CONTAINER BERTH SCHEDULING POLICY WITH VARIABLE COST FUNCTION

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Abstract: This paper presents a new mathematical formulation for the berth scheduling problem. The objective is to simultaneously minimize the total cost of vessels' late departures and waiting time, and maximize the benefits from vessels' early departures. It is assumed that different vessels, belonging to the same or different liner shipping companies, have different contractual agreements, and thus different cost functions. We discuss the applicability and advantages of nonlinear cost functions, vis à vis the linear ones commonly used in the literature. A genetic algorithm based heuristic is proposed to solve the resulting problem and a number of computational examples are presented to critically assess the proposed berth scheduling policy and evaluate the effect of the assumed cost function on the spread and distribution of the total cost among all vessels.

Keywords: Marine Container Terminals, Berth Scheduling, Cost Function, Metaheuristic Optimization

INTRODUCTION

The berth scheduling problem (BSP) deals with the assignment of vessels to berth space in container terminals (Theofanis et al., 2009; Meisel, 2009; Bierwirth and Meisel, 2009). Among the various models found in the literature, three assumptions are usually observed: a) discrete vs. continuous berthing space; b) static vs. dynamic vessel arrival, and c) static vs. dynamic vessel handling time. In the discrete problem, the quay is viewed as a finite set of
berths (see for example Hansen et al., 2007; Imai et al., 1997; Imai et al., 2001; Imai et al., 2003; Monaco et al., 2007). In the continuous case, vessels can berth anywhere along the quay (see for example Park and Kim, 2003; Kim and Moon, 2003; Guan and Cheung, 2004; Imai et al., 2005; Moorthy and Teo, 2006; Meisel and Bierwirth, 2009). The majority of the published research considers the discrete case (Theofanis et al., 2009; Meisel, 2009; Bierwirth and Meisel, 2009). In the static arrival problem, all vessels to be served are already in the port at the time scheduling begins. In the dynamic arrival problem, not all vessels to be scheduled for berthing have arrived, although arrival times are known in advance. The majority of the published work on berth scheduling considers the latter case. Finally, in the static handling time problem, vessel handling time is considered as a known input, whereas in the dynamic formulation, vessel handling time is a variable, usually assumed a function of the quay cranes that will operate on the vessel and the distance between the berthing position and a location in the yard. (Bierwirth and Meisel, 2009). Technical restrictions, such as berthing draft, inter-vessel and end-berth clearance distance are further assumptions that have been adopted in some studies, in an attempt to bring problem formulation closer to real world conditions. The introduction of technical restrictions to existing berth scheduling models is rather straightforward and is therefore not attempted here.

The paper presents a new formulation for the discrete space - dynamic arrival time berth scheduling problem (DBSP). The objective is to simultaneously minimize the total cost from vessels’ late departures and waiting time, and maximize the benefits from vessels’ early departures. Vessel departure times are often determined through contractual agreements between the terminal operator and the carrier, in relation to the ship’s arrival at the port and the total number of containers to be (un)loaded. To date, models found in the literature have assumed that penalties (or premiums in case of early departures) for late departure continue to increase indefinitely with time (usually on an hourly basis). This is not always the case and in practice two additional cases can be observed: a) if a vessel is tardy, the terminal operator pays a fixed penalty irrespective of the length of the delay, and b) the terminal operator pays a fixed penalty up to a point in time and he switches to an hourly rate after this. These three cases may be observed simultaneously, as different vessels may have different contractual agreements. All three cases are therefore included in this paper, assuming that different vessels, belonging to the same or different carriers, have different contractual agreements, and thus different cost functions. Each vessel is included in the objective function under one of the three policies (i.e. a vessel cannot be subject to two different policies). The effect of the three policies is evaluated and preliminary results are presented. At the same time, we also evaluate the effect of nonlinear cost functions - as opposed to linear ones used to date - on the distribution of costs among vessels. A genetic algorithm (GA) based heuristic is employed to solve the resulting problem. An indicative number of computational examples are presented to discuss the different policies and their effect on the cost function.

