

SOME APPLICATIONS OF THE MULTIPLE SERVER QUEUEING MODELS IN PORTS

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Abstract

The aim of this paper is analysis of basic port operational process through various models of the queueing theory. The comparison of described and developed models has been realized in relation with the essential characteristics of the system, i.e. operating port parameters. The results of the analyses show a series of statistical parameters which form fundamental indicators of the system. The advantages and disadvantages of the given models are pointed out in 2D and 3D graphical presentations.

This paper deals with the anchorage-ship-berth link in ports. The vessels movement process in port is analysed by using methods of queueing theory models, i.e. multichannel servicing systems with limited waiting queue, unit bulk arrival in the systems and cancellation of the whole group (model I) or partial system completion (model II). All the main characteristics of the systems are given.

Keywords: Port systems; Anchorage-ship-berth link; Queueing models; Systems performances

Topic area: A2 Maritime Transport and Ports

1. Introduction

The adequate choice of the queueing theory system models with their valid application provides the preconditions for the utilization of capacities installed in these port segments in the usage of analytical models. The purpose of this work is to give the comparative analysis of various models of multichannel system with limited waiting queue, the unit bulk arrival in the system and cancellation of the whole group or partial system completion. This means the determination of fundamental parameters of the system in the analysis of anchorage-ship-berth link in ports.

This paper discusses the anchorage-ship-berth link at the port as a system of queueing theory with bulk arrivals. The stochastic characteristics of the link operation are as follows:

- time of arrival of single ships or in bulk (barge tows) in the port can't be precisely given.
- the transshipment service time, (loading/unloading) is a random variable depending on handling capacities of berths, carriage of barges, hydro - meteorological conditions, the size of an arriving group, etc.
- the berths are not always occupied; in some periods there are no barges (the capacities underutilized); there are the time intervals of high utilization when the queues are formed.
- The operation of the anchorage-ship-berth link includes the following:
- waiting in the anchorage areas; if all anchorages are occupied, the vessels are canceled and have to go to another waiting area in that or some other port;

- vessels move from anchorage to berth;
- loading or unloading at the berth;
- towing of barges after the loading/unloading to the anchorage area or leaving of the port;

This cycle is called the turnaround time for the ships in the port.

Due to the influence of a great number of factors (water level, technical functioning of devices, meteorological conditions, the number of barges in tows, the distance between the anchorage and the berth, the position of vessels at the anchorage, etc.), these processes cannot ideally follow one another or have constant time intervals, but they are subjected to permanent changes. There are longer or shorter waiting time intervals due to the reasons mentioned which prolong the turnaround time and unsuitably influence the fleet operation.

In this paper the general models of ships traffic for the terminal are developed. The models focus on the anchorage-ship-berth link. The process is described by the stationary, multichannel queueing system. This system has the following characteristics: waiting areas are finite and given, unit bulk arrival into the system is assumed and arriving ships are not allowed into the system if $k > (n+m)-s$, where k is the number of ships arriving at the same time (bulk arrival).

2. Literature background

The application of queueing theory to shipping problems has been made by Plumlee 1966; Nicolaou 1967 and 1969; Miller 1971; Wanhill 1974; Agerschou, et al. 1983; Noritake and Kimura 1983; Berg-Andreassen and Prokopowicz 1992; Radmilović 1992; Zrnić 1996; Zrnić and Bugarić 1994 and Zrnić et al. 1999.

Apart from the classical theoretical references [e.g. (Bharucha-Reid, 1960), (Maisel, 1971), (Kleinrock, 1975), (Isaacson and Madsen, 1976), (Coper, 1981), (Chaudry and Templeton, 1983), (Cohen and Boxma, 1983), (Kleinrock and Gail, 1996)], used for describing the models in this paper, it was necessary to review some segments of papers [e.g. (Nicolaou, 1967 and 1969), (Miller, 1971), (Radmilović, 1992), (Zrnić and Bugarić, 1994), (Goswani and Gupta, 1998), (Zrnić et al., 1999), (Srinivasan et al., 2002), (Chaudry and Gupta, 2003)] in which some individual elements of optimization of the various operations have been considered.

3. Models formulation

The anchorage-ship-berth link is considered as bulk queueing system. In this case, customers are single ships, while the service channels are berths operating at the loading/unloading of cargo.

In the port anchorage-ship-berth link the following is assumed:

1. The applied queueing system is non-stationary, with finite waiting area at anchorage.
2. The sources of arriving pattern are not integral parts of anchorage-ship-berth link.
3. The service channels are berths with the similar or identical and independent cargo-handling capacities.
4. The units arrivals can be single ships and groups as barge tows. The arrival of service-seeking entities follows the Poisson distribution.
5. All barge tows and single ships at anchorage are waiting to be serviced.
6. The service time, i.e. loading/unloading time is a continuous random variable.
7. The size of an arriving group is a random variable.
8. The queue discipline is first come, first served (FCFS) by tows bulk and random within the tow bulk.

