Optimal Inspection and Replacement Policy using Stochastic Method for Deterioration Prediction

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Optimal Inspection and Replacement Policy using Stochastic Method for Deterioration Prediction

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Abstract

This paper introduces a Weibull hazard model formulated by the aggregated Markov method to forecast the expected life length of highway facilities and equipments and the inspection/replacement processes of tunnel lighting systems. First, the paper focuses on presenting a methodology to overcome the estimation bias issues caused by the incompleteness of the data set due to the lack of full life length information. The incompleteness of the data structure varies in different monitoring schemes. Hence, the necessity to develop an estimation methodology of the deterioration hazard models considering the available incomplete life length data from the observed samples arises. The paper also investigates the optimal inspection and replacement policy of tunnel lighting systems using a deterioration hazard model. The inspection/replacement intervals and the maximum life length of the light bulbs are considered as the major management policy variables, and the impacts of these management variables upon the life cycle costs and the fault probability of the light bulbs are investigated. Subsequently, the optimal inspection/replacement model to find the optimal policy that minimizes the life cycle costs given the levels of the fault risks is presented. The applicability of the methodology presented in this paper is examined against real world data concerning the facilities and equipments on the highways.

Keywords: asset management, hazard model, tunnel lighting system, life cycle cost, aggregated Markov process, Cost-Risk curve

1. Introduction

It is impossible to directly observe the deterioration process of a tunnel light bulb and only the existence of a faulty light bulb as a result of deterioration can be observed. Furthermore there exists uncertainty in the deterioration progress of a light bulb and it is impossible to predict its life definitely. A tunnel lighting system is constituted of many light bulbs. The maintenance of tunnel lighting systems requires not only the replacement cost of light bulbs but also the fixed cost pertinent to the maintenance works. Furthermore, inspection and replacement works on a highway causes negative externalities, such as delays and congestions due to the vehicular flow slowdowns. For this reason, it is possible to reduce the life cycle cost by synchronizing the timing of the inspection and the replacement of many light bulbs rather than managing individually each light bulb of the tunnel lighting system.

The inspection and replacement intervals and the maximum life length of the light bulbs are considered as the major management policy variables. In order to reduce the fault risk of the light bulbs it is necessary to conduct inspections and replacements frequently and shorten the maximum life length of the light bulbs. On the other hand, this will result in increased life cycle cost. Thus there is a trade-off relation between the fault risk and the life cycle cost of a tunnel’s lighting system.

In this paper, the deterioration process of tunnel light bulbs is expressed by a Weibull deterioration hazard model; the light bulbs’ fault probability, based on the time elapsed since the light bulb was first used, is computed using an aggregated Markov process model in order to analyze the influence on the inspection and replacement policy of the tunnel’s lighting system in view of the fault risk and life cycle cost. In addition, the optimal inspection and replacement model that minimizes the life cycle cost of tunnel lighting system under a given maintenance level is proposed. The second section of this paper presents the research fundamentals. In the third section the methodology of the Weibull hazard function estimation is described. The fourth section proposes methodologies for the formulation of the optimal inspection and replacement models.
for tunnel lighting systems and analyzes the trade-off relation between the fault risk and life cycle cost. The fifth section presents a case study as it was applied in real management works of a tunnel lighting system.

2. Research Fundamentals

(1) The hazard model estimation

This study was conducted to estimate the fault risk using a hazard model aimed at the management of road facilities, such as a road lighting system. When a hazard model is used for deterioration forecasting of road facilities some important details should be taken into consideration. First, the monitoring information about the deterioration condition of road facilities can be obtained by inspection. However, in the case that sufficient monitoring information exists, it is necessary to develop the estimation method of the deterioration hazard model according to the properties of the monitoring information. When continuous observation of the conditions of facilities is possible, e.g. by a sensor, perfect monitoring information can be obtained. In reality it is impossible to achieve uninterrupted monitoring of the conditions of many facilities. Actually, the deterioration condition can be observed only through inspections executed on certain time intervals. Consider a monitoring scheme where the inspections are carried out at a certain fixed time interval. The information obtained by such a fixed monitoring scheme is called fixed monitoring information.

Consider a monitoring scheme that monitors the condition of all facilities at the time instances $\tau_W$ and $\tau_T$, as shown in Figure-1. The time the facility starts being used is defined as $t = 0$, and two sample monitoring times are expressed as $W_i$ and $T_i (W_i < T_i)$. In the above scenario, as described in Figure-1, two cases can be considered: (i) In this case, considering facility $A$, failure has not occurred at the monitoring time $t = T_A$, but has occurred at time $t = T_A$. Although the exact time the fault occurred cannot be observed, information about the time interval where the fault occurred can be obtained, i.e. in period $(W_A, T_A]$. (ii) In this case, considering facility $B$, failure has not occurred during the monitoring interval $W_B, T_B$. This is an inefficient time interval (sample) from which we cannot extract any information on the fault occurrence. The only usable information derived here is that the life of the facility is longer than the monitoring period $T_B$. Section 3 (3) describes the estimation method of a deterioration hazard function based on such a fixed monitoring information.

