wctr'04
4-8 july 2004 / istanbul-turkey

REGULATION IN THE AUTOMOBILE INDUSTRY

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Abstract

We developed a model for car ownership and replacement, and use it to assess the effects of two policies that have been widely used to regulate the automobile industry: scrap value for old cars and taxes on gasoline price. These two policies have been used either to simulate sales of new cars or/and to reduce emission. In this model both policies hardly achieve these objectives.

Keywords: Car ownership and replacement; Scrap value for old cars; Taxes on gasoline price
Topic area: A1 – Road and Railway Technology Development
Ownership of cars – replacement cycle length – depreciation in quality – tax on gasoline – scrapping of old cars.

1. Introduction

Despite the lack of models that explain how the automobile market works, regulation policies have been frequently used. The difficulty to model this market arises from at least two facts. First, the dynamic oligopoly structure of the competition in this market makes speculations about the future of the industry a hard task since the theory of dynamic oligopoly is at very primitive stages. Second, the durable good property of cars and the interference of new and used car markets make the analysis of pricing and ownership decisions another challenging task. On the other hand, the importance of the automobile industry is evident. In particular, the market size and the volume of emissions of cars and their impacts on the economic activity and the environment are so important, that the decision maker tries frequently to regulate the industry. The general idea, is that if people replace their cars frequently the market for new cars is active and the economic activity is stimulated in whole, given the important size of the automobile industry. In the same time, new cars make less emissions than older ones, and pollution should be lower when people replace frequently their cars.

In this paper, we develop a model and use it to focus on the effects of two policies. First, the tax on gasoline which is frequently increased in order to make people consume less gasoline and then make less pollution. We show that this policy has a negative effect on cars ownership. That is, the higher is the price of gasoline, the higher is the replacement time and so is the average age of cars. Since older cars pollute more, the effect of the tax is negative in this respect. Second, we consider the scrap value policy, that has been used in the last two decades by European governments in order to reduce the replacement cost of old cars. The idea behind this policy that if someone is ordered a higher price than the market price for his car than he will scrap his car to get the price difference as a gain. We notice that the subsidy increases the value of old cars. A consumer can then delay the replacement of his because its possible future price will not be reduced. Indeed, even if the government uses the policy for a limited period, consumers will have an uncertainty about its future use, and the value of old cars will increase. In that case pollution will be higher and the market of new cars will be harmed. The same point has been pointed out by (Adda and Cooper, 1997). Their analysis however, lies on a hypothetical econometric basis.

The positive effects of the two policies are not clear. If they have to be used, care should be taken on how they are to be implemented. In particular, scrap value policy should be implemented in such a way that no uncertainty is introduced[1]. For the taxes on gasoline, the gain from travel distance reduction should be higher than (i) the loss from the increase in average vintage of the cars and, (ii) the reduction in the sales of new cars.

The model we consider abstracts from the difficulties outlined above. Indeed, it focuses on the consumers decision making under two main assumptions: (i) we consider the case of price rigidity and, (ii) the market for used cars is supposed to be competitive. Both assumptions do not seem to be restrictive with respect to our objectives (they have been assumed by the related literature as well. See (Berry et al. 1995), (Breshanan, 1987) or (Adda and Cooper, 1997) for examples. Considering these assumptions, we derive an optimal replacement strategy for each type of consumer and, the demand function for new cars. Before considering the main policies we do some comparative statics relative to the key parameters of the model. Of particular interest, is the depreciation parameter of the car.

The paper is organized as follows. The model is described in Section 2. In Section 3, the consumer's demand function is derived. In Section 4 and Section 5 analyze respectively the taxes on gasoline and scrap value effects. The appendix contains some proofs.

2. The model

An agent of type θ has utility θq_t from using a car of quality qt at the instant t. The more general case of a utility with constant term will be discussed in Section 3.5. The quality of a new car is q0. Without loss of generality we assume it is equal to one when there is one quality, i. e. q0 = 1. Starting from date 0, if the agent replaces his car optimally by a new one on the basis of periods of length θ, then his lifetime utility is,

$$
\begin{aligned}
B(\tau) &= \int_0^{\tau} \theta\ e^{-\lambda t}\ e^{-\beta t}\, dt + \int_{\tau}^{2\tau} \theta\ e^{-\lambda (t-\tau)}\ e^{-\beta t}\, dt + \dots \\
&\quad \dots + \int_{i\tau}^{(i+1)\tau} \theta\ e^{-\lambda (t-i\tau)}\ e^{-\beta t}\, dt + \dots \qquad (1) \\
&= \frac{\theta}{\beta+\lambda}\left(1 + \frac{1-e^{-\lambda\tau}}{e^{\beta\tau}-1}\right)
\end{aligned}
$$

for $\beta, \lambda > 0$. This function is monotone decreasing $(dB(\tau)/d\tau < 0)$ from $B(0) = \theta/\beta$ to $B(\infty) = \theta/(\beta+\lambda)$. Then, $\theta/(\beta+\lambda)$ is the benefit of an agent that has a car and never replaces it. From the simplified expression it is clear that $dB(\tau)/d\theta > 0$, and from the first expression we see that $dB(\tau)/d\lambda < 0$ and $dB(\tau)/d\beta < 0$. The role of β is clearly a negative one. The depreciation parameter ρ has two effects on the benefit. First, the higher is the car's quality depreciation, the lower is the benefit we get from the car. Second, the higher is the car's depreciation, the higher is the benefit we get from replacement. The two effects of ρ are in the opposite directions but the negative one dominates.

[1] 1Putting a higher bound on the vintage of the cars that can be scrapped can be a solution.

The cost of the same policy is the sum of the price paid for the first car plus the (discounted) difference between the prices of a new car and an old one,

$$C(\tau) = p + (p - p_s)\sum_{i=1}^{\infty} e^{-\beta i \tau} = p + \frac{p - p_s}{e^{\beta\tau} - 1}. \quad (2)$$

When τ is close to zero, the cost from replacement $(C(\tau))$ is infinite and the benefit is maximal and equal to θ / β, at this level the agent is permanently using a new car. The lifetime utility of owning a car and replacing it after a period of length τ is $B(\tau) - C(\tau)$.