9. The queue length or the number of waiting anchorage areas are finite and given.

The port terminal has the n berths for the service. The mean cargo handling rate per berth is μ . Apart from n ships which are possible to arrive on service, there are m spaces in the waiting queue. Ships arrive in batches according to a time homogenous Poisson process with mean arrival rate λ . The number of ships X which arrive to service at the same time is a random variable with distribution given by $a_k = P(X=k)$, $k \geq 1$, (k -number of vessels in group). Servicing process of ships or barges is analysed on the basic of two models of the queueing theory, such as:

- I. If the group of ships arriving into the system, finds s ships there, then in the case that $k \leq (n+m)-s$ the whole group will be accepted into the system, it means that in case $k > (n+m)-s$, the whole group would be canceled.
- II. In the case that group of k ships arriving into the system, finds s ships there, then for the case $k \leq (n+m)-s$ the whole group will be accepted, and it means that if $k > (n+m)-s$ system accepts $(n+m)-s$ ships form the group, the rest of $k - ((n+m)-s)$ ships will not be accepted.

The states of the systems for both models should be determined by the number of ships within the systems. The graphs of the states of the systems are given in Figure 1 and Figure 2. Then the following equations are obtained (Model I and Model II).

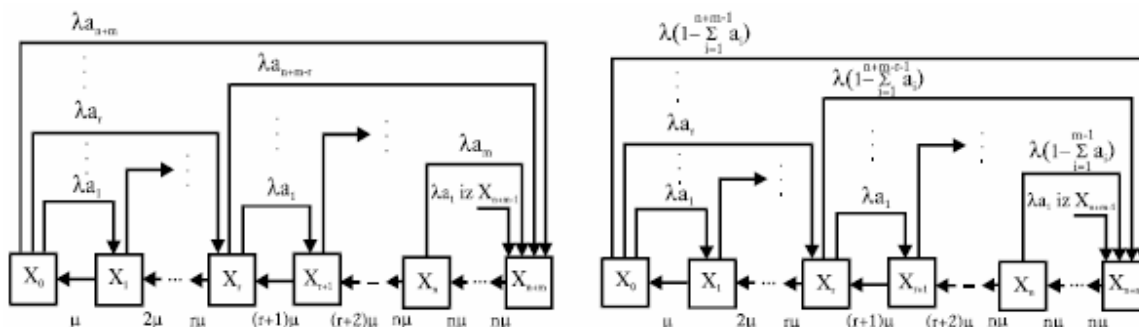


Figure 1. The graph of the state of the system I Figure 2. The graph of the state of the system II

On the basis of the graphs (Figs. 1 and 2) we obtain the following systems of equations of the probabilities of the state of the systems:

Model system I	Model system II
$p'_0(t) = \left(-\lambda \sum_{i=1}^{n+m} a_i \right) p_0(t) + \mu \varphi_1(t);$ \vdots $p'_r(t) = -\left(\lambda \sum_{i=1}^{n+m-r} a_i + r\mu \right) p_r(t) +$ $+ \lambda \sum_{k=0}^{r-1} a_{r-k} p_k(t) + (r+1)\mu \varphi_{r+1}(t), \text{ for } 1 \leq r \leq n-1;$ \vdots $p'_n(t) = -\left(\lambda \sum_{i=1}^m a_i + n\mu \right) p_n(t) +$ $+ \lambda \sum_{k=0}^{n-1} a_{n-k} p_k(t) + n\mu \varphi_{n+1}(t);$ \vdots $p'_{n+r}(t) = -\left(\lambda \sum_{i=1}^{m-r} a_i + n\mu \right) p_{n+r}(t) +$ $+ \lambda \sum_{k=0}^{n+r-1} a_{n+r-k} p_k(t) + n\mu \varphi_{n+r+1}(t), \text{ for } 1 \leq r < m;$ \vdots $p'_{n+m}(t) = -n\mu \varphi_{n+m}(t) + \lambda \sum_{k=0}^{n+m-1} a_{n+m-1} p_k(t);$	$p'_0(t) = -\lambda p_0(t) + \mu \varphi_1(t);$ \vdots $p'_r(t) = -(\lambda + r\mu) p_r(t) + \lambda \sum_{k=0}^{r-1} a_{r-k} p_k(t) +$ $+ (r+1)\mu \varphi_{r+1}(t), \text{ for } 1 \leq r \leq n-1;$ \vdots $p'_n(t) = (\lambda + n\mu) p_n(t) + \lambda \sum_{k=0}^{n-1} a_{n-k} p_k(t) + n\mu \varphi_{n+1}(t);$ \vdots $p'_{n+r}(t) = -(\lambda + n\mu) p_{n+r}(t) + \lambda \sum_{k=0}^{n+r-1} a_{n+r-k} p_k(t) +$ $+ n\mu \varphi_{n+r+1}(t), \text{ for } 1 \leq r < m;$ \vdots $p'_{n+m}(t) = -n\mu \varphi_{n+m}(t) + \lambda \sum_{k=0}^{n+m-2} \left(1 - \sum_{i=1}^{n+m-k-1} a_i \right) p_k(t);$
(1)	(2)

In the stationary state of work of the system ($\lambda = \text{const.}, \mu = \text{const.}, t \rightarrow \infty$) the systems of differential equations (1 and 2) should pass into the following system $(n+m+1)$ of the algebra equations (3 and 4):

