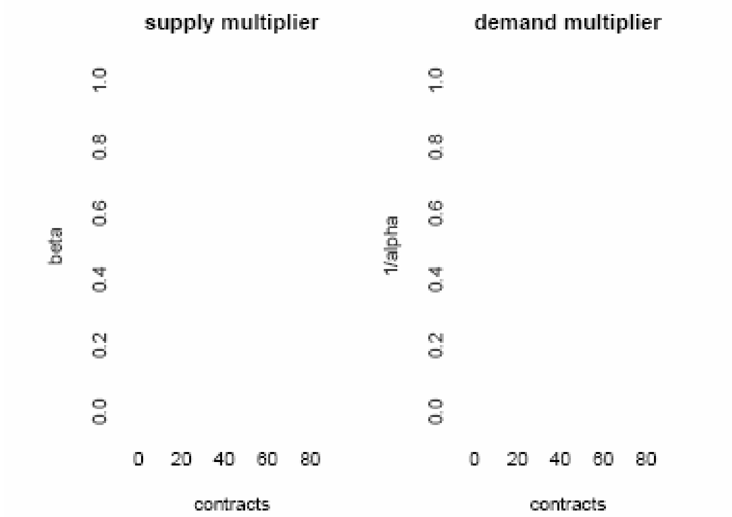

Figure 4: The demand and supply price multiplier

Remark : 14 (resp. 13) contracts lie on the upper (resp. lower) frontier and display an estimated extreme value of the multiplier, $\hat{\alpha}=1$ (resp. $\hat{\beta}=1$), i.e. an extreme supply (resp. demand) bargaining power. Furthermore, one firm is estimated as $\hat{\alpha}=\hat{\beta}=1$, corresponding to an outlying observation on the South-West of the graph.

When interpreting these numerical results, one should keep in mind that the demand and supply price multipliers and the bargaining power are unit-free coefficients, i.e. they are invariant under a change of units of the variables y_i or z_i whereas the measure of market imperfection ranging from 0 to 0.91 stands in the same unit as the price variable.

From the observation of the box-plots in Figure 5, we observe that 50% of the demand (resp. supply) bargaining power is greater (resp. lower) than 0.85 (resp. 0.15). This suggests that freight transport in Belgium is dominated by the demand. Furthermore, on average, the imperfection on the market lies around 0.45 EUR/km×ton. This would mean that, on average, the space of negocation for each firm turns around 0.45 EUR/km×ton. When interpreting this result, one should notice that the sampling procedure was notably targeted towards companies with a sizable turnover that may eventually be suspected of enjoying a more substantial demand bargaining power.

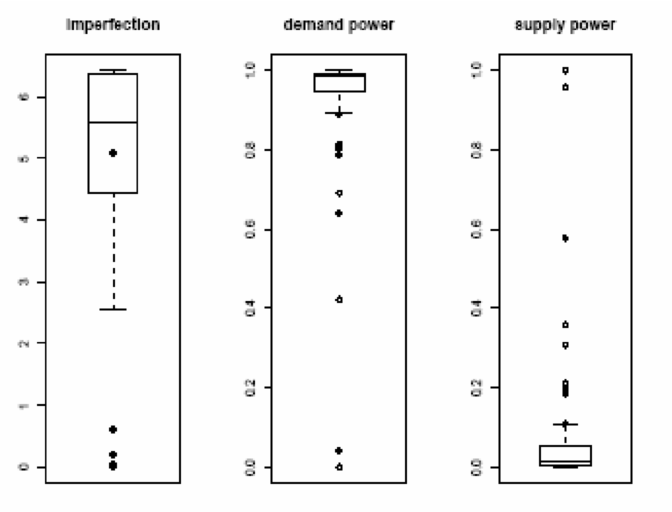

Figure 5: The market imperfection and bargaining powers

3.3. Importance of the attributes

3.3.1 Methodology and results

One method for assessing the influence of each attribute consists of comparing the two bid functions based on all the attributes, coded in the vector z, with those based on all those attributes but one. In this way, we can analyse and compare the imperfection and the bargaining powers when 4 or 5 attributes are taken into account.