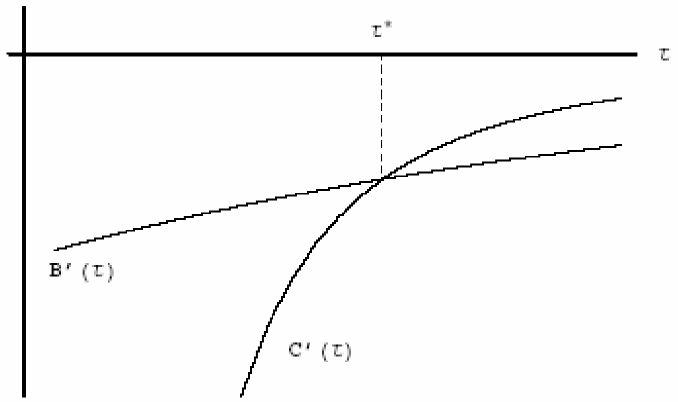

Figure 1: Optimal replacement time.

3. The consumer's decision

3.1. Existence and uniqueness of an optimal replacement time

The owner of the car seeks a replacement time cycle of length τ so that he maximizes his life time utility. We have, $C(0) = \infty > \theta/\beta = B(0)$. Increasing values of τ decreases both the cost of replacement (positive effect) and the benefit (negative effect), but the decrease in cost is higher, i.e. $|C'(\tau)| > |B'(\tau)|$. Optimal value for time of replacement is obtained by increasing gradually values of τ until gain from cost saving equals reduction in utility, so that we have,

$$B'(\tau^*) = C'(\tau^*). \quad (3a)$$

where τ^* is the optimal replacement time.

The two functions $B'(\tau)$ and $C'(\tau)$ are plotted on Figure 1 where the optimal value of τ corresponds to the intersection of the two curves. From (1) and (2), we have,

$$\frac{\theta}{\beta + \lambda} \frac{(\beta + \lambda)\, e^{\beta\tau^*} - \beta\, e^{(\beta+\lambda)\tau^*} - \lambda}{e^{\lambda\tau^*} \ \ (e^{\beta\tau^*} - 1)^2} = \frac{-\beta\,(p - p_s)\, e^{\beta\,\tau^*}}{(e^{\beta\,\tau^*} - 1)^2} \quad (3b)$$

The existence of a solution for this equation depends on parameters, θ, λ and β. The following lemma states the condition for existence and uniqueness of the optimal replacement time.

Lemma 1 An optimal replacement time exists and is unique if and only if,

$$\theta > (p - p_s)(\beta + \lambda). \qquad (4)$$

Proof. We wish to show that solution to (3) exists and is unique if and only if condition (4) holds. Notice that (3) could be written as,

$$\frac{\beta}{\lambda\, e^{-(\beta+\lambda)\tau} - (\beta+\lambda)\, e^{-\lambda\tau} + \beta} = \frac{\theta}{(p - p_s)(\beta + \lambda)}$$

after a little manipulation. Notice that given $\beta > 0$ and $\lambda > 0$, the left hand side of this equality LHS has a limit $\lim_{\tau \to 0^+} LHS = +\infty$ and $\lim_{\tau \to +\infty} LHS = 1$. Since LHS is a decreasing continuous function of τ (this could be checked by simple differentiation), it defines a one to one correspondence from $[0, \infty[$ onto $[1, \infty[$. Since the RHS is constant in θ, if RHS > 1 we have the result, i. e. iff (4) holds.

To understand the condition of Lemma 1, notice that in the expression of the benefit (1), we can distinguish two parts: $\theta / (\beta + \lambda)$ which is the benefit from owning a car without replacing it forever, i. e. $B(\infty) = \theta/(\beta + \lambda)$ and the benefit from replacement

$$\frac{\theta}{\beta + \lambda} \frac{1 - e^{-\lambda\tau}}{e^{\beta\tau} - 1}.$$

So B is the sum of the gain from owning a car and the gain from replacement frequency. But the replacement cost is $(p - p_s)/(e^{\beta\tau} - 1)$. Only if the condition (4) is satisfied, the marginal cost $C'(\tau)$ and marginal benefit $B'(\tau)$ curves will intersect[2].

The expression of the benefit from replacement is also useful to describe the role of parameters β and λ. Indeed, when λ is relatively high the replacement policy improves the benefit, since the gain from replacement is important. But, if λ is relatively low, the replacement has less effects on the benefit.

From here on, we refer to the lower bound as $\theta_{\inf}$, i. e. $\theta_{\inf} = (p - p_s)(\beta + \lambda)$.

Remark 1 For $\theta_{\inf}$ the benefit from owning a car is,

$$B(\tau) - C(\tau)|_{\theta=\theta_{\inf}} = \frac{p_s\,(1 + e^{\lambda\tau} - e^{(\beta+\lambda)\tau}) - p}{(e^{\beta\tau} - 1)\, e^{-\lambda\tau}} < 0$$

so the type $\theta_{\inf}$ does not buy a car. This result can be altered if we add a sufficiently high constant to the utility as in Section 3.5. In such a case an agent that has a car does not replace it and we have $\tau^* = \infty$, for any type $\theta \leq \underline{\theta}$.

We will use τ to refer at the replacement time as well as to its optimal value (instead of τ^*). It will be clear from the context, which variable is referred to.

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[2] Because the benefit will exceed the cost if,

$$\frac{\theta}{\beta + \lambda} \frac{1 - e^{-\lambda\tau}}{e^{\beta\tau} - 1} > \frac{p - p_s}{e^{\beta\tau} - 1}$$

which can be written,

$$\frac{\theta}{(\beta + \lambda)(p - p_s)} > \frac{1}{1 - e^{-\lambda\tau}} > 1.$$

3.2. Comparative statics for the optimal replacement time

We are interested at the role of key parameters θ, p, β and λ. The following lemma states a result for p, β and θ.