Model system I	Model system II
$\begin{aligned} & \left(-\lambda \sum_{i=1}^{n+m} a_i\right) p_0 + \mu p_1 = 0; \\ & \vdots \\ & -\left(\lambda \sum_{i=1}^{n+m-r} a_i + r\mu\right) p_r + \\ & + \sum_{k=0}^{r-1} a_{r-k} p_k + (r+1)\mu p_{r+1} = 0, \text{ for } 1 \leq r \leq n-1; \\ & \vdots \\ & -\left(\lambda \sum_{i=1}^m a_i + n\mu\right) p_n + \lambda \sum_{k=0}^{n-1} a_{n-k} p_k + n\mu p_{n+1} = 0; \\ & \vdots \\ & -\left(\lambda \sum_{i=1}^{m-r} a_i + n\mu\right) p_{n+r} + \\ & + \lambda \sum_{k=0}^{n+r-1} a_{n+r-k} p_k + n\mu p_{n+r+1} = 0, \text{ for } 1 \leq r < m; \\ & \vdots \\ & -n\mu p_{n+m} + \lambda \sum_{k=0}^{n+m-1} a_{n+m-k} p_k = 0; \end{aligned}$	$\begin{aligned} & -\lambda p_0 + \mu p_1 = 0; \\ & \vdots \\ & -(\lambda + r\mu) p_r + \lambda \sum_{k=0}^{r-1} a_{r-k} p_k + \\ & + (r+1)\mu p_{r+1} = 0, \text{ for } 1 \leq r \leq n-1; \\ & \vdots \\ & -(\lambda + n\mu) p_n + \lambda \sum_{k=0}^{n-1} a_{n-k} p_k + n\mu p_{n+1} = 0; \\ & \vdots \\ & -(\lambda + n\mu) p_{n+r} + \lambda \sum_{k=0}^{n+r-1} a_{n+r-k} p_k + \\ & + n\mu p_{n+r+1} = 0, \text{ for } 1 \leq r < m; \\ & \vdots \\ & -n\mu p_{n+m} + \lambda \sum_{k=0}^{n+m-2} \left(1 - \sum_{i=1}^{n+m-k-1} a_i\right) p_k = 0; \end{aligned}$
(3)	(4)

4. Numerical example

Consider previous described models for terminal with two berths, i.e. ($n=2$) and with two spaces in the waiting queue, i.e. ($m=2$). Suppose that the random variable X has geometric probability distribution function, with $a_k=(1-a)a^{k-1}$, $k=1,2,\dots$; $0 < a < 1$. Then the system (3) becomes:

$$\begin{aligned} & -\lambda \cdot b_4 \cdot p_0 + \mu \cdot p_1 = 0 \\ & -(\lambda \cdot b_3 + \mu) \cdot p_1 + \lambda \cdot a_1 \cdot p_0 + 2 \cdot \mu \cdot p_2 = 0 \\ & -(\lambda \cdot b_2 + 2 \cdot \mu) \cdot p_2 + \lambda \cdot (a_2 \cdot p_0 + a_1 \cdot p_1) + 2 \cdot \mu \cdot p_3 = 0 \\ & -(\lambda \cdot b_1 + 2 \cdot \mu) \cdot p_3 + \lambda \cdot (a_3 \cdot p_0 + a_2 \cdot p_1 + a_1 \cdot p_2) + 2 \cdot \mu \cdot p_4 = 0 \\ & -2 \cdot \mu \cdot p_4 + \lambda \cdot (a_4 \cdot p_0 + a_3 \cdot p_1 + a_2 \cdot p_2 + a_1 \cdot p_3) = 0 \end{aligned} \tag{5}$$

The solutions of the systems (5), by setting $\lambda/\mu = \rho$ and $\sum_{i=1}^j a_i = b_j$ is as follows:

$$p_0 = \left\{ 1 + \left[\frac{1}{8} \rho^4 \cdot b_1 \cdot b_2 \cdot b_3 \cdot b_4 + \frac{1}{8} \rho^3 \cdot b_1 \cdot b_2 \cdot b_4 - \frac{1}{8} \rho^3 \cdot b_1 \cdot b_2 \cdot a_1 + \frac{1}{4} \rho^3 \cdot b_1 \cdot b_3 \cdot b_4 - \frac{1}{4} \rho^3 \cdot b_1 \cdot b_4 \cdot a_1 - \frac{1}{4} \rho^3 \cdot b_3 \cdot b_4 \cdot a_1 + \frac{1}{2} \rho^3 \cdot b_2 \cdot b_3 \cdot b_4 + \frac{3}{2} \rho^2 \cdot b_3 \cdot b_4 + \frac{1}{2} \rho^2 \cdot b_2 \cdot b_4 - \frac{1}{2} \rho^2 \cdot b_2 \cdot a_1 - \frac{5}{4} \rho^2 \cdot b_4 \cdot a_1 + \frac{1}{4} \rho^2 \cdot b_1 \cdot b_2 - \frac{1}{4} \rho^2 \cdot b_1 \cdot a_1 - \frac{1}{4} \rho^2 \cdot b_1 \cdot a_2 - \frac{1}{2} \rho^2 \cdot b_4 \cdot a_2 - \frac{1}{4} \rho^2 \cdot a_1^2 + \frac{5}{2} \rho \cdot b_4 - \frac{3}{2} \rho \cdot a_1 - \rho \cdot a_2 - \frac{1}{2} \rho \cdot a_3 \right] \right\}^{-1};$$