As noted in Haralambides (2002), differences exist in berth scheduling policies between multi-user and dedicated terminals (see also Aversa et al., 2005). The berthing policies proposed here are better suited for public, multi-user, terminals but they could equally apply to dedicated terminals as part of a multi-objective or hierarchical formulation (Golias et al. 2009c; Golias et al., 2009d).

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1 The term cost function in this paper refers only to the tardy penalties and early premiums of departures and should not be confused with a vessel’s total cost function or that of the terminal operator (Haralambides, 2004).
The rest of this paper is structured as follows. The next section presents a brief description of models and objectives published to date on the DDBSP, followed by a section with the problem description and its mathematical formulation. The fourth section presents the proposed resolution algorithm used to solve the formulated problem. The fifth section presents results from a number of computational examples and the last section concludes the paper and suggests future research directions.

LITERATURE REVIEW

Imai et al. (1997) were the first to formulate the discrete space - static arrival time berth scheduling problem as an unrelated machine scheduling problem. They assert that, in ports with high throughput rates, optimal vessel-to-berth and vessel-to-time-slot assignments should be found that go beyond the first come first served (FCFS) rule. They thus develop a heuristic solution approach to solve the resulting problem. Imai et al. (2001) subsequently extended their 1997 work to address the DDBSP. A Lagrangian relaxation heuristic was proposed and computational experiments showed that the heuristic performs well in practice. Nishimura et al. (2001) address the same problem using Genetic Algorithms as a resolution approach. It should be noted that the resolution approach used by Nishimura (i.e. GA based heuristics) has been applied extensively by different research groups in the following years (Boile et al., 2009). Imai et al. (2003) modified and extended the formulation of Imai et al. (2001) by including service priority constraints. The problem was reformulated into a quadratic assignment problem, using a Lagrangian relaxation method, and a GA heuristic was proposed as the solution approach. Cordeau et al. (2005) consider the DDBSP, presenting two formulations (one based on the Multi Depot Vehicle Routing Problem with Time Windows and one based on the model by Imai et al. (2001)). A Tabu Search heuristic was proposed as the solution approach. Briano et al. (2005) outline the integration of a flexible simulator, representing the sea-side operations of a container terminal, with a linear programming model for improving berth assignment and yard stacking policies. Lokuge and Alahakoon (2007) present a unique approach, departing from all previous work. They use Artificial Intelligence (AI), and more specifically the Beliefs, Desires and Intention (BDI) agent architecture for a vessel berthing application system. Results showed a reduced average waiting time of vessels, while several other measures of port productivity were also presented. Similarly to earlier research, optimality of the final schedule was not guaranteed. Moorthy and Teo (2006) present a novel approach for the DDBSP, which for the first time incorporated the stochastic nature of vessel arrivals, with very promising albeit not optimal results. Han et al. (2006) present a non-linear modeling formulation and propose solution approaches based on GAs and a combination of GAs and simulated annealing. Neither approach guaranteed optimality. Monaco and Samara (2007) formulate the DDBSP as a dynamic scheduling problem of unrelated machines. They develop a new non-standard multiplier adjustment Lagrangian heuristic algorithm. Imai et al. (2007b) propose a bi-objective formulation to minimize ship delays and total service time. They propose a GA based algorithm, as well as a subgradient optimization procedure. The authors use a linear cost function. Lee and Chen (2009) propose a formulation for a case where berths are assigned to blocks of storage; a case between the discrete and continuous berthing space BSP, based on the FCFS principle. A neighborhood search based approach was proposed. The novelty of this research was that it could handle large scale problems within a small CPU time, although optimality, or a comparison of the final schedule with lower bounds, was not provided.
Imai et al. (2007a) address the berth allocation problem at a multi-user container terminal with indented berths for fast handling. A GA heuristic was again applied as the resolution approach. Imai et al. (2008) address a variation of the DDBSP at multi-user terminals, where vessels exceeding an expected waiting time limit are assigned to an adjacent terminal. A GA based heuristic was proposed for the resolution of the problem. Hansen et al. (2008) studied the DDBSP, considering the minimization of total costs of waiting and handling, as well as earliness or tardiness of completion, for all vessels. A Variable Neighborhood Search heuristic was proposed and compared with Multi-Start; a GA; and a Memetic Search Algorithm. Similarly to Imai et al. (2007b), the premium and cost function was linear. Golias et al. (2009d) formulated the berth scheduling problem as a bi-level unrelated machine scheduling problem with variable vessel release dates to accommodate an environmentally friendly BSP, while preserving the integrity of the ocean carriers’ schedule. An evolutionary algorithm based heuristic was proposed as a resolution approach. Golias et al. (2009b) studied the DDBSP with the objective of simultaneously minimizing delayed departures and maximizing early and on-time departures/berthing of vessels within a time window. An adaptive time window partitioning heuristic was proposed as the resolution approach that did not guarantee optimality of the final schedule. Cheong and Tan (2008) formulate a similar BSP as Imai et al. (2007b) but as a non-linear bi-objective problem with linear delay cost functions. Cheong et al. (2009) formulate the DDBSP as a non-linear three-objective problem minimizing total makespan, total waiting time, and deviations from a predetermined priority. Golias et al. (2009c) formulate the DDBSP for the first time as a multi-objective problem with n+1 objective functions (where n is less than or equal to the number of vessels) to provide customer-based differentiated services based on vessel service time. Finally, Golias et al. (2009a) propose a lambda-optimization based heuristic for the resolution of the DDBSP with promising results that guarantee local optimality in a pre-specified neighborhood.