Lemma 2 Given that condition (4) is satisfied, we have

$$\frac{d\tau}{dp} = -\frac{d\tau}{dp_s} > 0\ , \qquad \frac{d\tau}{d\theta} < 0 \ \ and \qquad \frac{d\tau}{d\beta} > 0$$

at the optimal level of replacement τ.

Proof. Condition (4) insures that τ is finite and satisfies equation (3). For the sensitivity of the result to p, we totally differentiate (3) by p and τ and simplify to get,

$$\frac{d\tau}{dp} = \frac{1}{(1-e^{-\lambda\tau})\,\theta - (p-p_s)(\lambda+\beta)}. \qquad (5a)$$

using again (3) to substitute for p we find after simplification,

$$\frac{d\tau}{dp} = \frac{\beta\, e^{(\beta+\lambda)\,\tau}}{\theta\,\lambda\,(e^{\beta\tau}-1)} > 0. \qquad (5b)$$

Following the same procedure and replacing p by θ we find,

$$\frac{d\tau}{d\theta} = -\frac{e^{(\beta+\lambda)\tau} + \lambda - (\beta+\lambda)\,e^{\beta\tau}}{(\beta+\lambda)\,\lambda\,\theta\,(e^{\beta\tau}-1)}. \qquad (6)$$

The sign of this quantity depends on the numerator. Notice that for $\tau = 0$ the numerator vanishes and for $\tau > 0$ the derivative of the numerator is $\beta(\beta+\lambda)\theta\,(e^{\beta\tau} - 1) > 0$, then $d\tau/d\theta < 0$.

Finally, for β we have,

$$\frac{d\tau}{d\beta} = \frac{\beta^2\, e^{(\beta+\lambda)\tau} - e^{\beta\tau}(\beta+\lambda)^2 + \lambda(\lambda+\beta^2\,\tau+\beta\,(2+\lambda\,\tau)}{\beta\,\lambda\,(\beta+\lambda)^2\,(e^{\beta\,\tau}-1)} \qquad (7)$$

and the sign of this derivative is the sign of the numerator. The last vanishes for $\tau = 0$ and has a derivative $\beta\,(\beta+\lambda)(\beta\, e^{(\beta+\lambda)\tau} + \lambda - (\beta+\lambda)\,e^{\beta\tau}) > 0$ and so $d\tau/d\beta > 0.$

The result of this lemma is clear from Figure (1). From (3), an increase of values of p has as effects only an increase on the cost of replacement. So the curve $C'(\tau)$ in Figure 1, moves upward and the intersection with the curve $B'(\tau)$ is obtained at a higher value.

The role of θ is similar but has the opposite effect. Increasing θ, keeps the curve $C'(\tau)$ unchanged and increases the benefit from using the same quality and so moves the curve $B'(\tau)$ upwards. Then, the two curves intersect for a lower τ.

Remark 2 From the proof of Lemma 1 it is possible to put a lower bound on $d\tau/dp$ and then on τ(p). Indeed[3], From (5b),

$$\frac{d\tau}{dp} = \frac{\beta\, e^{(\beta+\lambda)\,\tau}}{\theta\,\lambda\,(e^{\beta\tau}-1)} \geq \frac{1}{\theta}\left(\frac{\beta+\lambda}{\lambda}\right)^{\frac{\beta+\lambda}{\beta}}$$

and so,

[3] Since $e^{(\beta+\lambda)\,\tau}/(e^{\beta\tau}-1)$ has a minimum in $(1/\beta)\ln[(\beta+\lambda)/\lambda]$. Substituting this τ in dτ /dp and simplifying yields the result.

$$\tau(p) \geq \frac{p - p_s}{\theta}\left(\frac{\beta+\lambda}{\lambda}\right)^{\frac{\beta+\lambda}{\beta}}$$

since at p = ps the optimal replacement time is zero. This lower bound is decreasing in θ. It is also increasing in the ratio β/λ.

Notice that dτ/dp is decreasing in θ: agents with high demand reduces less the replacement time when the price increases. For the third parameter λ the sign of the derivative changes from a negative value for low τ to positive one when τ is high. Using the same approach in the proof of Lemma 2, we obtain for λ

$$\frac{d\,\tau}{d\,\lambda} = \frac{\beta\,\left[e^{(\beta+\lambda)\tau} + \lambda\,\tau(1 - 2\,e^{\beta\tau}) - 1\right] - \beta^2\,\tau\,e^{\beta\tau} + \lambda^2\,\tau\,(e^{\beta\tau} - 1)}{\lambda\,(\beta+\lambda)^2\,(e^{\beta\tau} - 1)} \qquad (8)$$

so the sign of the derivative is the opposite of the sign of the numerator in (8). When $\tau = 0$ the numerator is zero, and we have $\lim_{\tau\to 0}(d\,\tau/d\,\lambda) = -1/(2\,\lambda) < 0$. But notice also that $\lim_{\tau\to\infty}(d\,\tau/d\,\lambda) = \infty$. These details with others, put in the next proof, yield the following result.

Lemma 3 There is a λ^c such that dτ/dp < 0 if $\lambda < \lambda^c$ and dτ/dp > 0 if $\lambda > \lambda^c$.

Proof. We have,

$$\frac{d\,\tau}{d\,\lambda} = \frac{\beta\,\left[e^{(\beta+\lambda)\tau} + \lambda\,\tau(1 - 2\,e^{\beta\tau}) - 1\right] - \beta^2\,\tau\,e^{\beta\tau} + \lambda^2\,\tau\,(e^{\beta\tau} - 1)}{\lambda\,(\beta+\lambda)^2\,(e^{\beta\tau} - 1)}$$

so the sign depends on the numerator. Call this numerator u(τ). We have,

$$u'(\tau) = (\beta+\lambda)\,\left(\beta\,e^{(\beta+\lambda)\tau} - (\beta+\lambda)(1+\beta\,\tau)\,e^{\beta\tau} + \lambda\right)$$

and

$$u''(\tau) = \beta\,(\beta+\lambda)\,(e^{\lambda\tau} - \beta\,\tau - 2)\,e^{\beta\tau}.$$

This second order derivative satisfies,

$$u''(\tau) < 0 \text{ for } \tau < \tau^{\lambda,\beta} \text{ and } u''(\tau) > 0 \text{ for } \tau > \tau^{\lambda,\beta},$$

where $\tau^{\lambda,\beta}$ is a positive number. So $u'(\tau)$ will be negative decreasing for $\tau < \tau^{\lambda,\beta}$ and increasing for $\tau > \tau^{\lambda,\beta}$. These observations combined with the limits of dτ/dλ at 0 and the infinity, yields the result .