$$p_1 = \rho \cdot b_4 \cdot p_0;$$

$$p_2 = \frac{1}{2} \rho^2 \cdot b_3 \cdot b_4 \cdot p_0 + \frac{1}{2} \rho^2 \cdot b_4 \cdot p_0 - \frac{1}{2} \rho \cdot a_1 \cdot p_0;$$

$$p_3 = \frac{1}{4} \rho^3 \cdot b_2 \cdot b_3 \cdot b_4 \cdot p_0 + \frac{1}{4} \rho^2 \cdot b_2 \cdot b_4 \cdot p_0 - \frac{1}{4} \rho^2 \cdot b_2 \cdot a_1 \cdot p_0 + \frac{1}{2} \rho^2 \cdot b_3 \cdot b_4 \cdot p_0 + \frac{1}{2} \rho^2 \cdot b_3 \cdot p_0 + \frac{1}{2} \rho \cdot b_4 \cdot p_0 - \frac{1}{2} \rho \cdot a_1 \cdot p_0 - \frac{1}{2} \rho \cdot a_2 \cdot p_0 - \frac{1}{2} \rho^2 \cdot a_1 \cdot b_4 \cdot p_0;$$

$$p_4 = \frac{1}{8} \rho^4 \cdot b_1 \cdot b_2 \cdot b_3 \cdot b_4 \cdot p_0 + \frac{1}{8} \rho^3 \cdot b_1 \cdot b_2 \cdot b_4 \cdot p_0 - \frac{1}{8} \rho^3 \cdot b_1 \cdot b_2 \cdot a_1 \cdot p_0 + \frac{1}{4} \rho^3 \cdot b_1 \cdot b_3 \cdot b_4 \cdot p_0 - \frac{1}{4} \rho^3 \cdot b_1 \cdot b_4 \cdot a_1 \cdot p_0 + \frac{1}{4} \rho^3 \cdot b_2 \cdot b_3 \cdot b_4 \cdot p_0 - \frac{1}{4} \rho^3 \cdot b_3 \cdot b_4 \cdot a_1 \cdot p_0 + \frac{1}{4} \rho^2 \cdot b_1 \cdot b_4 \cdot p_0 - \frac{1}{4} \rho^2 \cdot b_1 \cdot a_1 \cdot p_0 - \frac{1}{4} \rho^2 \cdot b_1 \cdot a_2 \cdot p_0 + \frac{1}{4} \rho^2 \cdot b_2 \cdot b_4 \cdot p_0 - \frac{1}{4} \rho^2 \cdot b_2 \cdot a_1 \cdot p_0 - \frac{1}{4} \rho^2 \cdot b_4 \cdot a_1 \cdot p_0 - \frac{1}{4} \rho^2 \cdot a_1^2 \cdot p_0 + \frac{1}{2} \rho^2 \cdot b_3 \cdot b_4 \cdot p_0 - \frac{1}{2} \rho^2 \cdot b_4 \cdot a_1 \cdot p_0 - \frac{1}{2} \rho^2 \cdot b_4 \cdot a_1 \cdot p_0 + \frac{1}{2} \rho \cdot b_4 \cdot p_0 - \frac{1}{2} \rho \cdot a_1 \cdot p_0 - \frac{1}{2} \rho \cdot a_2 \cdot p_0 - \frac{1}{2} \rho \cdot a_3 \cdot p_0;$$

(6)

where $b_1 = a_1 = 1 - a$; $b_2 = a_1 + a_2 = 1 - a^2$; $b_3 = a_1 + a_2 + a_3 = 1 - a^3$; and $b_4 = a_1 + a_2 + a_3 + a_4 = 1 - a^4$.

Also, in stationary state of the work of the system, system (4) considered, with ($n = 2$ and $m = 2$) has the following form:

$$-\lambda \cdot p_0 + \mu \cdot p_1 = 0;$$

$$-(\lambda + \mu) \cdot p_1 + \lambda \cdot a_1 \cdot p_0 + 2 \cdot \mu \cdot p_2 = 0;$$

$$-(\lambda + 2 \cdot \mu) \cdot p_2 + \lambda \cdot (a_2 p_0 + a_1 p_1) + 2 \cdot \mu \cdot p_3 = 0;$$

$$-(\lambda + 2 \cdot \mu) \cdot p_3 + \lambda \cdot (a_3 p_0 + a_2 p_1 + a_1 \cdot p_2) + 2 \cdot \mu \cdot p_4 = 0;$$

$$-2 \cdot \mu \cdot p_4 + \lambda \cdot \sum_{k=0}^3 \left(1 - \sum_{i=1}^{3-k} a_i \right) \cdot p_k = 0;$$

(7)