(2) Inspection/Replacement Policy of Tunnel Lighting System

In the asset management of a tunnel lighting system, not only the replacement cost of each light bulb, but also the fixed cost of the inspection and replacement works is required. Furthermore, negative externalities such as road congestion, traffic slowdowns and driver confusion may occur as a result of the inspection and maintenance works. Therefore, by synchronizing the timing of the inspection with the repairing of the tunnel lighting system, e.g. the light bulb replacement, the fixed cost is reduced and as a result it becomes possible to reduce the life cycle cost. Consider the case where the tunnel lighting system consists of many light bulbs of the same kind and failures of light bulbs are observed periodically. In this case the inspection and replacement interval and the maximum life length of the light bulbs are considered as the management policy variables. If the inspection interval is extended, the fixed cost due to inspection is reduced thus enabling reductions in the replacement cost of light bulbs. On the contrary, extending the inspection interval causes an increase in the number of failed light bulbs and the fault risk increases as a result. The same trade-off relation applies also to the maximum life length of a light bulb. That is, the probability of fault of a light bulb that has been in use for a long period is high. Therefore, in order to control the probability of fault of a light bulb, it is desirable to replace the light bulb irrespective of being faulty when its usage interval reaches its maximum life length. However, if the maximum life length is shortened too much, the replacement cost will increase and as a result life cycle cost will also increase. In this way, there is a trade-off relation between the life cycle cost and the fault risk on one hand, which are management
Note) The dashed line expresses the $C - U$ curve to which inspection and replacement time $d$ was fixed to, and the maximum life length is changed. The vertical axis expresses the life cycle cost $C$, and the horizontal axis expresses fault probability $U$. The solid line shows the cost-risk curve which indicated the combination of efficient life cycle cost and fault risk level.

Figure-2 Cost and Risk curve

indexes, and the inspection interval and the maximum life length on the other hand.

In addition, considering the replacement policy of a tunnel lighting system, two different rules can be utilized: 1) the rule of individual (one by one) faulty light bulb replacement, and 2) the rule of all light bulbs replaced simultaneously (faulty and not). The former is a rule which replaces only a faulty light bulb during the inspection while the latter is a rule which replaces all light bulbs during every fixed inspection. When the rule of simultaneous replacement is adopted the replacement cost increases, because light bulbs that did not fail are also replaced. On the other hand, by having inspections at prescribed fixed intervals inspection costs are reduced. The decision on the choice of the replacement rule depends on the deterioration profile, replacement and inspection cost and other factors. However, the rule of simultaneous replacement can be considered as a special case of the more general one-by-one replacement rule. Therefore, in this study, the optimal inspection and replacement policy is formulated taking into consideration both of the rules.

(3) Optimal Inspection and Repair Policy

Let us consider two factors, life cycle cost and fault risk, as management indexes of a tunnel lighting system. The inspection and replacement interval $d$ and the maximum life length $md$ are considered as the inspection and replacement policy of the tunnel lighting system. The Life cycle cost and the fault probability are expressed by $C(d, m)$ and $U(d, m)$ respectively, depending on the inspection and repair strategy $\xi = (d, m)$. The dashed lines in Figure-2 illustrate the relation of $C(d, m)$ and $U(d, m)$ when the maximum life length $m$ is allowed to vary. As indicated in Figure-2, as the inspection and replacement interval $d$ increases, the life cycle cost $C(d, m)$ decreases and the fault probability $U(d, m)$ increases. The relationship between the life cycle cost and the fault risk is indicated by the $C - U$ line.

Each point on the envelope curve (solid line) indicates the minimum value of the life cycle cost when a fault risk is given. The envelope curve is called the cost-risk curve. Because there is uncertainty in the deterioration process, in order to eliminate the fault risk completely, a considerable increase in the life cycle cost must be incurred. Hence, allowance for the fault risk should be considered. If the maintenance level (fault risk level) can be determined, it is possible to compute the optimal inspection and replacement strategy that minimizes the life cycle cost for the level of maintenance. When the management level standards are decided, the benefits of the road users, the management flaws and the financial structure of the public sector must all be taken into consideration. The cost-risk curve indicated in Figure-2 shows the combination of the efficient life cycle cost and the fault risk and can provide useful information to be used toward the construction of an intelligent management system for managing tunnel lighting systems.

3. Deterioration hazard model

(1) Formulation of the deterioration hazard model

The deterioration process of a light bulb can be formulated using the hazard model. The period from an initial point in time, $t = 0$, to the present is defined as the monitoring period for $n$ facilities $i(i = 1, 2, \ldots, n)$ of the same type. The monitoring period $T_i$ changes with the facility type. Let us consider a given facility used continuously and a facility failure is observed by the periodical inspection during the monitoring period. The serviceable period of facility $i$ is expressed by $y_i$. The serviceable period and the monitoring period of each facility are in agreement due to the definition of the length of the serviceable period. The life expectancy of a given facility is assumed to be a stochastic variable having probability density function $f_i(\zeta)$ and cumulative distribution function $F_i(\zeta)$, where $\zeta_i$ is defined in the domain $[0, \infty)$. Using the cumulative probability $F_i(t)$, the probability $F_i(t)$ (referred
to as the survival function) of a transition during the time points interval $t = 0$ to $t \in [0, \infty]$ is defined by $\tilde{F}_i(t)$ given by

$$\tilde{F}_i(t) = 1 - F_i(t) \quad (1)$$

The conditional probability that a failure will occur at time $t$ during the time interval $[t, t+\Delta t]$ is defined as

$$\lambda_i(t)\Delta t = \frac{f_i(t)\Delta t}{\tilde{F}_i(t)} \quad (2)$$

where the probability density $\lambda_i(t)$ is referred to as the hazard function. By changing the form of the hazard function $\lambda_i(t)$ it is possible to formulate various kinds of hazard models. By differentiating both sides of equation (1) with respect to $t$;

$$\frac{d\tilde{F}_i(t)}{dt} = -f_i(t) \quad (3)$$

Equation (3) then becomes

$$\lambda_i(t) = \frac{f_i(t)}{\tilde{F}_i(t)} = -\frac{d\tilde{F}_i(t)}{dt} = \frac{d}{dt} \left( -\log \tilde{F}_i(t) \right) \quad (4)$$