Indeed, for λ = 0, there is no depreciation and the optimal replacement is τ = ∞. And from Lemma 1, for $\lambda = \theta/(p - p_s) - \beta$ the optimal replacement time is also τ = ∞. Then, the replacement time is decreasing for low values of λ and increasing for higher values.

3.3. The lowest type that buys

Consider an agent with type θ. From Lemma 1, there is an optimal replacement time denoted $\tau(\theta)$. The value of a new car with optimal replacement policy for the agent is,

$$V(\theta, \tau(\theta)) = B(\theta, \tau(\theta)) - C(\tau(\theta)) \qquad (9)$$

The agent buys a car only if V in 9 is positive: $V(\tau(\theta)) \geq 0$. Call $\underline{\theta}$ the lowest θ for which the agent buys a car. Then $\underline{\theta}$ is a solution to,

$$\theta = \frac{e^{\lambda\,\tau(\theta)}\left(e^{\beta\,\tau(\theta)}\,p - p_s\right)(\lambda+\beta)}{e^{(\beta+\lambda)\,\tau(\theta)} - 1} \qquad (10)$$

the solution to this equation could be found using fixed point iteration to the right hand side of (10). In Figure 2, we plot the replacement time for each agent and show whether he buys or not, for different prices. Dashed curves show the replacement time for an agent that doesn't buy.

For the comparative statics of $\underline{\theta}$ we will need the result of the following lemma.

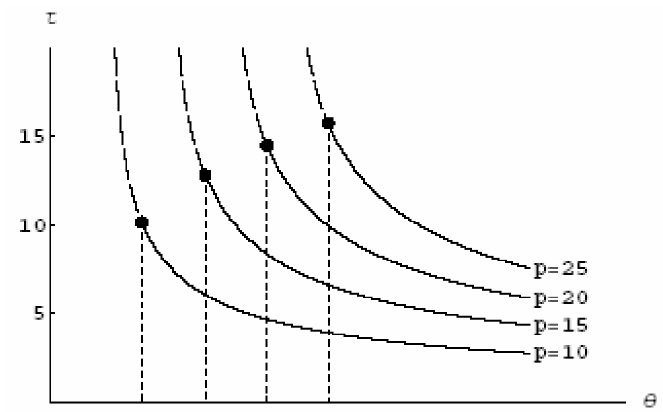

Figure 2: Replacement time as a function of price

Lemma 4 If a type θ buys a car, then $\theta > \beta\, p_s$.

Proof. For a type $\theta = \beta\, p_s$, the value of owning a car is,

$$\frac{-p_s\,\beta + p_s\,(\beta+\lambda)\,e^{\lambda\tau} + e^{(\beta+\lambda)\tau}\,(p_s\,\beta - p\,(\beta+\lambda))}{(\beta+\lambda)\,e^{-\lambda\tau}\,(e^{\beta\tau}-1)}$$

and has the sign of the numerator. The last is

$-(p-p_s)\,(\beta+\lambda) < 0$

and its derivative by τ is,

$(\beta+\lambda)\left[(p_s\,e^{\lambda\tau} - p\,e^{(\beta+\lambda)\tau})\lambda + (p_s - p)\,\beta\,e^{(\beta+\lambda)\tau}\right] < 0$

and such type never buys a car.

Of course, as p increases, $\underline{\theta}$ increases and the replacement curve moves upright. With higher price the replacement time is higher and less agents buy a car.

The comparative statics for $\underline{\theta}$ is summarized in the following lemma.

Lemma 5 We have:

$$\frac{d\,\underline{\theta}}{d\,p} > \beta+\lambda, \qquad \frac{d\,\underline{\theta}}{d\,p_s} < 0, \qquad \frac{\underline{\theta}}{\beta+\lambda} < \frac{d\,\underline{\theta}}{d\,\beta} \quad and \quad 0 < \frac{d\,\underline{\theta}}{d\,\lambda} < \frac{\underline{\theta}}{\beta+\lambda}.$$

Proof. See Appendix A.1.

Remark 3 The utility an agent of type θ gets from having a car and replacing it optimally at $\tau(\theta)$ is

$V(\theta,\tau(\theta)) = B(\theta,\tau(\theta)) - C(\tau(\theta))$

so by the envelope theorem we have,

$dV/d\theta = \partial V/\partial\theta = \partial B/\partial\theta$

and then, by substitution from the expression of the benefit, The gain of a type θ is,

$$V(\theta) = \int_{\underline{\theta}}^{\theta} \frac{1}{\beta+\lambda}\left(1+\frac{1-e^{-\lambda\tau(x)}}{e^{\beta\tau(x)}-1}\right)dx$$

since $V(\underline{\theta}) = 0$.

3.4. The demand function of an agent

In this section we describe the reaction function (or the demand function) of the consumer given a price set by the firm. We have to determine: Given a price p set by the firm, whether the type θ buys a car not and how frequently he will replace it (τ).

Since most of the equations are difficult to solve for τ, we will write inverse demand functions. This will help to construct a graphical illustration to the single firm problem.

First, solving equation (10) for p, we have a bound on the price for the consumer who buys,

$$p \leq e^{-\beta\tau} p_s + \theta \frac{1-e^{-(\beta+\lambda)\tau}}{\beta+\lambda} \qquad (11)$$

given a price p and the optimal τ, if the right hand side of (11) is higher than the left hand side then the agent does not buy a car, i.e. optimal response of the agent to the price of the firm doesn't give him a positive surplus. So (11) defines an upper bound on the price the firm can set so that the agent buys a new car.