The solution of the above system, by setting $\lambda/\mu = \rho$ and since X has geometric probability distribution ($a_k = (1-a)a^{k-1}$, $k=1,2,\dots$; $0 < a < 1$), has the form:

$$\begin{aligned}
 p_1^* &= \rho \cdot p_0^* \\
 p_2^* &= \left(\frac{1}{2} \rho^2 + \frac{1}{2} \rho - \frac{1}{2} \rho \cdot a_1 \right) \cdot p_0^* \\
 p_3^* &= \left(\frac{1}{4} \rho^3 + \frac{3}{4} \rho^2 - \frac{3}{4} \rho^2 \cdot a_1 + \frac{1}{2} \rho - \frac{1}{2} \rho \cdot a_1 - \frac{1}{2} \rho \cdot a_2 \right) \cdot p_0^* \\
 p_4^* &= \left[\frac{1}{8} \rho^4 + \frac{5}{8} \rho^3 - \frac{5}{8} \rho^3 \cdot a_1 + \rho^2 - \frac{5}{4} \rho^2 \cdot a_1 - \frac{3}{4} \rho^2 \cdot a_2 + \right. \\
 &\quad \left. + \frac{1}{4} \rho^2 \cdot a_1^2 + \frac{1}{2} \rho - \frac{1}{2} \rho \cdot a_1 - \frac{1}{2} \rho \cdot a_2 - \frac{1}{2} \rho \cdot a_3 \right] \cdot p_0^*
 \end{aligned} \tag{8}$$

and using the condition $\sum_{i=0}^4 p_i = 1$ we obtain

$$\begin{aligned}
 p_0^* &= \left\{ 1 + \left[\frac{1}{8} \rho^4 + \frac{7}{8} \rho^3 - \frac{5}{8} \rho^3 \cdot a_1 + \frac{9}{4} \rho^2 - 2 \cdot \rho^2 \cdot a_1 - \frac{3}{4} \rho \cdot a_2 + \right. \right. \\
 &\quad \left. \left. + \frac{1}{4} \rho^2 \cdot a_1^2 + \frac{5}{2} \rho - \frac{3}{2} \rho \cdot a_1 - \rho \cdot a_2 - \frac{1}{2} \rho \cdot a_3 \right] \right\}^{-1}
 \end{aligned} \tag{9}$$

4.1. Results of the comparative analyses of models I and II

The efficiency of operations and processes on anchorage-ship-berth link for models I and II has been comparatively analysed through the basic operating parameters such as: service probability (P_{ser} and P_{ser}^*), expected number of occupied berths (n_{oc} i n_{oc}^*), probability that berth is busy (P_{bus} , P_{bus}^*), cancellation probability (P_{can} and P_{can}^*), probability that all the berths are occupied (P_{ob} , P_{ob}^*), probability of existing of ships in queue (ships at anchor) (P_{eq} and P_{eq}^*), expected number of ships at anchor (N_w and N_w^*), expected time at anchor (t_w and t_w^* , in h), expected number of ships in the port (N_{ws} and N_{ws}^*) and expected time in the port (t_{ws} and t_{ws}^* , in h). The basic characteristics of the systems with the bulk arrival of units and the limited waiting queue are shown in the Table 1.

By using expressions for p_i , $i=4,3,2,1,0$, and p_i^* , $i=4,3,2,1,0$, determine all other characteristics of the system, i.e. operating terminal parameters. The solutions presented on the Figure 3 – Figure 20 are obtained by using MATLAB program.

Figure 3 – Figure 13 present the changes of operating parameters of anchorage-ship-berth link in the port for multiple-server in the function of ($\rho = 0-1$ and $a = 0,5$).

Figure 14 shows the changes of the probabilities of the states of the described systems depending on $\rho = 0-1$ and $a = 0,5$.

Figure 15 – Figure 20 show P_{ser} , P_{ser}^* , respectively (Figs. 15 and 16); P_{eq} , P_{eq}^* , respectively (Figs. 17 and 18) and N_{ws} , N_{ws}^* , respectively (Figs. 19 and 20) as the function of ρ ($\rho = 0 - 1$) and a ($a = 0.1 - 0.95$). This is presented in 3D graphs.

The results of the analysis show that the model I is more efficient than model II. From 2D and 3D graphical indicators one can conclude that all the displayed operating characteristics of model I are much better than those of model II.

Table 1. The Main Characteristics of System With Bulk Arrival of the Units and Limited Waiting Space

The System Characteristics (1)	Bulk Arrival of the Units to the Systems (2)
Probability of service - P_{ser}	$P_{ser} = \sum_{k=0}^{n+m-1} p_k$
Cancellation probability - P_{can}	$P_{can} = p_{n+m}$
Average number of occupied berths (servers) - n_{oc}	$n_{oc} = \sum_{k=0}^n k \cdot p_k + n \cdot \sum_{k=n+1}^{n+m} p_k$
Probability that berth is busy - P_{bus}	$P_{bus} = 1 - p_0$
Probability that all the berths are occupied - P_{ob}	$P_{ob} = \sum_{k=0}^m p_{n+k}$
Probability of existing of ships in queue (ships at anchor) - P_{eq}	$P_{eq} = \sum_{k=1}^m p_{n+k}$
Average number of ships at anchor - N_w	$N_w = \sum_{k=1}^m k \cdot p_{n+k}$
Average time which ship spends at a anchor - t_w	$t_w = \frac{N_w}{\lambda}$
Average number of ships in the port - N_{ws}	$N_{ws} = \sum_{k=1}^{n+m} k \cdot p_k$
Total average time which ship spends in the port (at anchor and on berth) - t_{ws}	$t_{ws} = \frac{N_{ws}}{\lambda}$