**MODEL FORMULATION**

From the above literature review it can be concluded that most of the studies dealing with the DDBSP have focused on total service and waiting time (total completion time), as well as on the costs/premiums from delayed/early departures. The latter objectives were introduced to take into account contractual agreements on the scheduled start or finish time of a ship’s cargo handling operations. Such arrangements can vary from berthing upon arrival, to guaranteed service time window, and/or guaranteed service productivity. Earliness or delays in cargo handling operations imply benefits or costs to both the terminal operator and the ocean carrier (Haralambides, 2002b). If cargo handling is completed after the agreed time (departure deadline), the operator may pay a penalty to the carrier, while, in the opposite (early departure), the carrier may pay a premium fee to the terminal operator. Early departure can help a carrier manage time to next port of call, simply by offering a buffer to compensate for time lost in other ports (Notteboom, 2006). Premiums paid by the carrier to the terminal operator, on the other hand, can be offset by reducing voyage costs through lower voyage speed and therefore fuel consumption. Slow-steaming is currently becoming an increasingly attractive policy among carriers, in their efforts to absorb superfluous excess capacity and thus maintain rates at an acceptable level. Industry observers also note that slow-steaming may even be statutorily determined in the future for environmental reasons.

As already mentioned above, research so far has assumed hourly based early departure premiums or tardy departure penalties. We assume here that different vessels, belonging to...
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the same or different liner shipping companies, have different contractual agreements and thus different cost functions. It is also assumed that only one of the following three agreements can apply to a ship (figure 1), where the terminal operator:

- pays (receives) a fixed penalty (premium) irrespective of the length of the delay (earliness) [cost policy 1].
- pays (receives) a linear hourly penalty (premium) according to the length of the delay (earliness) [cost policy 2].
- pays (receives) a constant penalty (premium) up to a point in time beyond which he pays (receives) an hourly penalty (premium) [cost policy 3].

![Figure 1 - Illustration of different cost/premium agreements](image)

In order to formulate the berth scheduling model (BSM) we need to define the following:
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Sets

$I$ : set of berths
$J$ : set of vessels
$J_1 \subseteq J$ : set of vessels under fixed premium/cost agreement for early/late departure
$J_2 \subseteq J$ : set of vessels under hourly premium/cost agreement for early/late departure
$J_3 \subseteq J$ : set of vessels under both fixed and hourly premium/cost agreement for early/late departure

Decision Variables

$x_{ij} \in \{0,1\}, \forall i \in I, j \in J$ = 1 if vessel $j$ is served at berth $i$ and 0 otherwise

$y_{ab} \in \{0,1\}, \forall a, b \in J, a \neq b$ = 1 if vessel $b$ is served at the same berth as vessel $a$ as its immediate successor and 0 otherwise