Considering that $\tilde{F}_i(0) = 1 - F_i(0) = 1$ and by integrating equation (4) we have;

$$\int_0^t \lambda_i(u)du = \left[ -\log \tilde{F}_i(u) \right]_0^t = -\log \tilde{F}_i(t) \quad (5)$$

Using the hazard function $\lambda_i(t)$, the probability $\tilde{F}_i(t)$ that the life expectancy of the facility will be bigger than $t$ is expressed by;

$$\tilde{F}_i(t) = \exp \left[ -\int_0^t \lambda_i(u)du \right] \quad (6)$$

By deciding the form of the hazard function $\lambda_i(u)$, it is possible to compute the survival function $\tilde{F}_i(t)$ and using $\tilde{F}_i(t) = 1 - F_i(t)$, the cumulative deterioration probability $F_i(t)$ can be computed.

(2) Weibull Hazard Function

The deterioration process depends on the property of each facility. Suppose that the difference in the deterioration process between facilities is expressed by an individual property, and the property of facility $i$ is expressed by the property vector $x_i = (x_i^0, x_i^1, \ldots, x_i^K)$ defined by $K+1$ characteristic variables $x_i^k (k = 0, \ldots, K)$, where $x_i^0 = y_i$ express the usage duration of the facility. The deterioration hazard function is expressed by $\lambda(x_i : \theta)$, where $\theta = (\theta^0, \ldots, \theta^K)$ indicates the vector of unknown parameters $\theta^k (k = 0, \ldots, K)$. The function form of the deterioration hazard function is specified by the Weibull hazard function;

$$\lambda(x_i : \theta) = \tilde{x}_i^{\tilde{\theta}} m y_i^{m-1} \quad (7)$$

where $\theta = (\theta^0, \tilde{\theta})$, $\tilde{\theta} = (\theta^1, \ldots, \theta^K)$ and in the case of the Weibull hazard function, $\theta^0 = m$, and $\tilde{x}_i = (x_i^1, \ldots, x_i^K)$. The probability density function $f(x_i : \theta)$ and survival function $\tilde{F}(x_i : \theta)$ are defined as;

$$f(x_i : \theta) = \tilde{x}_i^{\tilde{\theta}} m y_i^{m-1} \exp(-\tilde{x}_i^{\tilde{\theta}} y_i^m) \quad (8)$$

$$\tilde{F}(x_i : \theta) = \exp(-\tilde{x}_i^{\tilde{\theta}} y_i^m)$$

(3) Fixed Inspection Information

Suppose that facility failure monitoring is periodically carried out. The facility $i$ is opened to the public at time $t = 0$, and two periodical inspections at time $t = W_i$ and $t = T_i (T_i > W_i)$ are considered. Facility failure is not observed at $t = W_i$ while at inspection $t = T_i$ it is observed. In this case, although the exact time of failure cannot be observed, it can be deducted that the time of failure occurred during the time interval $(W_i, T_i)$. The probability $\pi(W_i, T_i, \tilde{x}_i : \theta)$ that the facility life is larger at least than $W_i$ and fails in period $(W_i, T_i)$ is defined by;

$$\pi(W_i, T_i, \tilde{x}_i : \theta) = \Pr\{T_i \geq \zeta_i \geq W_i\} = \tilde{F}(W_i, \tilde{x}_i : \theta) - \tilde{F}(T_i, \tilde{x}_i : \theta) \quad (9)$$

Next, the probability of not having a failure in any of the two inspection times is defined by;
\[
1 - \pi(W_i, T_i, \bar{x}_i : \theta) = \tilde{F}(T_i, \bar{x}_i : \theta) \quad (10)
\]

A dummy variable that expresses whether the facility \(i\) fails within period \((W_i, T_i)\) is defined by:

\[
d^f_i = \begin{cases} 
1 & \text{when } W_i \leq \zeta_i \leq T_i \\
0 & \text{when } \zeta_i > T_i 
\end{cases} 
\quad (11)
\]

The superscript \(f\) on the dummy variable means that the fixed periodical inspection scheme is considered. The conditional probability that inspection data \(d^f_i\) is observed by periodical inspection at time \(W_i, T_i\) is defined by:

\[
\ell^f(W_i, T_i, d^f_i, \bar{x}_i : \theta) = \pi(W_i, T_i, \bar{x}_i : \theta) d^f_i \{1 - \pi(W_i, T_i, \bar{x}_i : \theta)\}^{1-d^f_i} 
\quad (12)
\]