Second, solving (3) for p, we have

$$p = p_s + \theta \frac{\beta + \lambda e^{-(\beta+\lambda)\tau} - (\beta+\lambda)e^{-\lambda\tau}}{\beta(\beta+\lambda)} \qquad (12)$$

given a price p set by the firm, the solution to (12) in τ is the optimal replacement time of the consumer. Given a consumer of type θ, If a couple (τ (θ, p), p) solves (12) and satisfies (11), then this consumer buys a car and replaces at regular periods of length τ. If there is no such couple, then the consumer does not buy a car. The equality between the right hand sides of (11) and (12) yields the greatest value of τ given p for the agent who buys,

$$\overline{\tau} = \frac{1}{\lambda}\ln\left(\frac{\theta}{\beta p_s}\right) \qquad (13)$$

Substituting (13) in (12), we get the highest price for which the agent buys,

$$\overline{p} = \frac{\theta}{\beta+\lambda} + \frac{\lambda p_s}{\beta+\lambda}\left(\frac{\theta}{\beta p_s}\right)^{-\beta/\lambda} \qquad (14)$$

Notice that $\overline{\tau}$ is the point where the curve defined by (11) reaches its maximum level. It is increasing for $\tau < \overline{\tau}$ and decreasing for $\tau < \overline{\tau}$. Also, both functions intersect at $\tau = 0$. So, for $\tau < \overline{\tau}$ the reaction function is below the boundary curve for buying and for $\tau > \overline{\tau}$, the reaction function goes above. These points will be graphically illustrated in the next section.

From (13) we obtain $d\overline{\tau}/d\beta = d\overline{\tau}/dp_s < 0$ and $d\overline{\tau}/d\lambda < 0$. Since the reaction function of the consumer is increasing it follows that p varies in the same direction for these parameters.

There is three types. For $\theta < \theta_{\inf}$ the agent has no optimal replacement time. This type does not also buy a car. For $\theta_{\inf} < \theta < \underline{\theta}$ the agent has a unique optimal replacement time but does not buy a car. Finally, for $\theta > \underline{\theta}$ the consumer buys a car and has a unique replacement time. The $\overline{\tau}$ on the y-axis is the largest possible replacement time. For an agent that buys.

Notice also that we have always $\theta_{\inf} < \underline{\theta}$. This follows from the benefit in (1). This structure depends on the utility function we considered. Indeed if we add a constant term to this utility we can move $\underline{\theta}$ to the right and the ordering $\theta_{\inf} < \underline{\theta}$ can no longer hold. We comment on this point on the next subsection.

3.5. The utility with a constant term

The utility of the agent is $u(q) = A + \theta\, q$. With this utility function it is possible that all types, even $\theta = 0$, buy a car. As before, we assume that the quality of a new car is q0 = 1.

The benefit from owning a car for a periods of length τ is,

$$B(\tau) = \frac{A}{\beta} + \frac{\theta}{\beta + \lambda}\left(1 + \frac{1 - e^{-\lambda\tau}}{e^{\beta\tau} - 1}\right)$$

and the cost of replacement remains as before. Now the benefit is bounded above by (when τ vanishes),

$$\frac{A + \theta}{\beta}.$$

Finite replacement time requires the same condition as in Lemma 1, since the term with the constant has no effect on the replacement time.

The bound on the largest replacement time is now,

$$\overline{\tau} = \frac{1}{\lambda}\ln\left(\frac{\theta}{\beta\, p_s - A}\right)$$

and is defined as a real number only if $A < \beta\, p_s$. If this condition is not satisfied, then $\overline{\tau} = \infty$. Indeed, if it is not the case the bound on the value function of the consumer is increasing and has no intersection point with the reaction function of the consumer.

4. The demand for new cars as a function of gasoline price

An agent consumes a transportation good, which can be either public or private (vehicle). The utility he gets from the public transportation is fixed and equal to $\overline{u}$. The utility he gets from private transportation is the same as before except for the benefit which we assume now depending on the travel distance δ,

$$B(\tau) = \frac{\theta\, \delta^{\alpha}}{\beta + \lambda}\left(1 + \frac{1 - e^{-\lambda\tau}}{e^{\beta\tau} - 1}\right)$$

where $\alpha > 0$. Notice that *d* enters the utility in the same way as θ, so $d\tau/d\delta$<0. i.e. the more intensively the car is used, the more frequently it is replaced. The remaining lifetime revenue of the agent allows him to consume another good from which he gets utility $y + v(p^G)$, where p^G is the gasoline price and $v(p^G)$ is the indirect utility function as a function of p^G (assume v'(.) <0 and v''(.) >0).

The agent chooses the private transportation solution if he gets higher utility from it, and vice versa. Consider a type θ that consumes a private transportation good (a car). His lifetime utility can be written,

$$U = y + v(p^G) + B(\tau) - C(\tau) \qquad (15)$$

But, we have $g = \delta.\rho$: quantity of gasoline g is equal to distance δ by consumption rate ρ. Using Roy's identity and the envelope theorem, we obtain the quantity of gasoline demanded for the agent,

$$g = -v'(p^G)$$

so,

$$\delta = -v'(p^G)/\rho$$

and substituting for δ in (15), the utility can now be written,

$$U = y + v(p^G) + \left(\frac{-v'(p^G)}{\rho}\right)^\alpha \frac{\theta}{\beta+\lambda}\left(1 + \frac{1-e^{-\lambda\tau}}{e^{\beta\tau}-1}\right) - C(\tau)$$

Suppose that p^G increases, due to an increase in taxes, for example. There are two effects on the consumer's behavior,

1. An increase in the average replacement time, since $dv'(p^G)/dp^G > 0$ and so $d = \left(-v'(p^G)/\rho\right)^\alpha$ will decrease, and
2. Since $dv(p^G)/dp^G < 0$ the consumer that was indifferent between the private and public transportation will prefer the public transportation. It can be shown that if we allow for different cars qualities that are vertically differentiated, this effect induces a switch from high to low qualities.