To be more explicit, let us define $z_{\backslash\{j\}}$ as the vector z without the attribute z_j. If the j^{th} attribute is not particularly important in the negociation of the contracts, one may suspect that $P^*(z_{\backslash\{j\}})$ (resp. $P_*(z_{\backslash\{j\}})$) will be close to $P^*(z)$ (resp. $P_*(z)$), and as a consequence, $I(z_{\backslash\{j\}})$, $\pi_D(y,z_{\backslash\{j\}})$ and $\pi_S(y,z_{\backslash\{j\}})$ will be close to $I(z)$, $\pi_D(y,z)$ and $\pi_S(y,z)$. On the contrary, if the attribute z_j has some importance in the negociation of the contract, we would expect to observe significantly different bid functions and then, a different measure of imperfection and of bargaining powers.

When evaluating the multipliers, deleting one of the attributes means deleting one of the restrictions on the optimization problem (equations (5) and (6)), and eventually improving the optimal value relative to the complete set of attributes. Thus, if we write $\hat{\alpha}^j(y_i, z_{i\setminus\{j\}})$ and $\hat{\beta}^j(y_i, z_{i\setminus\{j\}})$ for the demand and supply multipliers when deleting the j^{th} coordinate, we always have:

$$\hat{\alpha}^j(y_i, z_{i\setminus\{j\}}) \geq \hat{\alpha}(y_i, z_i) \text{ and } \hat{\beta}^j(y_i, z_{i\setminus\{j\}}) \leq \hat{\beta}(y_i, z_i) \tag{10}$$

Moreover, relation (10) implies that $\hat{\alpha}^j - \hat{\beta}^j \geq \hat{\alpha} - \hat{\beta}$, *i.e.* the measure of imperfection (equation (7)) increases when deleting the j^{th} characteristic. These inequalities also hold for the mean or the median evaluated over the n observed contracts. These facts may be observed in the first 9 columns of Table 1, with a notably more substantial effect when the frequency is deleted. The effect, on the standard errors, of deleting one attribute is somewhat different: decreasing for the multipliers $1/\alpha$ and β but undetermined for the measure of imperfection.

The effect on the demand and supply bargaining power is not determinate because deleting the jth coordinate increases both the numerator and the denominator in equation (8) and (9). When deleting the flexibility, the reliability or the safety, a decreasing (resp. increase) impact on the mean of the demand (resp. supply) bargaining power is observed (columns 10 and 13 of Table 1). The opposite facts is observed for the frequency and the speed.

As a consequence of the relation (10), statistical tests are needed is order to test whether the observed differences on the price multipliers and then on the imperfection and on the bargaining powers between the restriced (4 attributes) and the unrestricted (5 attributes) optimization are significant.

3.3.2 Tests of significance, using a Bootstrap method

In order to assess whether the observed difference between the unrestricted (H_0) and the restricted (H_1) models is not due only to the removal of one constraint in the optimization (or minimization) problem, the following test could be applied (based on Simar and Wilson(2001)):

$$H_0 : \alpha^k(y_i, z_{i\setminus\{k\}}) = \alpha(y_i, z_i) \geq 1 \quad \forall((y_i, z_i))$$

$$H_1 : \alpha^k(y_i, z_{i\setminus\{k\}}) > \alpha(y_i, z_i) \geq 1 \quad \forall((y_i, z_i))$$

$$AND$$

$$H_0 : \beta^k(y_i, z_{i\setminus\{k\}}) = \beta(y_i, z_i) \leq 1 \quad \forall((y_i, z_i))$$

$$H_1 : \beta^k(y_i, z_{i\setminus\{k\}}) < \beta(y_i, z_i) \leq 1 \quad \forall((y_i, z_i))$$

So, the idea is to test whether the difference (if any) between the multipliers in the restricted and unrestricted models is significant.