If it is assumed that the occurrences of the failures of facility \(n\) are mutually independent, the log-likelihood function showing the simultaneous probability density of the fault pattern of each facility can be expressed with the following formula:

\[
\ln[\mathcal{L}(\theta)] = \ln \prod_{i=1}^{n} \ell^f(W_i, T_i, d^f_i, \bar{x}_i : \theta) \\
= \sum_{i=1}^{n} d^f_i \ln[\pi(W_i, T_i, \bar{x}_i : \theta)] \\
+ \sum_{i=1}^{n} (1 - d^f_i) \ln[1 - \pi(W_i, T_i, \bar{x}_i : \theta)] 
\quad (13)
\]

4. Formulation of the Optimal Inspection and Replacement Strategy

(1) Modeling Precondition

The historical deterioration and replacement process of a tunnel lighting system is described in Figure-3. In this figure, \(t\) represents real time. In Figure-3 it is shown that the tunnel is opened to the public at time \(t_0\) and failure occurs at time \(\tau = t_0 + \zeta\). The time the tunnel is opened to the public is assumed to be the origin on the time axis. The time represented by the sample time-axis, defined specifically for each facility, is referred to as a time point. In Figure-3, the failure of a light bulb occurs at time point \(\zeta\) on the sample time-axis. Under this condition, the life of a light bulb is \(\zeta\). However it can be observed by inspection whether a fault occurred. Information regarding the existence of a fault can be acquired through periodical inspections. In this case, periodical inspections at times \(t_0, t_0 + d, t_0 + 2d, \cdots\) on the time-axis are considered. The initial time is defined by \(t_0\) and the discrete time point for every time interval \(d\) from initial time \(t_0\) are defined as:

\[
t_i^d = t_0 + id \quad (i = 0, 1, \cdots) 
\quad (14)
\]

where \(i (i = 0, 1, 2, \cdots)\) expresses the time point. The facility in which the fault occurred at \(\tau\) is observed at time \(t_{i+1}^d = t_0 + (i + 1)d\) by the periodical inspection. That facility will be left in the failed state in the period \([\tau, t_{i+1}^d]\). If failure is observed by inspection at the time \(t_{i+1}^d\), that facility is reinstated immediately.

Suppose that the administrators manage \(N\) facilities simultaneously. An inspection and replacement strategy \(\xi\) for the tunnel lighting system is expressed by \(\xi = (d, m)\) using the inspection and replacement interval \(d\). Suppose that the existence of a fault is observed by periodical inspection at time \(t_0, t_1^d, t_2^d, \cdots, t_{d-1}^d\) according to the inspection and replacement strategy \(\xi\). Because there is uncertainty in the life of a light bulb and only faulty light bulbs that were observed during the inspection are replaced, there exist light bulbs with various usage histories during each periodical inspection. The usage duration of each light bulb is expressed as an integer multiple of \(d\) such as \(0, d, 2d, \cdots, md\) because of the periodical inspection. Thus, the numbers of the light bulbs classified by usage can be defined by equating the light bulbs’ usage with the current used time length. The light bulb which was found to be faulty is replaced, so the used time length of that light bulb at inspection time \(t_0\) returns to 0. The number of light bulbs classified by the usage duration are represented by the state variable vector \(n^\xi(t_0) = (n_0^\xi(t_0), \cdots, n_{m-1}^\xi(t_0))\). The state variable \(n_0^\xi(t_0)\) indicates the number of the light bulbs
having usage duration $jd$ at inspection time $t_0$. Therefore, the following equation is realized;

$$
\sum_{j=0}^{m-1} n_j^j(t_0) = N \quad (15)
$$

The inspection and repair process of a tunnel lighting system under the inspection and replacement strategy $\xi$ can be expressed as a series $n^\xi(t_0), n^\xi(t_1), n^\xi(t_2), \ldots$ of the state variable vector based on $t_0$.

(2) Modeling of the Inspection and Replacement Process

Suppose that the administrators manage $N$ light bulbs simultaneously with the inspection and replacement policy $\xi$, i.e. an inspection interval and a maximum life length are assigned. The number of light bulbs classified by the usage duration at inspection time $t_i^d$ are represented by the state variable vector $n_i^\xi(t_i^d) = (n_i^0(t_i^d), \ldots, n_i^{m-1}(t_i^d))$. As it was mentioned earlier, the state variable $n_j^\xi(t_i^d) (j = 0, \ldots, m - 1)$ expresses the number of light bulbs which have usage duration $jd$ at inspection time $t_i^d$. Thus, the relative frequency of a light bulb classified by usage duration at inspection time $t_i^d$ is represented by $\pi_j^\xi(t_i^d) = n_j^\xi(t_i^d)/N (j = 0, \ldots, m - 1)$ and the relative frequency vector is defined by $\pi_i^\xi(t_i^d) = (\pi_0^\xi(t_i^d), \ldots, \pi_{m-1}^\xi(t_i^d))$. Thus, the following equation is realized;

$$
\sum_{j=0}^{m-1} n_j^\xi(t_i^d) = 1 \quad (16)
$$

Let us focus on a light bulb which has usage duration $jd$ at the inspection time $t_i^d$. The probability that failure of this light bulb will not occur is expressed by $p_j^d$. The expected value of the relative frequency of the light bulb that failure will not occur at the inspection time $t_i^d$ and the usage duration will become $(j+1)d$ at the next inspection time $(j+1)d$ is computed as follows;

$$
\pi_{j+1}^\xi(t_i^{d+1}) = p_j^d \pi_j^\xi(t_i^d) \quad (17)
$$

On the other hand, if light bulb failure is observed at inspection time $t_i^{d+1}$, the light bulb will be replaced immediately and its usage duration will be reset (adjusted to 0) at the inspection time $t_i^{d+1}$. Then, the expected value of the relative frequency of the light bulb that its usage duration will become 0 at the inspection time $t_i^{d+1}$ is expressed as follows;

$$
\pi_0^\xi(t_i^{d+1}) = \sum_{j=0}^{m-2} (1 - p_j^d) \pi_j^\xi(t_i^d) + \pi_{m-1}^\xi(t_i^d) \quad (18)
$$