$f_j \in \{0,1\}, \forall j \in J$ = 1 if vessel $j$ is served as the first vessel

$l_j \in \{0,1\}, \forall j \in J$ = 1 if vessel $j$ is served as the last vessel

Auxiliary Variables

$HED_j \in \mathbb{R}^+ \forall j \in J_1, J_2$ : total hours the vessel $j$ departs before the requested deadline

$HLD_j \in \mathbb{R}^+ \forall j \in J_1, J_2$ : total hours the vessel $j$ departs after the requested deadline

$ed_j \in \{0,1\} \forall j \in J$ = 1 if vessel $j$ departs before the requested deadline and 0 otherwise

$ld_j \in \{0,1\} \forall j \in J$ = 1 if vessel $j$ departs after the requested deadline and 0 otherwise

$t_j, j \in J$ : start time of service for vessel $j$

$\alpha_j \in \mathbb{R}^+ \forall j \in J$ : positive number

Parameters

$ED_j \in \mathbb{R}^+ \forall j \in J$ : early requested departure deadline time of vessel $j$

(applicable only in the third cost policy)
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\[ LD_j \in R \forall j \in J : \] late requested departure deadline time of vessel \( j \)

(applicable only in the third cost policy)

\[ RDT_j \in R \forall j \in J : \] requested departure deadline time of vessel \( j \)

\[ hp_j \in R \forall j \in J : \] hourly premium if vessel \( j \) departs before the requested deadline

\[ (hp_j = 0, \forall j \in J_1) \]

\[ hc_j \in R \forall j \in J : \] hourly penalty if vessel \( j \) departs after the requested deadline

\[ (hc_j = 0, \forall j \in J_1) \]

\[ fp_j \in R \forall j \in J : \] fixed premium if vessel \( j \) departs before the requested deadline

\[ (fp_j = 0, \forall j \in J_2) \]

\[ fc_j \in R \forall j \in J : \] fixed penalty if vessel \( j \) departs after the requested deadline

\[ (fc_j = 0, \forall j \in J_2) \]

\[ wtc_j \in R \forall j \in J : \] hourly waiting time cost for vessel \( j \)

\[ S_i \in R \forall i \in I : \] time berth \( i \) becomes available for the first time in the planning horizon

\[ A_j \in R \forall j \in J : \] arrival time of vessel \( j \)

\( \alpha \): degree of non-linear cost function

The berth scheduling model (BSM) proposed in this paper can thus be formulated as follows:

\[
\min \left[ \sum_{j \in J} (wtc_j (t_j - A_j))^\alpha + \sum_{j \in J_1, J_2} \left[ (fc_j LD_j)^\alpha - (hp_j RDT - LD_j)^\alpha \right] + \sum_{j \in J_1} \left[ (hc_j [HLD + RDT - LD_j - HED])^\alpha - (hp_j HED + ED - RDT)^\alpha \right] \right]
\]

(1)

Subject To:

Decision variable constraints

\[
\sum_i x_{ij} = 1, \forall j \in J
\]

(2)

\[
f_b + \sum_{a \in b} y_{ab} = 1, \forall b \in J
\]

(3)
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\[ l_a + \sum_{b\neq a} y_{ab} = 1, \forall a \in J \] (4)

\[ f_a + f_b \leq 3 - x_{ia} - x_{ib}, \forall i \in I, a, b \in J, a \neq b \] (5)

\[ l_a + l_b \leq 3 - x_{ia} - x_{ib}, \forall i \in I, a, b \in J, a \neq b \] (6)

\[ y_{ab} \leq 1 - x_{ia} - x_{ib} \leq 1 - y_{ab}, \forall i \in I, a, b \in J, a \neq b \] (7)

\[ ed_j + ld_j = 1, \forall j \in J \] (8)

**Vessel start time estimation**

\[ t_j \geq A_j, \forall j \in J \] (9)

\[ t_j \geq S_j x_j, \forall i \in I, j \in J \] (10)

\[ t_b \geq t_a + \sum_i c_{ia} x_i - M (1 - y_{ia}), \forall a, b \in J, a \neq b \] (11)