The chosen statistic is the following:

$$S = \sum_{i=1}^{n} (\hat{\rho}_i - 1) \leq 0 \tag{11}$$

where $\hat{\rho}_i = \frac{\hat{\beta}^k(y_i, z_{i\setminus\{k\}})}{\hat{\beta}(y_i, z_i)} \leq 1$ or $\hat{\rho}_i = \frac{\hat{\alpha}(y_i, z_i)}{\hat{\alpha}^k(y_i, z_{i\setminus\{k\}})} \leq 1$

If H_0 is true, $\hat{\rho}_i = 1$, otherwise $\hat{\rho}_i < 1$. The null hypothesis is rejected if the appropriate critical value is lower than 0.05. $\hat{\beta}^k(y_i, z_{i\setminus\{k\}}))$ and $\hat{\beta}(y_i, z_i)$ are obtained by equation (6); $\hat{\alpha}^k(y_i, z_{i\setminus\{k\}}))$ and $\hat{\alpha}(y_i, z_i)$ by equation (5). The critical value is obtained by a bootstrap methodology proposed in Efron and Tibshirani (1993, chapter 16). Table 2 gives the value of the statistic S and his associated critical value. As all the critical values are higher than 0.05, we conclude that no statistical difference between the restricted and unrestricted models appears to be significant (the bid functions are not different). This would mean that none of the attribute seems to play a particularly important role in the bargaining process.

Table 1: Importance of the attributes

Deleted attribute		demand multiplier $1/\alpha$			supply multiplier β			Imperfection I			Demand power π_D			supply power π_S		
None	mean	0,071			0,373			0,455			0,705			0,294		
	median		0,209			0,192			0,471			0,863			0,136	
	std.error			0,022			0,426			0,266			0,367			0,367
Frequency	mean	0,027			0,252			0,788			0,741			0,211		
	median		0,137			0,104			0,804			0,898			0,101	
	std.error			0,00768			0,374			0,248			0,295			0,295
Speed	mean	0,010			0,349			0,508			0,725			0,274		
	median		0,167			0,070			0,518			0,852			0,099	
	std.error			0,0014			0,417			0,240			0,342			0,296
Flexibility	mean	0,059			0,224			0,688			0,643			0,311		
	median		0,176			0,026			0,524			0,862			0,137	
	std.error			0,017			0,392			0,274			0,355			0,355
Reliability	mean	0,069			0,273			0,493			0,682			0,317		
	median		0,188			0,122			0,500			0,847			0,152	
	std.error			0,022			0,383			0,269			0,350			0,350
Safety	mean	0,070			0,286			0,483			0,699			0,300		
	median		0,194			0,128			0,512			0,868			0,131	
	std.error			0,022			0,411			0,272			0,356			0,356

Table 2: Test for the importance of the attributes (on λˆand on ˆ

Deleted attribute	β : value of the statistic (S)	β : critical value		$1/\alpha$: value of the statistic (S)	$1/\alpha$: critical value
Frequency	-19.198	0.537		-23.280	0.818
Speed	-10.876	0.522		-6.344	0.688
Flexibility	-25.619	0.545		-3.558	0.794
Reliability	-9.738	0.515		-5.006	0.733
Safety	-14.735	0.521		-2.358	0.731

4. Conclusion

In this paper, we analyze the market for freight transport in an exploratory way. Considering that this market is all but perfect, it seemed interesting to highlight this characteristic in an econometric approach. The frontier analysis offers one possible alternative. This approach presents some differences compared to the classical methods as the logistic and hedonic regressions. Recognizing a high degree of market imperfection and the endogeneity of the attributes we analyze the joint distribution of the price and attributes of the contracts instead of their conditional distributions.

Concerning the results, we could conclude that market for freight transport is dominated by the demand but remember that this result should be influenced by the sampling procedure. Furthermore, this kind of analysis is sample-sensitive. As a consequence, a sensitivity analysis should be applied in order to assess the robustness of those results. The analysis should also be made on a more important database, after deleting the outliers. Concerning the importance of the attribute, this approach did not find that one particular attribute has more importance in the decision process.