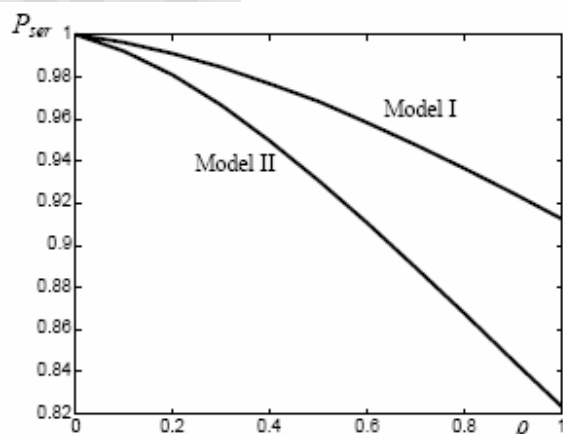


Fig. 3. Probability of service depending on ρ for model I and model II

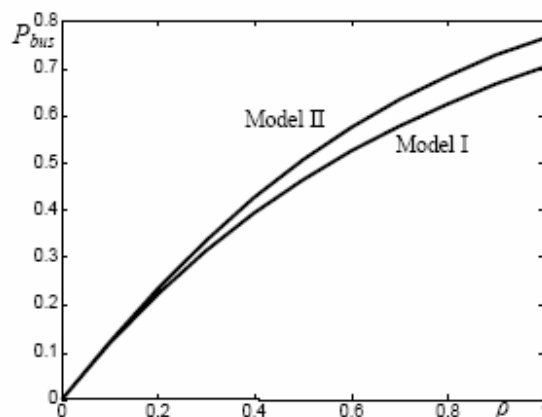


Fig. 6. Probability that berth is busy depending on ρ for model I and model II

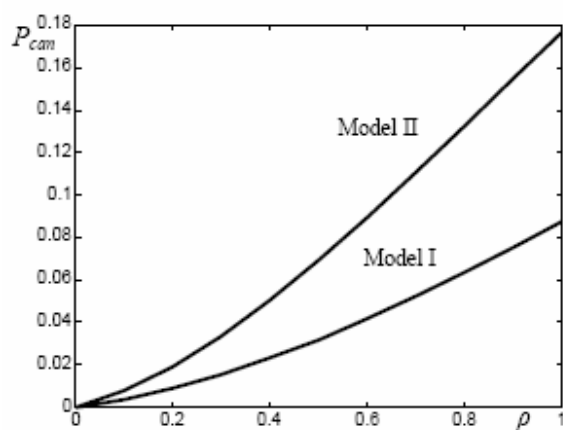


Fig. 4. Cancellation probability depending on ρ for model I and model II

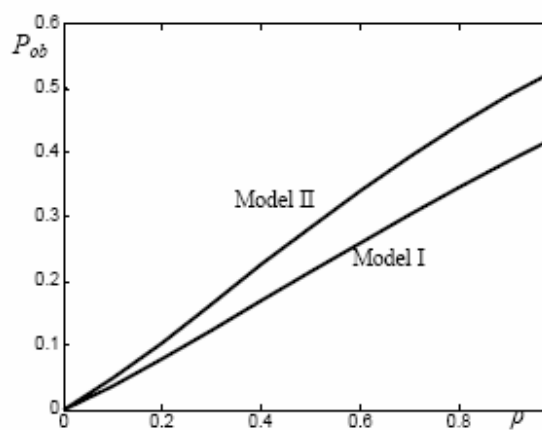


Fig. 7. Probability that all the berths are occupied depending on ρ for model I and model II

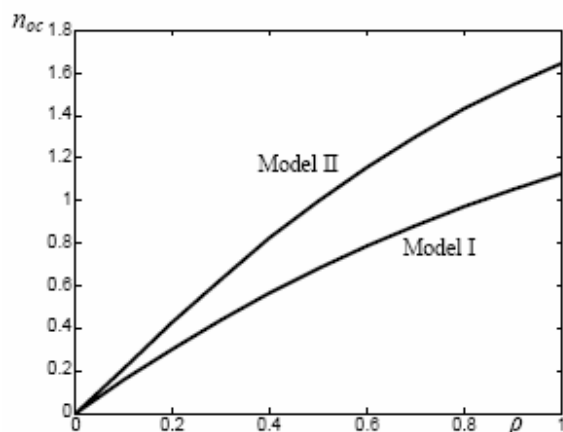


Fig. 5. Average number of occupied berths depending on ρ for model I and model II