**Early departure estimation**

\[ ED_j \geq t_j + \sum_i c_{ij} x_{ij} - M (1 - ed_j), j \in J \] (12)

\[ ED_j (1 - ed_j) \leq t_j + \sum_i c_{ij} x_{ij}, j \in J \] (13)

\[ HED_j \leq ED_j - ed_j - t_j - \sum_i c_{ij} x_{ij} + alpha_j, \forall j \in J \] (14)

\[ alpha_j \leq M (1 - ed_j), \forall j \] (15)

\[ alpha_j \leq t_j + \sum_i c_{ij} x_{ij}, \forall j \] (16)

**Late departure estimation**

\[ RTD_j \leq t_j + \sum_i c_{ij} x_{ij} + M (1 - ld_j), \forall j \in J \] (17)

\[ RTD_j (1 - ld_j) + ld_j M \geq t_j + \sum_i c_{ij} x_{ij}, \forall j \in J \] (18)

\[ HLD_j \geq t_j + \sum_i c_{ij} x_{ij} - RTD_j, \forall j \in J \] (19)
The objective function (1) minimizes the total cost of vessels' waiting time and late departures, and maximizes the total premiums from early departures. The first component of the objective function minimizes the total cost of vessels' waiting time. The second component minimizes the total tardy cost and maximizes the early departure premium for vessels under the fixed cost/premium policy. The third and fourth components of the objective function do the same for vessels under the hourly- and fixed and hourly cost/premium policies respectively.

It has often been argued (Gupta and Sen, 1983; Su and Chang, 1998; Schaller, 2002) that linear cost functions encourage schedules in which only a few ships contribute to the majority of the cost, with no regard to how the overall cost is distributed. To take this characteristic into account, a non-linear objective function (eq. 1) is adopted here, with each component raised to the power of \( \alpha^2 \). The assumption that a non-linear function would distribute the cost more evenly is evaluated in the next section through a number of numerical examples for different degrees of the non-linear function (including the linear case).

Constraint set (2) ensures that each vessel is served once, while constraint set (3) ensures that each vessel is either served first or is preceded by another vessel. In a similar manner, constraint set (4) ensures that each vessel is either served last or before another vessel. Constraint sets (5) and (6) ensure that only one vessel can be served first and last at each berth. Constraint set (7) ensures that a vessel can be served after another vessel, only if both are served at the same berth. Constraint set (8) ensures that a vessel will either depart early or late. Constraint sets (9) and (10) ensure that vessel service start time is greater than vessel arrival, or the time that the berth where the vessel will be served becomes available for the first time in the planning horizon. Constraint set (11) estimates the service start time of each vessel. Constraint sets (12) through (16) estimate the total hours of early departure. If a vessel \( j \) does not depart early from berth \( i \) (i.e. \( ED_j < t_j + \sum_i c_{ij}x_{ij} \)) then \( ed_j \) will take the value of zero so that equation (12) is feasible (i.e. \( ED_j \geq -M \)). In that case, equations (15) and (16) set \( \alpha_j \) to its upper bound (i.e. \( \alpha_j = t_j + \sum_i c_{ij}x_{ij} \)), equation (13) is feasible as an

\[ \sum_i c_{ij}x_{ij} \]

In the objective function, the constant penalties/premiums (i.e. second term) are raised to the same power as the rest of the components to avoid a diminishing influence (of this term) to the objective function (and thus vessel-to-berth assignment) as the degree (\( \alpha \)) increases.
inequality (i.e. $ED_j < t_j + \sum_i c_{ij}x_{ij}$), and equation (14) sets $HED_j$ to its lower bound (i.e. zero). If the vessel does depart early (i.e. $ED_j \geq t_j + \sum_i c_{ij}x_{ij}$), then $ed_j$ will take the value of one so that equation (13) is feasible (i.e. $0 \leq t_j + \sum_i c_{ij}x_{ij}$). In that case, equation (12) is feasible as well (i.e. $ED_j \geq t_j + \sum_i c_{ij}x_{ij}$). The case where vessel $j$, served at berth $l$, departs early is similar and thus omitted. Constraint sets (17) through (19) estimate the total hours of late departure. If vessel $j$ does not depart late from berth $i$ (i.e. $RTD_j \geq t_j + \sum_i c_{ij}x_{ij}$) due to constraint set (8) $ld_j$ is equal to zero. In this case constraint set (17) is feasible (i.e. $RTD_j \leq M$ and constraint set (19) sets $HLD_j$ equal to the lower bound (i.e. zero). The case where vessel $j$, served at berth $l$, departs late is similar and thus omitted.