Note that this analysis has been done on face-to-face interviews. As the procedure to get an interview is long and costly, the database is not huge and it has not been possible to make the analysis by disaggregating the data by mode of transport. This could be interesting too. Furthermore, some interviews had been deleted from the database due to missing data. Taking into account characteristics of the missing data could also refine the analysis.

Concerning the number of the characteristics, it has been noticed that the analysis could be made by taking only three characteristics into account. First, the coefficient of variation for the safety is very small (0,01). This characteristics is uninformative. Second, The economic interpretation of the frequency raises problems. Indeed, a high frequency can be associated either with a high cost (EUR/ton × km) reflecting the cost of a high level of service availability, or with a low cost, reflecting some return to scale. More specifically, the cost seems to depend more on the rate of utilization (of the material) than on the frequency of the service. For those reasons, a more detailed economic and econometric analysis will be done without the frequency and the safety.

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Appendix A: The Data Envelopment Analysis

Basic concepts

In economics, the production frontier represents the maximum quantity of output(s), y, which can be produced by a set of input(s), x. The production set gives all possible combinations of x which can produce y. If a firm lies on the production frontier, it is defined as efficient. If the firm is below the production frontier, it is defined as inefficient. Inefficient means that either the firm can increase its production with the same set of input(s) (output efficiency) or decrease the set of inputs in order to produce the same quantity of output (input efficiency). In Figure 6 the production frontier and input/output efficiencies are presented.

The frontier analysis will estimate this production set and gives a measure of output and/or output efficiency.

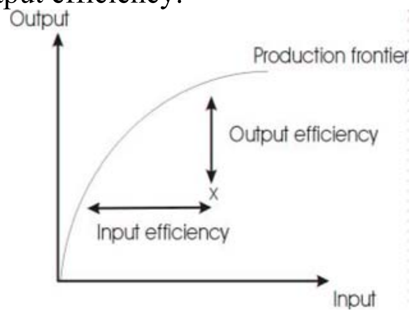

Figure 6: Frontier analysis and efficiency

If y is defined as the vector of outputs $\in \mathbb{R}_+^o$ and x as the vector of inputs $\in \mathbb{R}_+^s$, the production set can be represented as (Simar,Wilson 2003) :

$$\Psi = \{(x, y) \in R_+^{o+s} \mid x \text{ can produce } y\}$$

Also,

$\forall y \in \Psi, X(y) = \{x \in \mathbb{R}_+^s \mid (x, y) \in \Psi\}$ (all x which can produce a given y)

$\forall x \in \Psi, Y(x) = \{y \in \mathbb{R}_+^o \mid (x, y) \in \Psi\}$ (all y which can be produced by a given x)

Figure (7) represents $X(y)$ and $Y(x)$ in the 2-dimensional case.

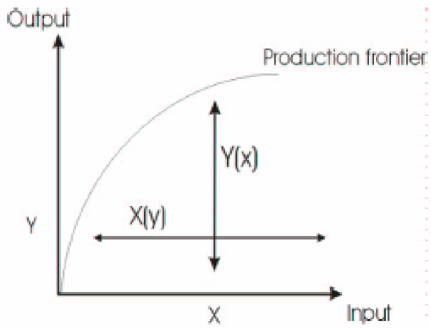

Figure 7: X(y) and Y(x) with 1 output -1 input

The efficient boundaries of X(y) and Y (x) are defined as :

$$effX(y) = \{x \mid x \in X(y), \quad x' \in X(y) \quad \forall x' < x, \quad x' \neq x\}$$
$$effY(x) = \{y \mid y \in Y(x), \quad y' \in Y(x) \quad \forall y' > y, \quad y' \neq y\}$$

The input efficiency is defined as :

$$\theta(x, y) = \inf\{\theta \mid \theta x \in X(y)\} \quad \leq 1$$

The output efficiency is defined as :

$$\lambda(x, y) = \sup\{\lambda \mid \lambda y \in Y(x)\} \quad \geq 1$$

In order to estimate Ψ, $\theta(x, y)$, $\lambda(x, y)$ at least 2 non parametrics methods have been developed : Data Envelopment Analysis (DEA) and Free Disposal Hull (FDH). In both methods, the idea is to find an estimation of the production set which envelops the data, depending on weak general assumptions on ψ. A common one is based on free disposal assumption of ψ. The main difference between DEA and FDH is that DEA assumes that the production set is convex (Figure 8).