The second term in equation (18) indicates that the light bulb that its usage duration will become $(m - 1)d$ will be replaced at the next inspection time unconditionally. By adopting a sufficiently large $m$, an assumption for limiting the degree of the Markov transition probability process, the analysis result is not influenced. Hence, the $m \times m$ transition probabilities matrix with inspection and replacement interval $d$ is defined by;

$$
P^\xi = \begin{pmatrix}
1 - p_0^d & p_0^d & \cdots & 0 \\
1 - p_1^d & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
1 - p_{m-2}^d & 0 & \cdots & p_{m-2}^d \\
1 & 0 & \cdots & 0
\end{pmatrix} \quad (19)
$$

When the relative frequency at the initial time is defined by $\pi_i^\xi(t_0)$, the expected relative frequency at an arbitrary time of periodical inspection is expressed by;

$$
\pi_i^\xi(t_i^d) = \pi_i^\xi(t_0) (P^\xi)^i \quad (20)
$$

where $(P^\xi)^i$ indicates the matrix $P^\xi$ multiplied $i$ times. By using the transition probabilities matrix $P^\xi$ with the inspection and replacement policy $\xi$ the deterioration and replacement process of the tunnel lighting system is formulated by equation (20). It is assumed that the inspection and replacement process is repeated for a long period and the relative frequency will reach a stationary state. The stationary probability vector about the share of the light bulbs classified by their usage duration is expressed by $\pi^\xi = (\pi_0^\xi, \ldots, \pi_{m-1}^\xi)$. The stationary probability is defined as $\pi^\xi$ which satisfies equation (21) below;
\[
\pi^\xi = \pi^\xi P^\xi
\]  \hfill (21)

(3) Transition Probabilities

The deterioration probability of a tunnel lighting system is expressed using the Weibull hazard function. The probability density function \( f(\zeta) \), which expresses the life of a light bulb, and the survival function \( \tilde{F}(\zeta) \) are defined by:

\[
f(\zeta) = \theta \alpha \zeta^{\alpha - 1} \exp(-\theta \zeta^\alpha) \\
\tilde{F}(\zeta) = \exp(-\theta \zeta^\alpha)
\]  \hfill (22)

The probability that the light bulb whose usage duration is \( \zeta = jd \) at inspection time \( t_i^d \) will not fail until the next inspection time \( t_{i+1}^d \) is defined by:

\[
p_i^d = \exp[-\theta \{(j + 1)^\alpha - j^\alpha\} d^\alpha]
\]  \hfill (23)

(4) Fault Risk Management Index

This paragraph defines the life cycle cost and the fault risk in the stationary state under the inspection and replacement policy \( \xi = (d, m) \). The stationary probabilities \( \pi^\xi \) are computed using the transition probabilities matrix \( P^\xi \).

The expected probability of failure of a light bulb \( U(d, m) \) at each inspection time under the stationary state is defined using equation (18);

\[
U(d, m) = \sum_{j=0}^{m-1} (1 - p_i^d) \pi_j^\xi \\
= \pi_0^\xi - \pi_{m-1}^\xi p_{m-1}^d
\]  \hfill (24)

The expected occurrence probability \( U(d, m) \) is an intuitive and comprehensible index. However, the expected occurrence probability index defines the expected value of the portion of the failed light bulbs as observed during the periodic inspections; it is not the portion of the failed light bulbs actually observed in each inspection. Naturally, it is possible that the portion of the failed light bulbs observed at each inspection gets a value greater than the expected value \( U(d, m) \).

The stationary probability classified by usage duration of each light bulb under the inspection and replacement policy \( \xi \) is defined as in formula (21). Here, the stationary probability that the light bulb has failed is defined by

\[ N \times \text{Bi}(n : N, \pi_F^\xi) = N \frac{C_n(\pi_F^\xi)^n(\pi_0^\xi)^{N-n}}{\text{Bi}(n : N, \pi_0^\xi)}
\]  \hfill (25)

At this point, let’s define the probability variable \( z = n/N \) that expresses the rate of failed light bulbs to the total number \( N \) of light bulbs, hereafter called the share of failed light bulbs. The share \( z \) of failed light bulbs observed in each periodic inspection follows a discrete probability distribution as shown in Figure-4. In this figure, the probability distributions in which the share of the failed light bulbs are expressed for each of the three cases to which the total number of light bulbs is changed. The Value at Risk (VaR) index is formulated as a management index of the fault risk in consideration of the probability distributions of the share of failed light bulbs. The probability that is above the permissible level \( \bar{U} \) (it’s hereafter called the fault control limit) that was generated by the share \( z \) of failed light bulbs, observed at the inspection time when the inspection and replacement \( \xi \) is adopted is expressed by;
\[ P^k(z \geq \bar{U}) = \sum_{n=\lceil N\bar{U}\rceil}^{N} Bi(n : N, \pi^k) \]  \hspace{1cm} (26)