**RESOLUTION ALGORITHM**

The BSM is a non-linear mixed integer problem (MIP) for $\alpha > 1$ and a linear MIP for $\alpha = 1$. In both cases, no exact resolution algorithm exists to date that can solve such problems in polynomial time (especially in real life instances which on average have a minimum of 10 to 15 vessels and 3 to 5 berths). To tackle this, the GA based heuristic, proposed by Golias (2007), is employed here as the resolution approach. The proposed heuristic consists of four parts: a) the chromosomal representation; b) the chromosomal mutation; c) the fitness evaluation; and d) the selection process. The GA uses an integer chromosomal representation, in order to exploit in full the characteristics of the problem. An illustration of the chromosome structure is given in figure 3 for a small instance of the problem with 6 vessels and 2 berths. As seen in figure 3, the chromosome has twelve cells. The first 6 cells represent the 6 possible service orders at berth 1 and the last 6 cells the 6 possible service orders at berth 2. In the assignment illustrated in figure 2, vessels 2, 4, and 5 are served at berth 1 as the first, second and third vessel respectively, while vessels 1, 3, and 6 are served at berth 2 as the first, second, and third vessel respectively.

![Figure 3 - Illustration of chromosome representation](image)

Four different types of mutation are applied, as part of the genetic operations, to all the chromosomes at each generation: insert, swap, inversion, and scramble mutations. Each of the four types of mutation, illustrated in figure 3, for the small example shown in figure 4, has its own characteristics in terms of preserving the order and adjacency information (Eiben and Smith, 2003).

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Since the case at hand is a minimization problem, the smaller the values of each objective function, the higher the fitness value. As discussed in Goldberg (1989), in minimization problems it is desirable to define the fitness function of a chromosome as:

\[ f_{pt}(x) = F_{max}^{pt} - z_{pt}(x) \]

where \( F_{max}^{pt} \) is the maximum value of the objective function \( z_{it} \), and \( f_{it} \) is the value of the fitness function of objective function \( i \) at iteration \( t \). However, this value is not known in advance; thus the largest value \( z_{pt} \) for each objective function at each iteration is chosen as the value of \( F_{max}^{pt} \). To find quality solutions for each objective function and, at the same time, retain a variety of different solutions, the Roulette Wheel Selection algorithm (Goldberg, 1989) was applied to select the children population of each generation.

**COMPUTATIONAL EXAMPLES**

Five problem instances are developed, where vessels with various handling volumes are served at a multi-user container terminal with five berths, a planning horizon of one week, and vessel inter-arrival time of 3 hours. The range of the remaining parameters considered here is chosen according to Golias et al. (2009c) and Hansen et al., (2008) and they are reported for purposes of consistency. Availability of berths for the first time in the beginning of the planning horizon (i.e. parameter \( S_i \)) is calculated using a uniform probability distribution with a minimum of zero and a maximum of 10 hours (Hansen et al., 2008). Delayed and early departure; hourly and constant penalties; and hourly waiting costs are calculated randomly, not based on actual cargo carried, as vessels carrying less than capacity cargo might belong to a carrier with higher priorities. Latest and earliest departure requests are generated randomly based on the formula by Hansen et al. (2008). Vessel handling volumes are generated randomly based on a uniform distribution pattern (Golias et al., 2009c) between 5 and 30 hours at the preferred berth (i.e. berth with the minimum handling time over all berths). The minimum vessel handling time at berth (excluding the preferred berth) is generated in relation to the berth with the minimum handling time, by increasing handling time proportionally to the distance from the preferred berth (Imai et al., 2001). The increase is based on a uniform probability distribution. Preferred berths for the vessels are chosen through the insertion of new berths in the existing berth schedule.
randomly. Experiments were performed on a Dual Core ASUS CM5570 computer with 6GB memory, using Matlab R2008b. The initial population for the GA based heuristic was obtained by using a first come first served (FCFS) rule, at the first available berth (similar to Hansen et al., 2008). The population size was set to 50 chromosomes and the heuristic stops if no improvement is observed for 100 iterations (i.e. new or improved schedules found).