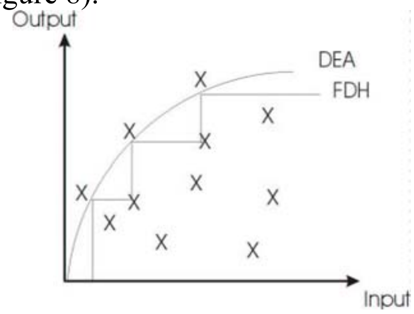

Figure 8: DEA and FDH methods

Intuitively, for economic applications, the DEA seems more appropriated (because convexity of the production set). So, the rest of this section is devoded to the DEA method. The estimated production set, $\hat{\Psi}_{DEA}$, is defined in the following way:

$$\hat{\Psi}_{DEA} = \{(x, y) \in \mathbb{R}_+^{o+s} \mid y \leq \sum_{i=1}^{n} \gamma_i y_i; x \geq \sum_{i=1}^{n} \gamma_i x_i; \sum_{i=1}^{n} \gamma_i = 1; \gamma_i \geq 0\}$$

where n is the number of observations.

And then,

$$\hat{\theta}(x_i, y_i) = \inf\{\theta \mid (\theta x_i, y_i) \in \hat{\Psi}_{DEA}\}$$
$$\hat{l}(x_i, y_i) = \sup\{\lambda \mid (x_i, \lambda y_i) \in \hat{\Psi}_{DEA}\}$$

So, “this method of measuring the technical efficiency of a firm consists in comparing it with a hypothetical firm which uses the factors in the same proportions. This hypothetical firm is constructed as a weighted average of two observed firms, in the sense that each of its inputs and outputs is the same weighted average of those of the observed firms, the weights being chosen so as to give the desired factor proportions.”(Farrell, 1957)

The linear programming for the output and input efficiencies are presented in Table 3, where

$i_n = (n \times 1)$ vector of 1

Table 3: Linear Programming

	Output efficiency	Input efficiency
Primal	$\max \lambda$	$\min \theta$
	s.t.	s.t.
	$i_n'\gamma = 1$	$i_n'\gamma = 1$
	$\lambda y_i - Y'\gamma \leq 1$	$Y'\gamma - y_i \geq 0$
	$-x_i + X'\gamma \leq 0$	$\theta x_i - X'\gamma \geq 0$
	$\gamma > 0$	$\gamma > 0$
Dual	$\inf \nu' x_i + v$	$\sup\{\mu' y_i + v\}$
	s.t.	s.t.
	$\mu' y_i \geq 1$	$\nu' x_i \leq 1$
	$X\nu - Y\mu + v i_n \geq 0$	$-X\nu + Y\mu + v i_n \leq 0$
	$\mu', \nu' \geq 0$	$\mu', \nu' \geq 0$
	v free	v free

$\gamma = (\gamma_1, \ldots, \gamma_n)'$

Y $= (n \times o)$ matrix of observed outputs

X $= (n \times s)$ matrix of observed inputs

x_i and y_i are respectively the observed inputs and outputs for the considered decision-maker (or firm).

$\nu \in \mathbb{R}^s_+$ and $\mu \in \mathbb{R}^o_+$ are the mutlipliers for the constraints in the primal problem.

For a review of the litterature on DEA, we refer to Seiford (1996) and Simar and Wilson (2000,2003). Examples of applications can be found in the following areas : post offices (see e.g. Deprins, Simar, Tulkens, 1984), transport (see e.g. Mouchart and Simar (2002)), electricity sector (see e.g. Fre, Grosskopf and Kokkelenberg (1989)), education(see e.g. Charnes, Cooper and Rhodes (1981)), hospital, bank.