where, \( \lceil N\bar{U}\rceil \) expresses the minimum integer of all the integers exceeding \( N\bar{U} \). As shown in Figure-4, the share \( z \) of the failed light bulbs observed at the inspection time is subjected to variations. We can deduct that the dispersion of the probability variable \( z \) becomes small, so that the number of light bulbs increases. The total \( \omega \) of the line segment length (probability) of the dashed line currently drawn on the domain of \( z \geq \alpha \) in the figure expresses the probability which is larger than \( \bar{U} = \alpha \) for the failed light bulbs share that was observed at the time of inspection and was set up as a fault control limit when the light bulbs total was \( N = 200 \). Since there is uncertainty in the deterioration process of a tunnel lighting system, the given fault control limit about the portion of the failed light bulbs cannot always be attained. The probability \( \omega \) is an index that expresses the fault risk of a light bulb, and is called the fault risk management level. Here, the VaR given the fault control level \( \omega \) and inspection and replacement policy \( \xi = (d, m) \) is expressed by;

\[ \text{VaR}_{\omega}(d, m) = \arg \max U \mid P^k(z \geq U) \leq \omega \] \hspace{1cm} (27)

where, \( \arg \) specifies \( U \) from the right-hand side of equation (27). Next, a set \( \Omega_{\omega}(\bar{U}) \) is defined by;

\[ \Omega_{\omega}(\bar{U}) = \{(d, m) \mid \text{VaR}_{\omega}(d, m) \leq \bar{U}\} \] \hspace{1cm} (28)

A set \( \Omega_{\omega}(\bar{U}) \) means “a set of inspection and replacement policies which can suppress the generating share of failed light bulbs below the fault control limit \( \bar{U} \) under the fault risk management level \( \omega \)”. Thus, the fault risk of a light bulb can be expressed using two parameters, i.e. the fault risk management level \( \omega \) and the fault control limit \( \bar{U} \).

\[ C(d, m) = \frac{(e+f)N\pi^k + eL + h}{dN} \] \hspace{1cm} (29)

where, \( N \) number of light bulb
\( e \) inspection cost per length of a tunnel
\( L \) length of a tunnel
\( c \) light bulb cost
\( f \) replacement unit cost
\( h \) cost for traffic restriction

The expected occurrence probability index of a failed light bulb is equivalent to a special case of the VaR index. Therefore, the fault risk of the tunnel’s lighting system can be defined using the VaR index \( \text{VaR}_{\omega}(d, m) \). Hence, an optimal inspection and replacement model that minimizes the life cycle cost under the fault control limit \( \bar{U} \) and fault risk management level \( \omega \) can be formulated by;

\[ \min_{d,m} \{C(d, m)\} \] \hspace{1cm} (30)

subject to \( (d, m) \in \Omega_{\omega}(\bar{U}) \)

5. Empirical Analysis

(1) Outline

The optimal inspection and replacement model proposed by this research is applied to the Towada road office asset management of the tunnel lighting system of the Tohoku Expressway. The tunnel lighting fault history database contains data on the dates when the system’s light bulbs are installed and begin their usable life and about their failure dates for a period of 12 years, i.e. from 1983 until 1995. In this expressway, for illuminating the tunnel, two types of light bulbs are installed: a low-pressure sodium-vapor light bulb type and a high-pressure sodium-vapor light bulb type. Furthermore, tunnel light bulbs are also classified into light bulbs for basic lighting and for relief lighting. Basic lighting light bulbs are installed throughout the tunnel and they aim at providing the necessary illumination required for the tunnel’s users to be able to have proper front view of the road. On the other hand, relief lighting light bulbs are installed in order to ease the impact of the illumination difference between the tunnel’s inside and the outside, especially at the tunnel’s entrance during daytime. The relief lighting circuitry varies the light bulbs’ intensity according to the outdoor
Table-1 the average lighting time per day (hours)

<table>
<thead>
<tr>
<th></th>
<th>Basic lighting</th>
<th>Relief lighting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daytime</td>
<td>11.7</td>
<td>Fine-1</td>
</tr>
<tr>
<td>Night</td>
<td>16.2</td>
<td>Fine-2</td>
</tr>
<tr>
<td>Midnight</td>
<td>24.0</td>
<td>Cloudy-1</td>
</tr>
<tr>
<td>Emergency</td>
<td>24.0</td>
<td>Cloudy-2</td>
</tr>
</tbody>
</table>

Figure-5 Survival Function (low-pressure sodium-vapor)

luminosity. As it can be seen in Table -1, the average lighting time per day varies with the type of lighting, the weather and the hour of the day. From the database, 7,145 sample data points were acquired, and the Weibull degradation hazard model was estimated based on them. For the Weibull deterioration hazard model the following were taken as explaining variables: 1) the type of the light source for lighting, 2) Lighting time, 3) the position of a light bulb in the tunnel is considered as a substitution-explaining variable. The Weibull hazard function was estimated based on those explaining variables. As a result, the explaining variable shown below, in addition to used time $T$, was also adopted as an explaining variable that satisfied the significance conditions of an explaining variable with mark conditions and t-test.