We summarize these results in the following proposition.

Proposition 1 Given a utility described by (15), a price increase of p^G has two effects: (a) the optimal replacement time τ increases for all types and, (b) $\underline{\theta}$ decreases.

The first effect is clearly negative, since older cars pollute more. The second effect however is not clearly negative or positive, since lower quality cars can be either more or less polluting. But, what is clear is that the demand for new cars will decrease.

The effect of this tax is described in Figure 3. A continuum of consumers is distributed over the x-axis $(0, \overline{\theta})$. The density of this distribution is f . The demand for new cars is indicated by the shaded area $f(\theta)/\tau(\theta)$. If the price of gasoline increases, the optimal replacement time τ increases for all types and then, the curve $f(\theta)/\tau(\theta)$ moves downward. Also, $\underline{\theta}$ increases. As a result the demand for new cars (the shaded area) is reduced.

So a policy that aims to reduce pollution through a gasoline taxation is not necessarily successful to reach its target, and is harmful for the market of new cars. The first reason is that the direct effect of reducing the usage of cars will indeed reduce the travel distance for each agent, but will increase the replacement time, and so the average age of the car fleet will be higher. The second reason is that consumers will switch to other alternatives and the demand for new car will decrease, harming the producers of new cars.

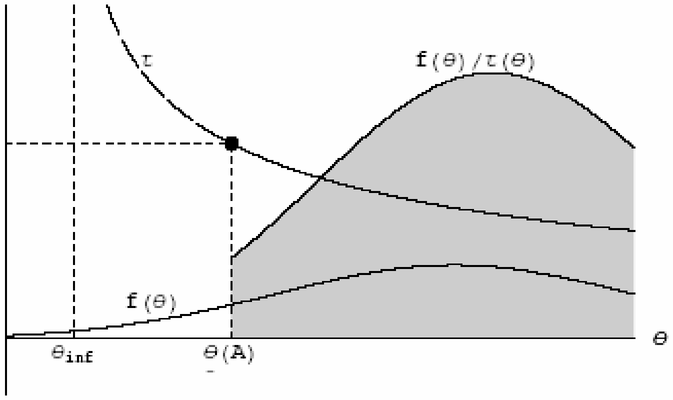

Figure 3: Agents' decision as a function of type

This policy is dominated by one that targets directly the replacement of the oldest cars[4] . In the following section we turn to such a policy, and highlight the difficulty to conduct a successful result.

5. The scrap value policy

The use of scrap value is usually justified for at least two reasons. The first one is that it stimulates the automobile industry in times of recession. Second, by the renewal of old cars, the average age is lower and then pollution should also be lower. In this discussion, we will notice that it is not guaranteed to achieve the first objective (which can put difficulties to achieve the second objective too).

5.1. Effects of a scrap value

We assume that the market for used cars is competitive. Each agent that has a car aged a can sell it at a price $p_s(a) = k\ e^{-\lambda a}$ $(0 < k < 1)$. We restrict our discussion to the type of agents that buy only new cars and each period makes a decision about either to keep the car or sell it in the second hand market. The Bellman equation for this problem is,

$$V(a) = \max \left\{ \begin{array}{c} u(a,\lambda) + \beta V(a) \\ u(1,\lambda) + p_s(a) - p + \beta\, V(2) \end{array} \right\} \qquad (16)$$

which is similar to (16), except for $p_s(a)$. To guarantee a finite replacement time, assume that there is a maximum age n after which a car must be sold (we can force this by letting $u(n,\lambda) = -\infty$ for example). This problem has a stationary solution that implies a replacement of any car which age is higher than n^*. At the optimum, the value of each car is decreasing with age. It is interesting to notice also that the replacement occurs at n^* because $V(n^*+1)$ is low. This decrease in the value function is the result of two factors: (1) the car's quality depreciates at a rate λ and so will be the utility of having such a car, and (2) because the reselling price $p_s(n^* + 1)$ is low. Notice that an increase in either the quality or the reselling price of a car of age $n^* + 1$ will increase its value.

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[4] All over the world, fifty per cent of the pollution is generated by ten per cent of the oldest cars.

Now, suppose that the government introduces a scrap value to be applicable for any car that is aged above a given number of years. Assume also that this scrap value is relatively low. The consumer's problem will be,

$$V(a) = \max \left\{ \begin{array}{c} u(a,\lambda) + \beta V(a+1) \\ u(1,\lambda) + p_s(a) - p + \beta V(2) \\ u(1,\lambda) + s - p + \beta V(2) \end{array} \right\} \qquad (17)$$

because the consumer will have a third opportunity: to scrap his car and obtain s. But since $p_s(a)$ decrease and s is fixed, this will be as if the reselling price of used cars is always greater or equal to s. This will rise the value of old cars. Consequently some consumers will choose to keep their cars instead of replacing. The presence of a scrap value rises the replacement time because it increases the first line of (17). This policy doesn't reach the objective of reducing the replacement time. Notice however that we are comparing the case where there is no scrap value with the case where there is a relatively small scrap value. A relatively high scrap value will indeed reduce the replacement time, but requires high government expenditures.

Suppose now that the government introduces for one period a scrap value to be applicable for some age. The value function will not be altered by this policy since after the period has elapsed it always be will be obtained from (16). Then, for this period each agent will compare the keep decision with the maximum of scrap and sell decisions, and then choose the alternative that gives him the maximum gain. The only effect is that some consumers[5] will find it profitable to scrap their cars. And then during that period the number of replacement should be higher. For the later periods the number of replacements will be lower, since old cars have been scrapped. The overall effect will however be positive.

Lemma 6 In the presence of scrap value the lowest type that buys is lower than in the case of competitive second hand market, i.e. the number of consumers that buy a car is higher.

Proof. In the presence of scrap value, type $p(\beta + \lambda)$ has utility,

$$\frac{s - p e^{-\lambda \tau}}{e^{\beta \tau} - 1}$$

which is positive for sufficiently high τ (i.e. for $\tau \geq 1/\lambda \;\ln(p/s)$). By the continuity of the utility on θ, the result follows.