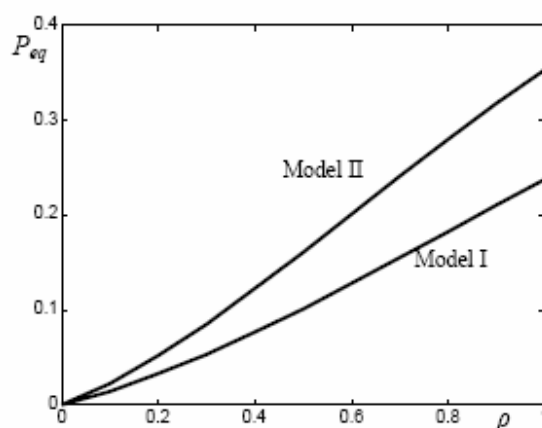


Fig. 8. Probability of existing of ships in queue depending on ρ for model I and model II

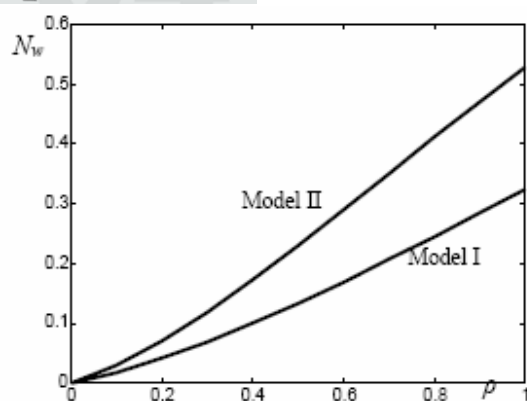


Fig. 9. Average number of ships at anchor depending on ρ for model I and model II

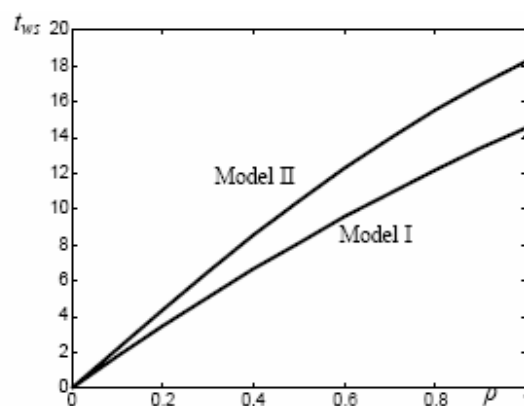


Fig. 12. Total average time which ship spends in the port depending on ρ for model I and model II

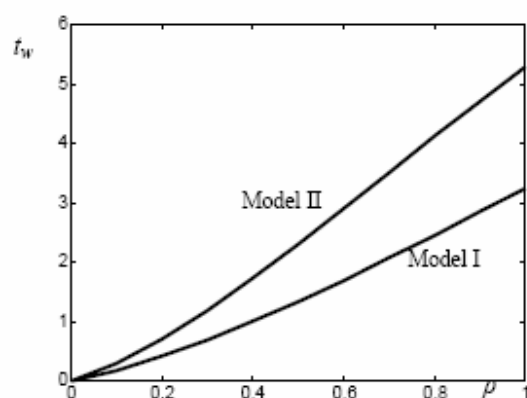


Fig. 10. Average time which ship spends at a anchor depending on ρ for model I and model II

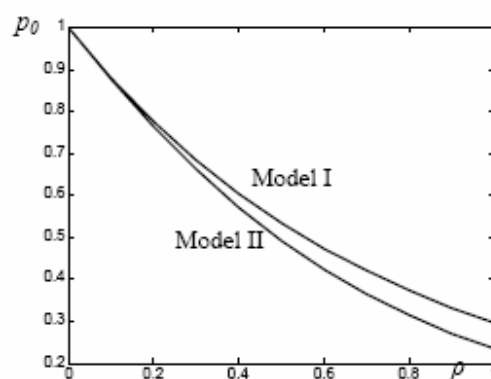


Fig. 13. Probability of having zero ships in the port depending on ρ for model I and model II

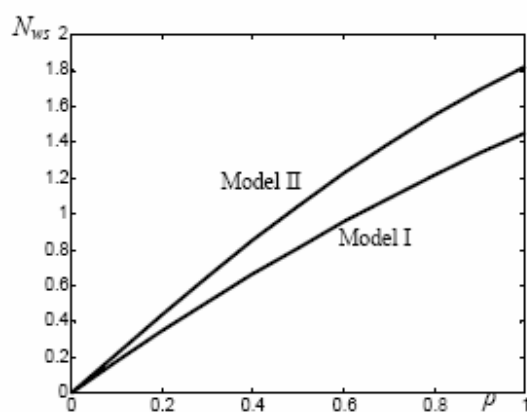


Fig. 11. Average number of ships in the port depending on ρ for model I and model II

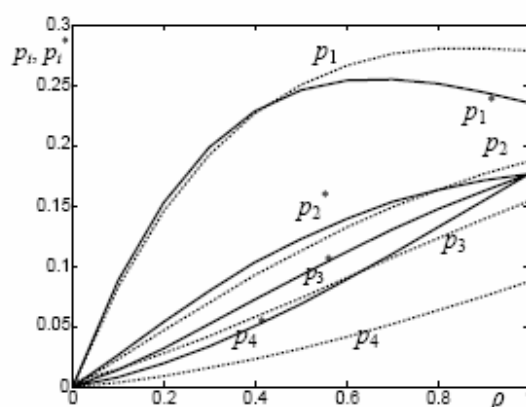


Fig. 14. Probability of states of systems depending on ρ for model I and model II