$$x = \begin{cases} 
1 & \text{low - pressure sodium – vapor lamp} \\
0 & \text{high – pressure sodium – vapor lamp} 
\end{cases}$$

$$y = \text{mean lighting time per day}$$

The estimated Weibull deterioration hazard model is expressed as;

$$\lambda(\xi) = (0.0154 + 0.0045x + 0.00042y)^{1.44} \xi^{0.44} \quad (31)$$

$$\begin{align*}
(13.96) & \quad (6.42) & \quad (6.95) & \quad (80.97) 
\end{align*}$$

Figure-6 Fault risk management level and VaR value

where, the numerical values in parenthesis expresses t-values, and the null hypothesis that the t-value of the parameters that do not explain the model for each explaining variable were rejected when the level of significance was 0.95. The log-likelihood of the Weibull deterioration hazard model is -16,870, and the initial likelihood was -99,223; the likelihood ratio is 0.83. The survival function for every lighting type, except the low-pressure sodium-vapor light bulb type, were created based on the Weibull deterioration hazard model, as shown in Figure-5. By using a Weibull deterioration hazard model, the transition probabilities under the inspection and replacement policy $\xi$ is defined by,

$$p_{ij}^\xi = \exp[-(0.0107 + 0.0031x + 0.00029y)] (32)$$

$$\{(j + 1)^{1.44} - j^{1.44}\}d^{1.44}$$

By using formula (31), the transition probability matrix $P^\xi$ can be defined according to lighting light bulb classification and the lighting type.

(2) Cost – Risk Curve

The stationary probability $\pi^F$, $\pi^R$ can be computed using the transition probabilities matrix $P^\xi$ under the inspection and replacement policy $\xi$. Figure-4 shows the share distribution of the failed light bulbs of the low-pressure sodium-vapor type in the case of $d = 6$ months and $md = 24$ months as inspection and replacement policies and the stationary state of the basic lighting during midnight. The points $\alpha$ and $\beta$ of this figure show the VaR index VaR$_{0.05}(6, 4)$ and the expected occurrence probability $U(6, 4)$ of failed light bulbs when the light bulb population is $N = 200$. As shown in this figure, also in the stationary state, the risk exists in the share of

- 10 -
the failed light bulbs for each period. Furthermore, in Figure-6, in the case of low-pressure sodium-vapor light bulbs and basic lighting (midnight), considering light bulb populations of \( N = 50, 100, 200 \), the relation between the fault risk management level \( \omega \) and VaR index \( \text{VaR}_\omega(6, 4) \) and \( \text{VaR}_\omega(12, 2) \) is shown, that is the case when the inspection and replacement interval is \( d = 6, 12 \) and the maximum life length is \( md = 24 \). The points \( \alpha \) and \( \alpha' \) in the figure show the expected occurrence frequency \( U(6, 4) \) and \( U(12, 2) \), respectively. We can see that \( \text{VaR}_\omega \) increases, so that the permissible probability \( \omega \) becomes small. The change of \( \text{VaR}_\omega(6, 4), \text{VaR}_\omega(12, 2) \) becomes smaller than the change of \( \omega \), so that the number \( N \) of samples increases.

In order to compute the life cycle cost, information on the parameters, \( c, e, f, h \), in formula (29) is required. In this study, the recorded data of the Towada road office is used. The light bulbs unit prices are different for the low-pressure sodium-vapor light bulb and the high-pressure sodium-vapor light bulb. Traffic restriction cost means the regulation cost per direction in the tunnel for inspection and replacement. Traffic restriction cost depends on the length of the tunnel. Based on the above data, an inspection and replacement unit time interval \( d \) of one month for a maximum of two years was assumed. The value \( m = 1, 2, \cdots, [60/d] \) was adopted for the parameter \( m \) showing the maximum life length \( md \). Here, \( [60/d] \) expresses the minimum integer exceeding \( 60/d \). The \( C - U \) curve was computed for each inspection and replacement interval \( d \) according to the light bulb classification and the lighting type, as mentioned earlier. In that case, \( \text{VaR}_{0.05}(d, m) \) is adopted as a fault risk management index.

Figure-7 shows the \( C - U \) curve when the number of light bulbs is \( N = 100 \), the light bulb type is low-pressure sodium-vapor light bulb, and the basic lighting (midnight) scheme is considered. The \( C - U \) curve shows how the relation between the life cycle cost and the fault risk level \( \text{VaR}_{0.05}(d, m) \) changes, when inspection and replacement interval \( d \) is fixed and \( m \) is changed. This figure shows the relation of the trade-off when the life cycle cost becomes small so that the parameter \( md \) showing inspection and replacement interval \( d \) and the maximum life length \( md \) becomes large, but the fault risk becomes large. Furthermore, the cost and the risk curve, which is an envelope curve of the \( C - U \) curve, defined for different \( d \) and all shown together in this figure. It can be seen that when the fault control limit is kept to below 0.1 the life cycle cost increases quickly. On the other hand, even if the fault control limit is increased to 0.2 or more the life cycle cost still cannot be controlled. In other words, when a fault probability limit is 0.2 or more, even if the budget is reduced slightly, the fault probability of the tunnel lighting system will increase sharply.