Notice however that this does not mean that the demand for cars is higher since this depends on the replacement cycles too.

Proposition 2 Let $\tau^*(\theta)$ be the optimal replacement time for a type θ. The reselling price of a car aged $\tau^*(\theta)$ is $p_s(\tau^*(\theta))$. Suppose the government offers a scrap value s, such that $p > s > p_s(\tau^*(\theta))$, for any car aged more than $\underline{\tau}$. Assume $\underline{\tau} < \tau^*(\theta)$ and denote by $\tau^s(\theta)$ the optimal replacement time in the presence of the scrap value. Then we have two possibilities: (a) $\tau^s(\theta) > \tau^*(\theta)$, or (b) $(b)\ \tau^s(\theta) < \tau^*(\theta)$.

Proof. In appendix A.2.

The problem with this policy is that once used one time it will introduce some uncertainty in the consumers' expectation. In the future they will put some positive probability on the use of a scrap value. The value of old cars will then increase dependently on the expectation of the agents. The outcome in this case will be an immediate increase in sales for the period scrap value is

[5] Those who have age a such that,
$u(1,\lambda) + s - p + \beta V(2) > u(a,\lambda) + \beta V(a) > u(a,\lambda) + \beta V(a+1) > u(1,\lambda) + p_s(a) - p + \beta V(2)$

introduced, but in the long term, uncertainty introduced by the scrap value will increase the replacement time. A negative effect on government revenue will follow.

Consider the following example. A type θ = .35 has to decide first of his replacement policy where there is no scrap value: s = 0 (assume parameters ∂ = .05,ρ= .1,k = .9,p = 20,n = 10,q0 = 10). This implies a stationary situation where he buys a car and replaces it each 11 years. Now there is a scrap value s = 7 that is applied for cars older than 10 years. The subsidy is higher than the reselling price in this case. The utility he gets from a car 10 years old is higher, but the optimal replacement time increases to 13 years. Figure 4 shows two curves: The bold one reflects the effect of the subsidy. It shows that the utility of the agent increases for all vintages higher than 10 years, but nothing guarantees the maximum will be reached at that vintage.

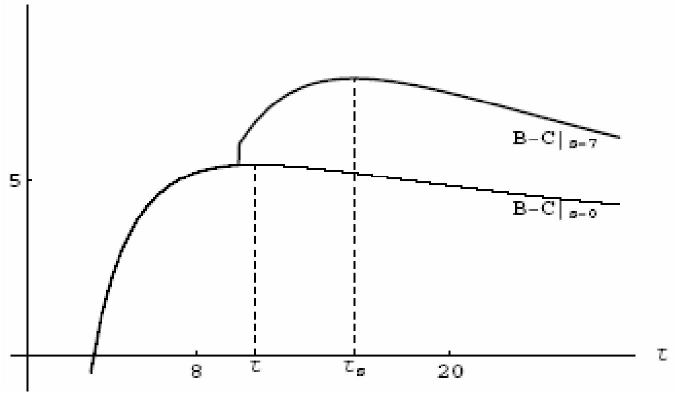

Figure 4: The effect of a subsidy on the agent's utility

5.2. Related literature

The impacts of this policy have been addressed by (Adda and Cooper, 1997) in order to evaluate its effects on the automobile industry.

The paper of (Adda and cooper,1997) is mainly an empirical investigation based on a dynamic discrete choice model. They focus on the French case. The authors concluded that the scrapping value policy, increases sales in the short run, but decreases car production considerably in the long run, and so the benefit of the policy is very hypothetical.

The model used is of dynamic discrete choice, similar to the model we discussed in last section. They consider one type of consumers (unique value of θ) and a competitive producers. To match empirical fluctuations observed in the automobile market, heterogeneity is introduced through a hazard function that summarizes effects such as stochastic revenue, taste and car wrecks. While the model allows for a second hand market, the equilibrium is picked so that this market is not active and the consumer decides, each period either to keep using his car or scrap it for a new one.

In a simple deterministic version of the model, at equilibrium the individual consumer's problem is,

$$V(a) = \max \left\{ \begin{array}{c} u(a,\lambda) + \beta V(a) \\ u(1,\lambda) + p_s - p + \beta V(2) \end{array} \right\} \quad (18)$$

where p is the price of a new car and ps is the scrapping value. The consumer problem has a unique solution that implies a replacement of the car at finite cycle of length J. Aggregation across the entire population is then possible. The total sales in period t is,

$$S_t = \sum_{a=1}^{N} h(a)G_t(a) \qquad (19)$$

where $h(a)$ = 1 if $a \geq J$ and zero elsewhere, and $G_t(a)$ is the number of cars aged *a* in period t. The dynamic induced by (19) is however cyclical. At the estimation process this model is enriched by stochastic chocks so its output gets close to empirical observations.

The result obtained depends on the fact that consumers have uncertainty about the scrap value. Indeed, after the introduction of a scrap value, the replacement time is higher and car sales should increase as well, but if the scrap value is introduced with uncertainty, the consumers will on average delay their replacement policy, and the market sales will be reduced substantially after the increase in the first period. In the same time of this decrease in sales the introduction of a scrap value has a negative effect on government revenue revenues. The government collects VAT and pays subsidies. The net effect of the scrap value on these is not clear but in the empirical investigation of this paper, the negative effect seems to be dominant.

References

Adda, J. and Cooper, R., 1997. Balladurette and juppette: a discrete analysis of scrapping subsidies, NBER working paper 6048, May 1997

Adda, J. and Cooper, R., 2000. The dynamics of car sales: A discrete choice approach, mimeo, May 2000

Berry, S., Levinson J., and Pakes, A., 1995. Automobile Prices in Market Equilibrium, Econometrica, 63, 841-890.

Bresnahan, T., 1987. Competition and Collusion in the American Automobile Industry: The 1955 Price War, Journal of Industrial Economics, 35, 457-482.

Esteban, S., 1999. Semi durable goods and imperfect competition, mimeo, November 1999.