Berth Scheduling Policy Evaluation

Using the dataset described in the previous subsection, we performed two types of experiments. The first focuses on the effect of the degree of the non-linear cost function, and the second on the effect of the cost policy on the distribution of total cost among vessels. To perform these experiments, for each of the five datasets presented above, 16 different berth schedules were considered. Each berth schedule was obtained by using a different combination of cost policy and degree of cost function, keeping the remaining data of each dataset unchanged (i.e. handling time; arrival time; hourly and constant penalties; etc.). Four different degrees were used for the cost function, with values of $\alpha=1$, $2$, $3$, and $4$. For the type of cost policy, we assume the following four different cases:

a) Case 1: all vessels fall under cost policy 1;
b) Case 2: all vessels fall under cost policy 2;
c) Case 3: all vessels fall under cost policy 3;
d) Case 4: each vessel falls randomly under one of the three cost policies, based on a uniform probability distribution.

It should be noted that although schedules were obtained using the non-linear cost function (i.e. $\alpha=1$, $2$, $3$, and $4$), the cost values used in the evaluation were based on the linear cost function of each schedule (i.e. $\alpha=1$). To obtain a measure of uniformity of the cost distribution among all vessels, a uniform distribution was fitted, using as observations the cost of each vessel from each schedule. Figure 4 shows the range of the estimated parameters of the fitted uniform distribution (i.e. minimum and maximum value) for the 16 different schedules for each dataset. Figure 5 shows the variance of the fitted uniform distributions for the 16 different schedules of each dataset. The x-axis represents the degree of the objective function, with total cost measured on the y-axis. Each graph plots four lines, one for each case previously presented.

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We observe that, in general, as the degree of the non-linear function increases (i.e. moving right on the x-axis) the spread and variance of total cost among vessels decreases in all cases. This becomes more noticeable, as the total cost difference between the smallest and largest value of the fitted distribution increases (e.g. figures 4 and 5, datasets 1 and 2, cost policies 2 and 3), supporting our initial assumption that non-linear cost functions spread the cost among all vessels more evenly. On the other hand, one notes that objective function values (shown in figure 6 for the 16 schedules per dataset), increase, in general, with the degree of the cost function. In figure 6, the y-axis shows the value of each objective function as a percentage of its maximum value among the four different degrees, for the same case. For example, for dataset 1, the objective function values for schedules with cost functions of degrees $\alpha=1$, $\alpha=2$, and $\alpha=3$ (for the second case) were 21%, 7%, and 2% lower than the objective function value of the berth schedule with $\alpha=1$ (for the same case).
CONCLUSIONS

We have presented a mathematical formulation for the berth scheduling problem where the total cost from vessels’ late departure and waiting time is minimized, and total benefits from early departures maximized. Different cost functions for each vessel were used, to represent different contractual agreements. To the best of our knowledge this formulation and cost policies are attempted for the first time in the published literature. We have also discussed the applicability and effectiveness of nonlinear cost functions, and evaluated the assumption that a non-linear function would distribute costs more evenly among vessels. A GA based heuristic was used to solve the resulting problem, and a number of computational examples showed that higher order degrees of the cost function provide smaller deviations of the spread and variance of total cost among vessels but result in a higher cumulative cost.
Future research could focus on: a) performing a larger number of computational examples with increased size (i.e. up to ten berths and seven hours of vessel inter-arrival times), b) investigate a formulation that explicitly models the variability and dispersion of costs among vessels and the total cost (i.e. a bi-objective or a bi-level formulation), and c) introduce a time window departure request whereby if the vessel departs within a time window, after its arrival, no penalties or premiums are applicable.

Figure 6 - Changes in the objective function value
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Container berth scheduling policy with variable cost function

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