(3) Optimal Inspection and Replacement Policy

The fault risk management level of a tunnel lighting system can be determined by specifying the fault risk management level \( \omega \) and the fault control limit \( U \). If these parameters are changed, the optimal inspection and replacement policy will also change. The optimal inspection and replacement policy is expressed by \( \xi^*(U, \omega) = (d^*(U, \omega), m^*(U, \omega)) \). Figure-8 shows the relation between the fault control limit \( U \) and the optimal inspection and replacement policy \( \xi^*(U, \omega) \) to a different fault risk management level \( \omega \) considering a light
bulb population of 100 light bulbs. As shown in Figure-7, the cost and risk curves are defined as an envelope curve of the $C - U$ curve, defined to a different inspection and replacement interval. If the fault risk management level $\omega$ is fixed, as long as it will stop on a $C - U$ curve with the same optimal inspection and replacement policy, the optimal maximum life length $m^*$ becomes large and thus increases the fault control limit. Special attention is needed when defining the envelope curve of the $C - U$ curve from cost and risk curve that differ. If the fault control limit is increased further, the $C - U$ curve which Makes up the cost and risk curves will shift to the $C - U$ curve corresponding to the biggest optimal inspection and replacement interval $d^*$. Thus, if the fault control limit is enlarged, the optimal inspection and replacement policies will change one by one following the order: 1) $d^*$ is fixed and $m^*$ becomes large, 2) $d^*$ becomes large. So, the optimal inspection and replacement policy is ranked, as shown in the vertical axis. Figure-8 shows the relation between a fault control limit and the optimal inspection and replacement interval. The number in the parenthesis in this figure expresses the optimal policy $(d^*, m^*)$. As shown in this figure, we can deduct that the inspection and replacement interval $d^*$ and the maximum life length $m^*d^*$ become smaller when the risk management level becoming severer ($\omega$ becoming smaller or the fault control limit $U$ becoming smaller). Thus, the graph showing the relation between the risk management parameter $U, \omega$ and the optimal inspection and replacement policy $\xi^*(U, \omega)$ is called a fault risk management figure.

The fixed cost, which in turn has an impact on the life cycle cost varies with the number $N$ of light bulbs. The relation of the number of light bulbs and the optimal inspection and replacement policy is analyzed as follows. Figure-9 shows the change in the life cycle cost per light bulb, hereafter called the unit life cycle cost, when the fault risk management level is fixed to $\omega = 0.05$ , the fault control limit is fixed to $U = 0.1 , 0.2$, and the number of light bulbs is allowed to vary.

This figure shows how the optimal inspection and replacement policy changes are expressed together by the change in the number $N$ of light bulbs. The vertical axis of Figure-9 expresses the measure of ranking of the optimal inspection and replacement policies, just like Figure-8, and the horizontal axis express the logarithm of the number of light bulbs.

If the number $N$ of light bulbs becomes large enough as shown in this figure, even if the number $N$ of light bulbs changes, the unit life cycle cost, the optimal inspection and replacement policy will hardly change. Therefore, if the aim of the administrators is towards maintaining a practical sufficiently large number of samples, it becomes very important to determine a suitable fault control limit under the given fault risk management level $\omega$. On the other hand, since the effect of the fixed cost for inspection and replacement shows up when there are few light bulbs, the unit life cycle cost per light bulb becomes large.

Moreover, since the change of the share of failed light bulbs becomes large, in order to keep a given fault control limit under a certain fixed fault risk management level, more frequent inspections and replacements are required. For example, when a fault control limit is set around 0.5, if the number of light bulbs becomes 20 or less, the effect of fixed cost will prevail and the rule of simultaneous replacement will be chosen.

Finally, Figure-10 shows the relation of the number of light bulbs and the fault control limit when the rule of simultaneous replacement is chosen as the optimal inspection and replacement policy.
replacement policy. The limiting number of light bulbs where the rule of simultaneous replacement or the rule of on by one replacement is chosen is called the critical light bulb number to a given fault control limit. The solid line and the dashed lines of Figure-10 show the relation of the fault control limit and the critical light bulb number when the fault control level is \( \omega = 0.05, 0.5 \) respectively. As shown in this figure, when the size of the tunnel lighting system is small, it becomes efficient to omit an inspection and carry out the simultaneous replacement periodically. However, if the fault control limit becomes small and the periodical inspection along with the rule of the one by one replacement policy become rational, the critical light bulb number will become small. In other words, even for small tunnel having a small number of light bulbs, it becomes rational to adhere to a periodical inspection and utilize the rule of one by one replacement.

6. Conclusions

This research presented the deterioration process of tunnel light bulb expressed by the Weibull deterioration hazard model and the inspection and replacement process of a tunnel lighting system which consists of many light bulbs expressed by the aggregated Markov process model. Furthermore, an optimal inspection and replacement model was formulated to analyze the influence the two management variables called the inspection and replacement interval and the maximum life length of a light bulb on the life cycle cost and a fault risk. As a result, when the fault risk is given as a control level, it became possible to search for the optimal inspection and replacement policy of a tunnel lighting system which minimizes the life cycle cost. Moreover, the validity of the methodology proposed by this research was tested positively using actual data from a tunnel lighting system. The methodology proposed by this research is applicable to the asset management of many other types of facilities which their deterioration process is described by a Weibull deterioration hazard model.

In the future, it is necessary to develop an asset management methodology of facility systems which consists of many facilities of the same kind. It will be beneficial to develop a practical asset management system application for tunnel lighting systems which utilizes the core model of optimal inspection and replacement developed in this research. For that purpose, development of a database system which records the inspection and replacement processes and a managerial accounting system that can carry out the asset-management task will be required.

REFERENCES