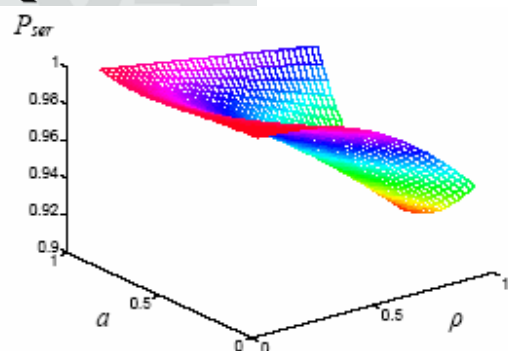


Fig. 15. Probability of service depending on a and ρ for model I

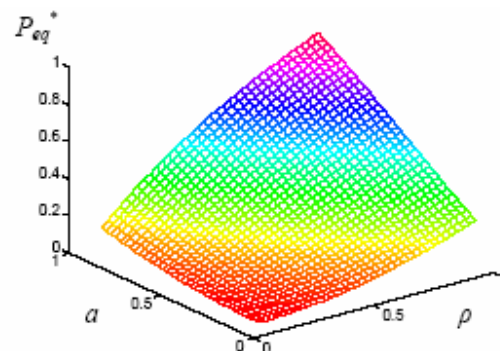


Fig. 18. Probability of existing of ships in queue depending on a and ρ for model II

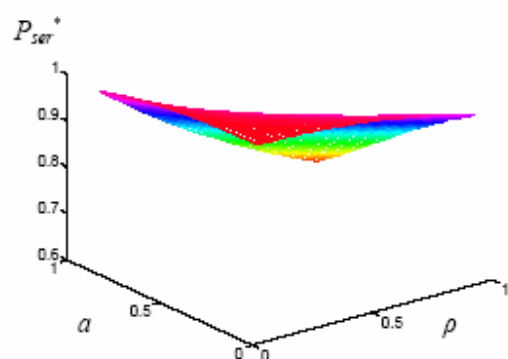


Fig. 16. Probability of service depending on a and ρ for model II

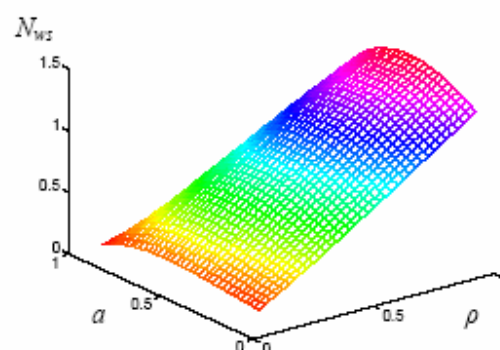


Fig. 19. Average number of ships in the port depending on a and ρ for model I

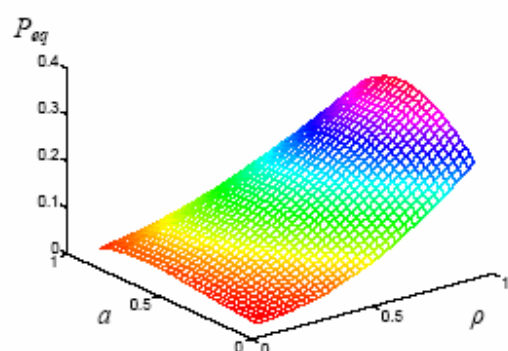


Fig. 17. Probability of existing of ships in queue depending on a and ρ for model I

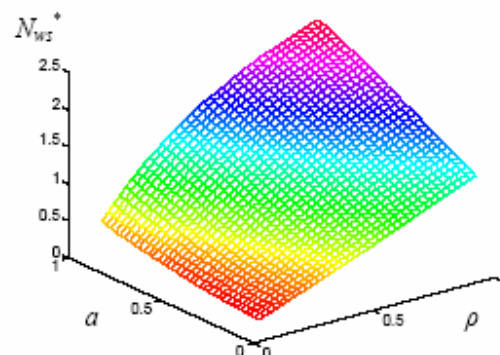


Fig. 20. Average number of ships in the port depending on a and ρ for model II

5. Conclusion

The analytical expressions obtained for the state probabilities of the stationary system bring forth useful optimizations of the operating parameters of the anchorage-ship-berth link. Through these systems, i.e. multiple-server queueing models, the determination of the optimal working order is possible. In the same way, the optimum number of berths, optimum berth efficiency and optimum berth capacity are directly conditioned by the values of the basic operating parameters in the port.

The results of the analysis show that the model I with the cancellation of the whole group is better in the view of the efficiency of the system than the model II with partial system completion. Considering 2D and 3D graphical indicators we can conclude that the values for ρ above 0.2 all the displayed working characteristics of model I are much better than those of model II.

The decision about whether to apply model I or model II from the aspect of terminal operators ought to be made in accordance with economical analysis.

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