Esteban, S. and Shum, M., 2001. Durable-Goods Oligopoly with secondary markets: Theory and an empirical application to the automobile Market, mimeo, July 2001.

Goldberg, P. K., 1995. Product Differentiation and Oligopoly in International Markets: The Case of the U.S. Automobile Industry, Econometrica, 63, 891-951.

Tomat, G. M., 2002. Durable goods, price indexes and quality change: An application to the automobile prices in Italy, 1988-1998", European Central Bank Working Paper No. 118, January 2002.

A. Proofs

A.1. Proof of Lemma 5

From (14) we have:

$$p = \frac{\theta}{\beta + \lambda} + \frac{\lambda p_s}{\beta + \lambda} \left(\frac{\theta}{\beta p_s} \right)^{-\beta/\lambda}$$

first, differentiate this equality by p to get,

- $\frac{d\underline{\theta}}{dp} = \frac{\beta+\lambda}{1-\left(\frac{\theta}{\beta p_s}\right)^{-\frac{\beta+\lambda}{\lambda}}} > \beta + \lambda$
- $\frac{d\underline{\theta}}{dp_s} = \frac{\underline{\theta}(\beta+\lambda)}{\beta p_s - \theta\left(\frac{\theta}{\beta p_s}\right)^{\beta/\lambda}} < 0$
- $\frac{d\,\underline{\theta}}{d\beta} = \frac{\underline{\theta}}{\beta+\lambda} \left(1 - \frac{1}{\underline{\theta}} \frac{p_s\,(\beta+\lambda)\,\ln(\underline{\theta}/\beta p_s)}{\beta \quad p_s/\underline{\theta} - (\underline{\theta}/\beta p_s)^{\beta/\lambda}}\right) > \frac{\underline{\theta}}{\beta+\lambda}$
- $0 < \frac{d\underline{\theta}}{d\lambda} = \frac{\underline{\theta}}{\beta+\lambda} \left(1 + \frac{\beta+\lambda}{\lambda} \frac{\beta p_s}{\underline{\theta}} \frac{\ln(\underline{\theta}/\beta p_s)}{\beta p_s/\underline{\theta} - (\underline{\theta}/\beta p_s)^{\beta/\lambda}}\right) < \frac{\underline{\theta}}{\beta+\lambda}$

the first three are immediate. For the last we should show that the second member in the bracket is between 0 and 1. consider the function

$$f(x) = \frac{\beta + \lambda}{\lambda} \frac{(1/x)\ \ln(x)}{(1/x) - x^{\beta/\lambda}}$$

corresponding to this term when we replace $\theta/p_s\beta$ by x. The function f has the following properties: (a) $\lim_{x\to 1} f(x) = -1$, for $x > 1$, $f'(x) > 0$ and $f(x) < 0$ so for x> 1, −1 <f(x) < 0. The result then follows from the expression of $d\underline{\theta}/d\lambda = \underline{\theta}/(\beta + \lambda)[1 + f(\underline{\theta})]$

A.2. Proof of proposition 2

When there is only a competitive market for used cars, the agent maximizes,

$$B(\tau) - \frac{p - k\,p\,e^{-\lambda\tau}}{e^{\beta\tau} - 1} - p \qquad (20)$$

over θ. Instead if a scrap value policy is introduced (s,θ) he maximizes,

$$B(\tau) - \min\left\{\frac{p - k\,p\,e^{-\lambda\tau}}{e^{\beta\tau} - 1} - p, \frac{p - p_s}{e^{\beta\tau} - 1} - p\right\} \qquad (21a)$$

over θ.

Now consider an agent θ who has an optimal replacement time $\tau^{\theta} > \underline{\tau}$ such that $k\,p\,e^{-\lambda\tau^{\theta}} < s$. For such a type, clearly (21a) reduces to,

$$B(\tau) - \frac{p - p_s}{e^{\beta\tau} - 1} - p \qquad (21b)$$

and to see if he did reduce his replacement or not, we compare solutions to (20) and (21b).

The first order condition for (20) is,

$$B'(\tau) + \frac{(p - k\,p\,e^{-\lambda\tau})(e^{\beta\tau} - 1)'}{(e^{\beta\tau} - 1)^2} - \frac{(p - k\,p\,e^{-\lambda\tau})'(e^{\beta\tau} - 1)}{(e^{\beta\tau} - 1)^2} = 0 \qquad (22)$$

and to (21b)

$$B'(\tau) + \frac{(p-s)\ \left(e^{\beta\tau}-1\right)'}{\left(e^{\beta\tau}-1\right)^2} = 0 \qquad (23)$$

Consider first $s = k\,p\,e^{-\lambda\tau}$. For this level of scrap value, the left hand side of (22) is lower than that of (23). It follows that the replacement time for this case will be higher. This replacement time could be reduced only if,

$$\frac{(p-s)\ \left(e^{\beta\tau}-1\right)'}{\left(e^{\beta\tau}-1\right)^2} < -\frac{\left(p-k\,p\,e^{-\lambda\tau}\right)'\left(e^{\beta\tau}-1\right)}{\left(e^{\beta\tau}-1\right)^2} + \frac{\left(p-k\,p\,e^{-\lambda\tau}\right)\ \left(e^{\beta\tau}-1\right)'}{\left(e^{\beta\tau}-1\right)^2}$$

which can be written also as,

$$\frac{p-s}{p-k\,p\,e^{-\lambda\tau}} < 1 - \frac{\left(p-k\,p\,e^{-\lambda\tau}\right)'}{p-k\,p\,e^{-\lambda\tau}}\,\frac{\left(e^{\beta\tau}-1\right)}{\left(e^{\beta\tau}-1\right)'} = 1-\varepsilon \qquad (24)$$

where ε is the elasticity of the second hand market price by the discounting effect in replacement. The higher is ε, the higher is the required scrap value.

Condition (24) holds when either, (i) θ is very high (i.e. θ close to $p(\beta+\lambda)$ or, (ii) s is sufficiently high, so the left hand side is sufficiently low (of course if both (i) and (ii) are satisfied the condition holds).